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CHIRAL EFFECTIVE LAGRANGIANS AND VECTOR MESON DOMINANCE*

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Using the technique of nonlinear realizations we discuss the structure of the chiral effective Lagrangians. The only restrictions on the possible form of the Lagrangian we consider are phenomenological ones: the Vector Meson Dominance (VMD), universality and KSFR relation. We consider in more detail the SU(2) case (nonanomalous part). From this general point of view we review a number of Lagrangians recently proposed in literature.

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1. *Introduction*

There is a recent interest in the old Skyrme idea of baryons as soliton excitations of mesonic fields [1]. In order to realize this idea one has to write first an effective Lagrangian describing the low-energy physics of mesons. Unfortunately, due to the unsolved dynamical problems of the QCD, which do not permit to relate the phenomenological parameters to those of QCD the rigorous derivation of such a low energy Lagrangian is still lacking (there are, however, some attempts in this direction, see Ref. [2]). Therefore, in order to construct the effective Lagrangian, some authors invoke other apriori principles like the

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hidden symmetry [3] or gauging the chiral group [4]. The question arises how those principles are connected (if at all) with the QCD dynamics.

We would like to present here a slightly different point of view.

First, following Weinberg [5] we notice, that any scattering amplitude satisfying the conditions of being unitary, analytic, Lorentz covariant, and having the appropriate symmetry can be reconstructed (by the usual Feynman rules) from the general, hermittean, local, Lorentz invariant Lagrangian, having the same symmetry. Of course, all particles we consider should be included into this Lagrangian from the very begining. We have only the algorithm for an amplitude construction with given poles, thresholds etc., rather than a theory predicting the bound states etc.

Secondly, the form of the Lagrangian is defined by using only principles strongly supported by phenomenology and, by some theoretical arguments:

(a) The PCAC scheme,

(b) the vector meson dominance hypothesis (VMD).

Here PCAC means that the underlying symmetry is the chiral $SU_L(N)\times SU_R(N)$ symmetry, broken spontaneously to the diagonal $SU(N)$ (and, of course, broken also explicitly by giving the mass to the Goldstone bosons in order to produce the partial conservation of the axial current) [6, 7].

The VMD principle is introduced by imposing the field-current identity in the way proposed by Kroll, Lee and Zumino [8, 9].

The resulting Lagrangian is of course, still an infinite sum of terms with increasing number of derivatives. Proceeding in the standard way to describe the low energy region we restrict our analysis to the terms with as small as possible number of derivatives. In order to eliminate the off-diagonal terms in the kinetic part of the Lagrangian one has to perform axial field redefinition (see Sect. 2). We demand this redefinition not to produce the terms containing more derivatives than those already included into the Lagrangian. The minimal number of derivatives satisfying this condition is four.

The aim of this paper is the explicite realization of the program. We derive the most general (nonanomalous part [10] of) Lagrangian using systematically the technique of nonlinear realizations and exploiting fully freedom allowed by symmetry principles and VMD. This unique framework allows us to discuss Lagrangians appearing frequently in literature. The Lagrangians obeying condition (a) emerge as particular cases of our Lagrangian (see Tables I and II). We are also motivated in part by some inconsistent statements made in different papers. In contradistinction to the standard approach (cf. Refs [16, 17, 20]) we proceed as follows. By imposing the VMD principle in a way as general as possible we derive the whole family of Lagrangians with VMD differing in terms breaking the local chiral symmetry (Sect. 2). For the field-current identity is the off-shell condition the important problem of choice the physical ϱ_μ-field is also automatically solved.

In the sequel we restrict ourselves to the SU(2) case [10] (a possible generalization to SU(N) case is briefly discussed in Sect. 4).

The material contained in this paper is an extended version of that presented during the Krakow Workshop on Skyrmion and Anomalies [21].

2. *The effective Lagrangians*

2.1. Nonlinear realization of $SU_L(2)\times SU_R(2)$

The nonlinear realizations [7] seem to be natural framework guaranteeing the appropriate pattern of the symmetry breaking. Thus we present the basic notions and formulae.

