PARTON-LIKE FORMULATION OF THE STRING-EFFECT

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A model of coherent soft gluon production at large angle (string effect) admitting parton-like interpretation is proposed. Comparison with earlier approaches used in Monte-Carlo simulations is made.

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One of the most interesting features of the Quantum Chromodynamics (QCD) is the presence of the colour interference between partons — gluons and quarks. It is convenient to divide all interference effects into two classes [1]. Effects that originate from soft coherent emission at small angle in the same jet belong to the first class. The second class is formed by effects of the coherent emission of a soft particle at large angle by the whole ensemble of the hard particles (e.g. so called string effect [2]). The coherence phenomena found in QCD at the parton level have been also observed in hadronic distributions [3]. This means that the careful analysis of the coherence among partons is very important. By comparing results of calculations at the perturbative level with distribution of hadrons one can learn more about hadronisation process.

The presence of the colour interference makes a Monte-Carlo (MC) simulation for parton cascades much more subtle since it perturbs quasi-classical, probabilistic picture of parton evolution. Up till now the coherence of class one has been successfully incorporated to the MC simulation for the e^+-e^- annihilation [4] by using the angular ordering within a jet. Whether the class-two interference can be properly included in MC programs is still being debated.

In this paper we show that probabilistic formulation of the string effect suitable for the parton — shower MC simulation is also possible. The model presented here deals with a soft gluon emission by three hard partons. For definiteness we shall consider the deep inelastic scattering process (a similar analysis applies for all other processes related by crossing):

$$\gamma^* + q \rightarrow q' + b + \tilde{g}, \tag{1}$$

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where the momentum of incoming (outgoing) quark is denoted by p ($\tilde{p}$), b is either a hard gluon (g) or a hard real photon (γ) with momentum l and $\tilde{g}$ is a soft gluon with momentum k. We shall compare the results of our model with two approaches: A) no interference, and B) naive angular ordering, which were used in earlier versions of the MC programs [4, 9]. Throughout this paper we shall deal with the normalized angular distribution of soft gluon emission for a given topology of hard partons defined as follows

$$H(\vartheta, \varphi) = \left(\frac{\alpha_s}{8\pi^2}\right)^{-1} \frac{1}{\sigma_h} \omega^3 \frac{d\sigma}{d^3k}. \tag{2}$$

Here σ_h is the differential cross section for the hard process

$$\gamma^* + q \to q' + b \tag{3}$$

ω denotes the energy of the soft gluon, ϑ and φ its azimuthal and polar angle and α_s is the strong coupling constant.

In the soft gluon limit it has been pointed out [2] that the soft gluon distribution H takes the following form:
1) for $b = g$

$$H(\vartheta, \varphi) = \left(C_F - \frac{C_A}{2}\right)\langle p, \tilde{p}\rangle + \frac{C_A}{2}(\langle p, l\rangle + \langle \tilde{p}, l\rangle), \tag{4}$$

where $C_A = N_c$ (3 for SU (3)), $C_F = (N_c^2-1)/2N_c$,

$$\langle i, j\rangle = \frac{\xi_{ij}}{\xi_i \xi_j}, \tag{5}$$

$\xi_{ij} = i \cdot j/(\omega_i\omega_j) = 1-\cos\vartheta_{ij}$, $\xi_i = i \cdot k/(\omega_i\omega) = 1-\cos\vartheta_{ik}$, $i, j = p, \tilde{p}, l$ denote momenta of hard partons and ω_i their energies;
2) for $b = \gamma$

$$H_\gamma = C_F\langle p, \tilde{p}\rangle. \tag{6}$$

The factor $\langle i, j\rangle$ can be interpreted in terms of a colour string spanned by partons i and j, and in the collinear limit it can be approximated by the independent emission:

$$\langle i, j\rangle = \begin{cases} \dfrac{1}{\xi_i} & \text{for} \quad \vartheta_{ik} \to 0 \\ \dfrac{1}{\xi_j} & \text{for} \quad \vartheta_{jk} \to 0. \end{cases} \tag{7}$$

The distributions H and H_γ (integrated over the polar angle φ measured from the reaction plane of the hard process) are shown in Fig. 1. The azimuthal angle ϑ is measured from the direction of the quark q. The quark, hard gluon (photon) and struck quark were fixed at $\vartheta = 0°$, 54° and 134° degree respectively in the γ^*q center of mass system. One