The element of dynamical group $G = SU_L(2)\times SU_R(2)$ is denoted by $g = (g_L, g_R)$; the algebraic subgroup H is the diagonal SU(2) subgroup (the isospin group). The preferred (Goldstone) fields transforms as follows:

$$(g_L, g_R)(\xi, \xi^\dagger) = (\xi', \xi'^\dagger)(h, h) \tag{1}$$

or

$$g_L\xi = \xi^\dagger h, \qquad g_R\xi^\dagger = \xi'^\dagger h. \tag{2}$$

The fields ξ parametrize the coset space G/H which is topologically equivalent to SU(2). We introduce the gauge fields transforming as follows

$$A_\mu'^{L(R)} = g_{L(R)}A_\mu^{L(R)}g_{L(R)}^{-1} + g_{L(R)}\partial_\mu g_{L(R)}^{-1}. \tag{3}$$

It is convenient to define the vector and axial vector fields as follows:

$$V_\mu = \tfrac{1}{2}(A_\mu^L + A_\mu^R), \qquad A_\mu = \tfrac{1}{2}(A_\mu^L - A_\mu^R). \tag{4}$$

In the fundamental representation of (g_L, g_R) the generators read:

$$\vec{T}^L = (i\tfrac{1}{2}\vec{\tau}, 0), \qquad \vec{T}^R = (0, i\tfrac{1}{2}\vec{\tau}). \tag{5}$$

Writing:

$$(\xi^\dagger, \xi)\partial_\mu(\xi, \xi^\dagger) \equiv (\omega_\mu+\eta_\mu, \omega_\mu-\eta_\mu),$$

$$(\xi^\dagger, \xi)(\partial_\mu + V_\mu + A_\mu)(\xi, \xi^\dagger) \equiv (\tilde{\omega}_\mu+\tilde{\eta}_\mu, \tilde{\omega}_\mu-\tilde{\eta}_\mu), \tag{6}$$

we get:

$$\eta_\mu = \tfrac{1}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger),$$

$$\omega_\mu = \tfrac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger), \tag{7}$$

and

$$\tilde{\eta}_\mu = \eta_\mu + \tfrac{1}{2}(\xi^\dagger A_\mu^L\xi - \xi A_\mu^R\xi^\dagger),$$

$$\tilde{\omega}_\mu = \omega_\mu + \tfrac{1}{2}(\xi^\dagger A_\mu^L\xi + \xi A_\mu^R\xi^\dagger). \tag{8}$$

The forms $\tilde{\eta}_\mu$ and $\tilde{\omega}_\mu$ have the following transformation properties under the local and global chiral transformations

$$\tilde{\eta}_\mu' = h\tilde{\eta}_\mu h^{-1}, \qquad \tilde{\omega}_\mu' = h\tilde{\omega}_\mu h^{-1} + h\partial_\mu h^{-1}. \tag{9}$$

On the other hand, under the global chiral transformations forms η_μ and ω_μ transform as follows

$$\eta'_\mu = h\eta_\mu h^{-1}, \qquad \omega'_\mu = h\omega_\mu h^{-1} + h\partial_\mu h^{-1}. \tag{10}$$

The forms ω_μ and $\tilde{\omega}_\mu$ can be used to construct the covariant derivatives of η_μ and $\tilde{\eta}_\mu$

$$D_\mu \eta_\nu \equiv \partial_\mu \eta_\nu + [\omega_\mu, \eta_\nu],$$
$$\tilde{D}_\mu \tilde{\eta}_\nu \equiv \partial_\mu \tilde{\eta}_\nu + [\tilde{\omega}_\mu, \tilde{\eta}_\nu]. \tag{11}$$

The transformation properties of the covariant derivatives $D_\mu \eta_\nu$ and $\tilde{D}_\mu \tilde{\eta}_\nu$ are the same as forms η_μ and $\tilde{\eta}_\mu$, respectively (Eqs (9) and (10)).

Finally we can form the field tensor

$$F_{\mu\nu}(\tilde{\omega}) = \partial_\mu \tilde{\omega}_\nu - \partial_\nu \tilde{\omega}_\mu + [\tilde{\omega}_\mu, \tilde{\omega}_\nu]. \tag{12}$$

It is easy to check that the following identities hold (the Cartan-Maurer equations)

$$\tilde{D}_\mu \tilde{\eta}_\nu - \check{D}_\nu \tilde{\eta}_\mu = \tfrac{1}{2}[\xi^\dagger F_{\mu\nu}(A^{\mathrm{L}})\xi - \xi F_{\mu\nu}(A^{\mathrm{R}})\xi^\dagger],$$
$$F_{\mu\nu}(\tilde{\omega}) + [\tilde{\eta}_\mu, \tilde{\eta}_\nu] = \tfrac{1}{2}[\xi^\dagger F_{\mu\nu}(A^{\mathrm{L}})\xi + \xi F_{\mu\nu}(A^{\mathrm{R}})\xi^\dagger],$$
$$D_\mu \eta_\nu - D_\nu \eta_\mu = 0, \qquad F_{\mu\nu}(\omega) + [\eta_\mu, \eta_\nu] = 0. \tag{13}$$

In order to fascilitate the comparison with literature we present below the explicit expression for the forms in two other parametrizations (see also the Appendix).

A. Introducing the new SU(2) variable

$$U = \xi^2 \tag{14}$$

we obtain the following transformation rule:

$$U' = g_{\mathrm{L}} U g_{\mathrm{R}}^{+}. \tag{15}$$

The following relations hold:

$$\xi \tilde{\eta}_\mu \xi^\dagger = \tfrac{1}{2}(A_\mu^{\mathrm{L}} - U A_\mu^{\mathrm{R}} U^\dagger - U\partial_\mu U^\dagger),$$
$$\xi \tilde{\omega}_\mu \xi^\dagger + \xi \partial_\mu \xi^\dagger = \tfrac{1}{2}(A_\mu^{\mathrm{L}} + U A_\mu^{\mathrm{R}} U^\dagger + U\partial_\mu U^\dagger). \tag{16}$$

B. One can also use the σ-model parametrization $(\sigma, \vec{\phi})$ defined by:

$$U = \xi^2 = \sigma \mathbf{1} + i\vec{\phi}\vec{\tau}, \qquad \sigma^2 + \vec{\phi}^2 = 1. \tag{17}$$

$(\sigma, \vec{\phi})$ transforms as the four-vector under the SO(4) $(\mathrm{SU(2)_L} \times \mathrm{SU(2)_R})$ group. The Cartan forms read

$$\eta_\mu = \frac{i}{2}\vec{\tau}\left(\partial_\mu \vec{\phi} - \frac{1}{\sigma+1}\vec{\phi}\partial_\mu \sigma\right),$$

$$\omega_\mu = \frac{i}{2}\vec{\tau}(\vec{\phi} \times \partial_\mu \vec{\phi})\frac{1}{\sigma+1}, \tag{18}$$

and

$$\tilde{\eta}_\mu = \eta_\mu + \frac{i}{2}\vec{\tau}\left(\sigma\vec{A}_\mu + \vec{\phi}\times\vec{V}_\mu + \frac{1}{\sigma+1}\vec{\phi}(\vec{\phi}\vec{A}_\mu)\right),$$

$$\tilde{\omega}_\mu = \omega_\mu + \frac{i}{2}\vec{\tau}\left(\sigma\vec{V}_\mu + \vec{\phi}\times\vec{A}_\mu + \frac{1}{\sigma+1}\vec{\phi}(\vec{\phi}\vec{V}_\mu)\right). \tag{19}$$

To construct the Lagrangian invariant under the global (or local) transformations of $SU_L(2)\times SU_R(2)$ we have only to choose the function of η_μ, $F_{\mu\nu}(\omega)$, $D_\mu\eta_\nu$ (or $\tilde{\eta}_\mu$, $F_{\mu\nu}(\tilde{\omega})$, $\tilde{D}_\mu\tilde{\eta}_\nu$) invariant under the group $SU(2)_V$ (diagonal subgroup).