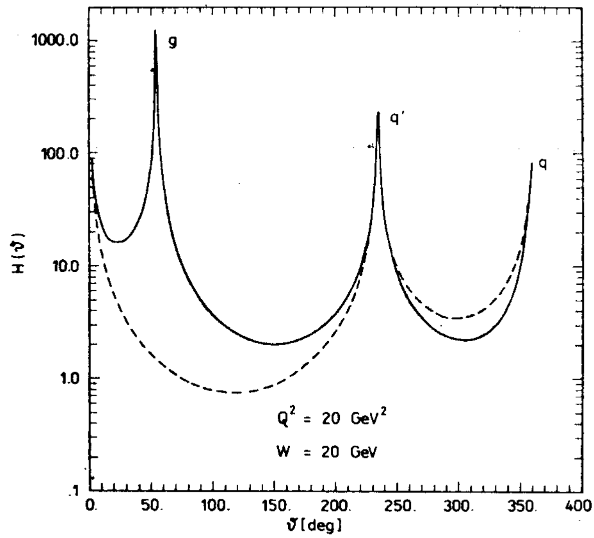

Fig. 1. Soft production by the three (solid line) and two (dashed line) colour sources having the same angular topology. The depletion of the region between quarks (opposite to the hard gluon or hard photon) as a result of the negative coherence is seen

can observe the string effect — the depletion of soft gluon production in the direction opposite to the hard gluon in H (i.e. between qq') compared with H_γ in the same region.

Now we shall discuss beforementioned two approaches to the soft distribution (Eq. (4)), followed by the discussion of our model.

A. No interference

Following the idea of an independent emission off a colour source all interference effects are neglected altogether. Each parton is assumed to emit independently from others. Therefore one gets the following approximation:

$$H^{\mathrm{A}}(\vartheta, \varphi) = \frac{C_{\mathrm{F}}}{\xi_p} + \frac{C_{\mathrm{F}}}{\xi_{\tilde{p}}} + \frac{C_{\mathrm{A}}}{\xi_l}. \tag{8}$$

B. Naive angular ordering

One can try to account for the string effect in a way analogous to the treatment of coherence of class one. This amounts to consider an independent emission by three (or two in the case of hard photon production) hard partons but this time restricted to cones with opening angles limited by the nearest colour source:

$$H^{\mathrm{B}}(\vartheta, \varphi) = \frac{C_{\mathrm{F}}}{\xi_p}\,\theta(\xi_{\mathrm{m}1}-\xi_p)+\frac{C_{\mathrm{F}}}{\xi_{\tilde{p}}}\,\theta(\xi_{\mathrm{m}2}-\xi_{\tilde{p}})+\frac{C_{\mathrm{A}}}{\xi_l}\,\theta(\xi_{\mathrm{m}3}-\xi_l), \tag{9}$$

where $\theta(x)$ is a step function and

$$\xi_{\mathrm{m}1} = \min(\xi_{p\tilde{p}}, \xi_{pl}), \quad \xi_{\mathrm{m}2} = \min(\xi_{\tilde{p}l}, \xi_{p\tilde{p}}), \quad \xi_{\mathrm{m}3} = \min(\xi_{pl}, \xi_{\tilde{p}l}).$$

The interference effect simply reduce the allowed phase space for the soft radiation. This approximation has been used in an earlier version of the model of Marchesini and Webber [4].

C.

We would like to propose a model which interpolates between the model B and the soft limit formula (4). A closer examination of expression (4) shows that the restrictions on the phase space for soft gluon emission from quark and gluon are different. To see this let us start from the following decomposition:

$$\langle i,j\rangle = \tfrac{1}{2}[i,j]+\tfrac{1}{2}[j,i],$$

where $[i,j]$ and $[j,i]$ contain the leading singularity for $\vartheta_{ik}\to 0$ and $\vartheta_{jk}\to 0$, respectively:

$$[i,j] = \frac{\xi_{ij}}{\xi_i\xi_j} - \frac{1}{\xi_j} + \frac{1}{\xi_i}. \tag{10}$$

The function $[i,j]$ is positive for $\vartheta_{ik} < \vartheta_{ij}$. The destructive colour interference occurs for $\vartheta_{ik} > \vartheta_{ij}$, where $[i,j]$ changes sign, and its integral over the azimuthal angle ϕ around the momentum i vanishes, i.e.:

$$\int \frac{d\phi}{2\pi}[i,j] = \frac{2}{\xi_i}\,\theta(\xi_{ij}-\xi_{ik}). \tag{11}$$

Therefore, we propose the following approximation:

$$H^{C} = \frac{C_F}{\xi_p}\,\theta(\xi_{pl}-\xi_p)+\frac{C_A}{2\xi_l}\,\theta(\xi_{pl}-\xi_l),$$

$$\frac{C_F}{\xi_{\tilde{p}}}\,\theta(\xi_{\tilde{p}l}-\xi_{\tilde{p}})+\frac{C_A}{2\xi_l}\,\theta(\xi_{\tilde{p}l}-\xi_l). \tag{12}$$

This expression can be interpreted in the following way. Half of the gluon radiation (second term) is colour connected to the quark q and the other half (last term) to the quark q'. The interference occurs only between colour-connected partons and the angular restrictions on the soft radiation off a given hard partons is determined only by the position of a colour--connected partner. Note that the H^C contains correct normalized collinear singularities but dynamically suppressed terms of order $C_F - C_A/2 = -1/N_C^2$ have been neglected. Note also that when hard particles are in a symmetric configuration ($\xi_{pl} = \xi_{p\tilde{p}} = \xi_{\tilde{p}l}$) then (12) reduces to (9).