2.2. The principle of VMD

According to Kroll, Lee and Zumino [8, 9] the Lagrangian having the global isospin symmetry and satisfying the field-current identity

$$j_\mu^{e-m} = \frac{m_\varrho^2 e}{g_\varrho}\varrho_\mu^3$$

should be of the form

$$L(\vec{\varrho}_\mu, \ldots) = \mathscr{L}(\vec{\varrho}_\mu, \ldots) + \tfrac{1}{2} m_\varrho^2 \vec{\varrho}^2, \tag{20}$$

where $\mathscr{L}$ is the part which is invariant under local isospin transformations

$$\varrho'_\mu = h\varrho_\mu h^{-1} + \frac{1}{g_\varrho} h\partial_\mu h^{-1}, \qquad \varrho_\mu = \frac{i}{2}\vec{\tau}\vec{\varrho}_\mu \tag{21}$$

and, for remaining fields denoted symbolically by (...) the transformation looks as follows

$$(\ldots)' = D(h)(\ldots).$$

For L should be also globally chiral invariant, the nontrivial problem is to reconcile both conditions. This includes in particular the proper choice of the physical ϱ_μ-field to be discussed later.

2.3. The effective Lagrangian

Following the recipe presented above, we can write the general Lagrangian containing the terms up to the fourth order in the forms as follows:

$$L = L_{\text{local}} + L_{\text{global}}, \tag{22}$$

where

$$L_{\text{local}} = a\,\text{Tr}\,(\tilde{D}_\mu\tilde{\eta}_\nu - \tilde{D}_\mu\tilde{\eta}_\nu)^2 + b\,\text{Tr}\,F_{\mu\nu}(\tilde{\omega})^2$$
$$+ c\,\text{Tr}\,(F_{\mu\nu}(\tilde{\omega}) + [\tilde{\eta}_\mu, \tilde{\eta}_\nu])^2 + d^{(-)}\,\text{Tr}\,([\tilde{\eta}_\mu, \tilde{\eta}_\nu])^2$$
$$+ d^{(+)}\,\text{Tr}\,(\tilde{\eta}_\mu\tilde{\eta}^\mu)^2 - d\,\text{Tr}\,(\tilde{\eta}_\mu\tilde{\eta}^\mu),$$

$$L_{\text{global}} = \alpha_0 \operatorname{Tr}[(\tilde{\eta}_\mu-\eta_\mu)^2+(\tilde{\omega}_\mu-\omega_\mu)^2]+\alpha_1 \operatorname{Tr}(\tilde{\eta}_\mu-\eta_\mu)^2$$

$$+(\alpha_2-\alpha_1)\operatorname{Tr}(\eta_\mu\eta^\mu)+\delta_1 \operatorname{Tr}([\eta_\mu,\eta_\nu])^2$$

$$+\delta_2 \operatorname{Tr}([(\tilde{\eta}_\mu-\eta_\mu),(\tilde{\eta}_\nu-\eta_\nu)])^2+\gamma \operatorname{Tr}([(\tilde{\omega}_\mu-\omega_\mu),(\tilde{\omega}_\nu-\omega_\nu)])^2,$$

and a, b, c, $d^{(-)}$, $d^{(+)}$, d, α_0, α_1, α_2, δ_1, δ_2, γ are arbitrary coefficients.

In L_{local} (invariant under the local transformations of the chiral group) the term $\operatorname{Tr}(\tilde{D}_\mu\tilde{\eta}_\nu+\tilde{D}_\nu\tilde{\eta}_\mu)^2$ has been omitted, because it contains the second order derivatives of fields. On the other hand we admitted the term $\operatorname{Tr}(\tilde{\eta}_\mu\tilde{\eta}^\mu)^2$ despite the fourth power of time derivative. This results in some problems with quantization. In L_{global} (invariant only under the global transformations of the chiral group), we have written explicitly only those terms which survive after imposing VMD principle (the first three terms) or arise in the Lagrangians appearing in the literature. The omitted terms could be easily constructed from η_μ, $\tilde{\eta}_\mu$,$(\tilde{\omega}-\omega)_\mu$, $F_{\mu\nu}(\tilde{\omega})$ and $\tilde{D}_\mu\tilde{\eta}_\nu-\tilde{D}_\nu\tilde{\eta}_\mu$. Now we have to consider the restrictions following from VMD condition. Let us recall, that, in order to satisfy the VMD, L should allow the representation (20):

$$L = \mathscr{L}-m_\varrho^2 \operatorname{Tr}\varrho_\mu\varrho^\mu,$$

with $\mathscr{L}$ invariant under the gauge transformations (21).

Performing these transformations in the Lagrangian L we get:

$$\Delta L = L'-L = \frac{m_\varrho^2}{g_\varrho^2}(2g_\varrho \operatorname{Tr}(\varrho_\mu h^\dagger\partial^\mu h)-\operatorname{Tr}(h^\dagger\partial_\mu h)^2).$$

On the other hand:

$$\Delta L = \Delta L_{\text{global}}.$$

One easily finds that the transformation rule for L can be reconciled with its structure only if L_{global} has the form:

$$L^{\text{VMD}}_{\text{global}} = -\left(\frac{m_\varrho}{g_\varrho}\right)^2 \operatorname{Tr}[(\tilde{\eta}_\mu-\eta_\mu)^2+(\tilde{\omega}_\mu-\omega_\mu)^2]$$

$$+\alpha\left(\frac{m_\varrho}{g_\varrho}\right)^2 \operatorname{Tr}[\tilde{\eta}_\mu-\eta_\mu)^2-\eta_\mu^2] + \text{other terms at}$$

$$\text{most linear in } \eta_\mu \text{ and } (\tilde{\omega}-\omega)_\mu. \tag{23}$$

The terms not written out explicitly are of a higher order in derivatives. In the sequel we neglect these terms, since the generalization taking them into account is straightforward.

The corresponding ϱ_μ-field reads:

$$\varrho_\mu = \frac{1}{g_\varrho}(V_\mu+\tfrac{1}{2}\alpha(\xi^\dagger\tilde{\eta}_\mu\xi-\xi\tilde{\eta}_\mu\xi^\dagger)), \tag{24}$$

where

$$V_\mu = \tfrac{1}{2}\,[\xi^\dagger(\tilde{\omega}_\mu-\omega_\mu-\tilde{\eta}_\mu+\eta_\mu)\xi+\xi(\tilde{\omega}_\mu-\omega_\mu+\tilde{\eta}_\mu-\eta_\mu)\xi^\dagger]$$

is the vector part of the linear multiplet (V_μ, A_μ) and $\alpha \equiv \alpha_1\left(\frac{g_\varrho}{m_\varrho}\right)^2$. The solution of Eq. (24) with respect to V_μ reads:

$$\vec{V}_\mu = \frac{1}{1-(1-\sigma^2)\alpha}\,[g_\varrho\vec{\varrho}_\mu-\alpha(\vec{\phi}\times(\partial_\mu\vec{\phi}+\sigma\vec{A}_\mu)+g_\varrho\vec{\phi}(\vec{\phi}\vec{\varrho}_\mu))]. \tag{25}$$

The final form of the L_{global} satisfying the VMD conditions, is:

$$L^{\text{VMD}}_{\text{global}} = -\left(\frac{m_\varrho}{g_\varrho}\right)^2 \text{Tr}\,[(\tilde{\eta}_\mu-\eta_\mu)^2+(\tilde{\omega}_\mu-\omega_\mu)^2-\alpha((\tilde{\eta}_\mu-\eta_\mu)^2-\eta_\mu^2)]. \tag{26}$$

Insofar we have identified the physical fields for the pseudoscalar Goldstone bosons (modulo the transformations allowed by the equivalence theorems [7]) and for the vector mesons. Now we have to consider the axial-vector fields. We have no principles to determine the form of the physical axial field $\vec{\mathscr{A}}_\mu$ apart from obvious ones: it should be an axial-vector and should transform according to the adjoint representation of the diagonal subgroup of the chiral group and quadratic part of the Lagrangian should be diagonal in fields. The most natural way to fulfil these requirements is to choose the axial-vector part of the linear multiplet of the gauge fields shifted appropriatly to get rid off the nondiagonal terms in quadratic part of Lagrangian:

$$\vec{A}_\mu = g_A\vec{\mathscr{A}}_\mu+\frac{1}{f_\pi}\,S(\sigma)\,(\partial_\mu\vec{\pi}-g_\varrho\vec{\varrho}_\mu\times\vec{\pi}), \tag{27}$$

where $\vec{\pi} = f_\pi\vec{\phi}$ and $S(\sigma)$ is a function of the field $\sigma = \sqrt{1-\vec{\phi}^2}$. The nondiagonal terms are eliminated if:

$$S(1) = d\left[(\alpha-1)\left(\frac{m_\varrho}{g_\varrho}\right)^2-d\right]^{-1}. \tag{28}$$

Apart from the condition (28) the form of $S(\sigma)$ is arbitrary. In most papers it is simply a constant: $S(\sigma) = S(1)$. However, we would like to stress, that the above arbitrariness does not influence (according to the equivalence theorems [7]) on shell S-matrix elements. On the other hand, by a proper choice of $S(\sigma)$ (to be discussed below) one can make the off-shell extension of the S-matrix as smooth as possible.

The final form of the Lagrangian is obtained after the normalization of the kinetic terms. As a result we obtain the following relations:

$$a = \frac{1}{2g_A^2}, \quad b+c = \frac{1}{2g_\varrho^2}, \quad d = \left(\frac{m_A}{g_A}\right)^2+(\alpha-1)\left(\frac{m_\varrho}{g_\varrho}\right)^2,$$

$$S(1) = -1-(\alpha-1)\left(\frac{m_\varrho g_A}{m_A g_\varrho}\right)^2$$

and

$$f_\pi^2 = \left(\frac{m_\varrho}{g_\varrho}\right)^2 \left[1-(\alpha-1)^2\left(\frac{m_\varrho g_A}{m_A g_\varrho}\right)^2 - \beta\right], \tag{29}$$

where we denote $\alpha \equiv \left(\frac{m_\varrho}{g_\varrho}\right)^2 \beta$. VMD demands β to vanish. Note, that the last relation taken for $\alpha = \beta = 0$ coincides with the famous Weinberg's sum rule [11].

Summing up this section we write down the properly normalized Lagrangian:

$$\begin{aligned} L = {} & \frac{1}{2g_A^2}\,\mathrm{Tr}\,(\tilde{D}_\mu\tilde{\eta}_\nu - \tilde{D}_\nu\tilde{\eta}_\mu)^2 + \left(\frac{1}{2g_\varrho^2} - c\right)\mathrm{Tr}\,(F_{\mu\nu}(\tilde{\omega}))^2 \\ & + c\,\mathrm{Tr}\,(F_{\mu\nu}(\tilde{\omega}) + [\tilde{\eta}_\mu, \tilde{\eta}_\nu])^2 + d^{(-)}\,\mathrm{Tr}\,([\tilde{\eta}_\mu, \tilde{\eta}_\nu])^2 + d^{(+)}\,\mathrm{Tr}\,(\tilde{\eta}_\mu^2)^2 \\ & - \left(\frac{m_A}{g_A}\right)^2 \mathrm{Tr}\,(\tilde{\eta}_\mu)^2 - 2(\alpha-1)\left(\frac{m_\varrho}{g_\varrho}\right)^2 \mathrm{Tr}\,(\eta_\mu\tilde{\eta}^\mu) + (\beta-1)\left(\frac{m_\varrho}{g_\varrho}\right)^2 \mathrm{Tr}\,(\eta_\mu)^2 \\ & - \left(\frac{m_\varrho}{g_\varrho}\right)^2 \mathrm{Tr}\,(\tilde{\omega}_\mu - \omega_\mu)^2 + \delta_1\,\mathrm{Tr}\,([\eta_\mu, \eta_\nu])^2 + \delta_2\,\mathrm{Tr}\,([(\tilde{\eta}-\eta)_\mu, (\tilde{\eta}-\eta)_\nu])^2 \\ & + \gamma\,\mathrm{Tr}\,([(\tilde{\omega}-\omega)_\mu, (\tilde{\omega}-\omega)_\nu])^2. \end{aligned} \tag{30}$$