Now let us discuss numerically the consequences of the described approximations.

The distributions H, H^A, H^B and H^C (averaged over the polar angle measured from the reaction plane of the hard process) are shown in Fig. 2. We see that the model A does not account for the discussed depletion. On the other hand the model B overestimates the coherence effects. In the most of the phase space the model C is a reasonable approximation of the formula (4). The observed discrepancies near the quark (q) direction are due

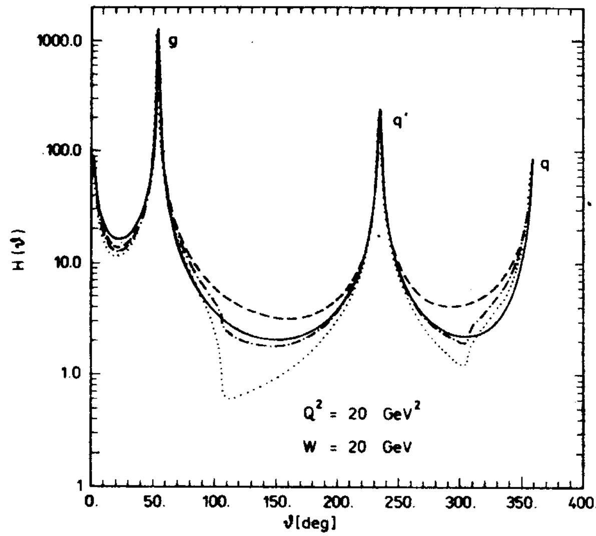

Fig. 2. Azimuthal distribution of the soft gluon production by three hard particles integrated over the polar angle. Positions of the incoming (q) outgoing (q′) quarks and hard gluon (g) are depicted. H — solid, H^A — dashed, H^B — dotted and H^C — dashed-dotted line

to the neglect of the azimuthal correlations present in $[i, j]$. One can try to take also these correlations into account [6, 8].

There are two convenient ways of investigating the string effect in a more quantitative way. The first one is to compare the soft gluon production in the region between qq' (opposite the hard gluon) in H^i with the corresponding production in H_γ, i.e.:

$$R_1^i = \frac{\int_{0.3}^{0.7} H^i(x)dx}{\int_{0.3}^{0.7} H_\gamma(x)dx}. \quad (13)$$

Alternatively, one can compare the soft production in regions between qq' and q'g in the same process, i.e.

$$R_2^i = \frac{\int_{0.3}^{0.7} H^i(x)dx}{\int_{0.3}^{0.7} H(y)dy}. \quad (14)$$

In the expressions above $x = \phi_1/\vartheta_{p\tilde{p}}$, $y = \phi_2/\vartheta_{\tilde{p}l}$, where $\phi_1(\phi_2)$ is the angle between the quark q (q') and the projection of the soft gluon momentum onto the hard process plane, $H^i = H, H^A, H^B, H^C$ and integration of H^i and H_γ over the polar angle φ in (13) and (14) is assumed. In Table I the ratios R_1 and R_2 for a few energies W[1] are shown. The Q^2

[1] We use standard kinematical variables for deep inelastic scattering process (3): W — total energy in the γ^*q cms and Q^2 — virtual mass of an exchanged photon γ^*.

TABLE I

W [GeV]	R_1	R_1^C	R_1^B	R_1^A	R_2	R_2^C	R_2^B	R_2^A
20	0.66	0.64	0.51	1.17	0.76	0.84	1.42	0.86
40	0.67	0.66	0.43	1.26	0.76	0.83	1.16	0.88
100	0.69	0.67	0.41	1.35	0.78	0.82	1.06	0.89

dependence of R_1 and R_2 is negligible. We observe that the model C provides the best approximation. Although it appears that $R_2^A \cong R_2^C$, a glance at Fig. 2 shows that absolute normalization of H^A is completely wrong.

The implementation of the proposed approximation C into MC program is straightforward [6, 8]. Each term of Eq. (12) is positive define and has obvious parton-like interpretation. In fact it suffices to take the formula of Eq. (12) into account instead of an appropriate Altarelli-Parisi splitting function only at first branching following a hard one. The reason is that, as a result of class one coherence, in the soft gluon approximation the emission of soft gluons at large angles by the jet can be reduced to the emission from the "parent" of the cascade [7].

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Note added. After completion of this paper we have received preprint [8], where a similar approach to the string effect has been discussed and its implementation to the Monte Carlo program successfully achieved.

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