For further discussion we retained here the terms proportional to β, δ_1, δ_2 and γ which are not allowed by VMD. The forms $\tilde{\eta}_\mu$ and $\tilde{\omega}_\mu$ expressed in terms of physical fields defined above (Eqs (25) and (27)) read

$$\begin{aligned} \tilde{\eta}_\mu = {} & \frac{i}{2}\vec{\tau}\Big\{[1-\alpha(1-\sigma^2)]^{-1}\Big[g_A\sigma\vec{\mathscr{A}}_\mu + \frac{1}{f_\pi}(1+\sigma S(\sigma))\,(\partial_\mu\vec{\pi} - g_\varrho\vec{\varrho}_\mu\times\vec{\pi}) \\ & + \frac{1}{f_\pi^3}\,\frac{1-\alpha(1+\sigma)}{\sigma(\sigma+1)}\,\vec{\pi}((1+\sigma S(\sigma))\,(\vec{\pi}\partial_\mu\vec{\pi}) + g_A f_\pi\sigma(\vec{\pi}\vec{\mathscr{A}}_\mu))\Big]\Big\}, \\ \tilde{\omega}_\mu = {} & \frac{i}{2}\vec{\tau}\Big\{g_\varrho\vec{\varrho}_\mu + \frac{g_A}{f_\pi}\,\frac{1-\alpha}{1-\alpha(1-\sigma^2)}\,\vec{\pi}\times\vec{\mathscr{A}}_\mu \\ & + \frac{1}{f_\pi^2}\,\frac{1}{\sigma+1}\,[1-\alpha(1-\sigma^2)]^{-1}[\sigma S(\sigma) + (1+S(\sigma))\,(1-\alpha-\alpha\sigma)] \\ & \times\vec{\pi}\times(\partial_\mu\vec{\pi} - g_\varrho\vec{\varrho}_\mu\times\vec{\pi})\Big\}. \end{aligned} \tag{31}$$

The main result of our paper is the Lagrangian (30) with $\beta = \delta_1 = \delta_2 = \gamma = 0$ given in terms of physical fields (modulo Eqs (18) and (31)). In Appendix we present, in more familiar notation, the equivalent form of the Lagrangian (30). This is the most general chiral symmetric Lagrangian of the fourth order in Cartan forms satisfying the VMD condition. Its construction fully exploits a freedom allowed by symmetry considerations. It follows from Eqs (25) and (30) that there exists the one-parameter family (characterized by α) of solutions to the VMD condition. This is in contradistinction with the literature (cf. Refs [13, 17, 20]) where the Lagrangians satisfying VMD condition correspond to $\alpha = 0$.

We want to underline that our Lagrangian is a generalization of all chiral invariant Lagrangians proposed in the literature. Thus we can discuss them as particular cases of our approach.

3. *Discussions and comparisons with other approaches*

3.1. The VMD, KSFR relation and universality

As we have stated above the VMD condition implies (see Eq. (30)) $\beta = \delta_1 = \delta_2 = \gamma = 0$ and the proper choice of the ϱ_μ field (Eq. (24)). Let us notice that the relation between g_ϱ and $g_{\varrho\pi\pi}$ does depend on the definition of the ϱ_μ field. For choice made above, i.e. consistent with the VMD condition it reads

$$g_{\varrho\pi\pi} = g_\varrho \frac{(\alpha-1)^2\left(\dfrac{m_\varrho g_A}{g_\varrho m_A}\right)^2 - 1}{1-(\alpha-1)^2\left(\dfrac{m_\varrho g_A}{g_\varrho m_A}\right)^2 - \beta}.$$

So for $\beta = 0$ the VMD and universality hold simultaneously irrespective of the value of α. Due to this freedom the KSFR relation $2g_{\varrho\pi\pi}^2 \cdot f_\pi^2 = m_\varrho^2$ can be satisfied without imposing any further constraint on masses m_ϱ, m_A and coupling constants g_ϱ, g_A. It demands only

$$(\alpha-1)^2 = \tfrac{1}{2}\left(\frac{m_A g_\varrho}{g_A m_\varrho}\right)^2.$$

For the standard choice i.e. $\alpha = 0$ (compare with Table I) the KSFR relation leads to the additional constraint $2m_\varrho^2 g_A^2 = m_A^2 g_\varrho^2$. Obviously, the KSFR relation can hold without demanding VMD and universality. It is then equivalent to the relation

$$2\left[1-(\alpha-1)^2\left(\frac{m_\varrho g_A}{g_\varrho m_A}\right)^2\right]^2 = 1-\beta-(\alpha-1)^2\left(\frac{m_\varrho g_A}{m_A g_\varrho}\right)^2.$$

We want to stress again that the consequent use of the technique of nonlinear realizations is the best approach to construct the maximal number of independent invariants out of which the effective Lagrangian is built.

Comparing our general Lagrangian (Eq. (30)) with those given in the literature (see Table I), we note in particular, that the existence of the additional arbitrary parameter C is recognized only by Ogievetsky and Zupnik [13] (see also Sect. 3.2).

TABLE I

The comparison of different effective Lagrangians (using the original notation) containing vector and axial — vector mesons. Each invariant enters into Lagrangian with appropriate coefficient shown in the

Table; here $M = \left(\frac{m_\varrho}{g_\varrho}\right)^2$, $\left(N = \frac{m_A}{g_A}\right)^2$

Invariants	L Eq. (30)	L^{VMD} Eq. (30)	Ref. [16]	Ref. [13] or Eq. (35)	Ref. [17]	Ref. [20]
$\text{Tr}\,(\tilde{D}_\mu\tilde{\eta}_\nu - \tilde{D}_\nu\tilde{\eta}_\mu)^2$	$\frac{1}{2g_A^2}$	$\frac{1}{2g_A^2}$	$\frac{1}{2g^2}$	$\frac{1}{2g_A^2}$	$\frac{1}{2g_A^2}$	$\frac{1}{4g^2}$
$\text{Tr}\,(F_{\mu\nu}(\tilde{\omega}))^2$	$\frac{1}{2g_\varrho^2} - c$	$\frac{1}{2g_\varrho^2} - c$	0	$\frac{\varepsilon^2}{2g_\varrho^2}$	0	0
$\text{Tr}\,(F_{\mu\nu}(\tilde{\omega}) + [\tilde{\eta}_\mu, \tilde{\eta}_\nu])^2$	c	c	$\frac{1}{2g^2}$	$\frac{1-\varepsilon^2}{2g_\varrho^2}$	$\frac{1}{2g_\varrho^2}$	$\frac{1}{4g^2}$
$\text{Tr}\,([\tilde{\eta}_\mu, \tilde{\eta}_\nu])^2$	$d^{(-)}$	$d^{(-)}$	0	$-\frac{\varepsilon^2(1-\varepsilon^2)}{2g_\varrho^2}$	0	0
$\text{Tr}\,(\tilde{\eta}_\mu\tilde{\eta}^\mu)^2$	$d^{(+)}$	$d^{(+)}$	0	0	0	0
$\text{Tr}\,(\tilde{\eta}_\mu\tilde{\eta}_\mu)$	$-N$	$-N$	$-(a+c)f_\pi^2$	$-N$	$-N$	$-\frac{1}{2}\left(\frac{m_A}{g}\right)^2$
$\text{Tr}\,(\eta_\mu\tilde{\eta}^\mu)$	$2(1-\alpha)M$	$2(1-\alpha)M$	$2af_\pi^2$	$2M$	$2M$	$\left(\frac{m_\varrho}{g}\right)^2$
$\text{Tr}\,(\eta_\mu\eta^\mu)$	$(\beta-1)M$	$-M$	$-(a+d)f_\pi^2$	$-M$	$-M$	$-\frac{1}{2}\left(\frac{m_\varrho}{g}\right)^2$
$\text{Tr}\,(\tilde{\omega}_\mu - \omega_\mu)^2$	$-M$	$-M$	$-bf_\pi^2$	$-M$	$-M$	$-\frac{1}{2}\left(\frac{m_\varrho}{g}\right)^2$
$\text{Tr}\,([\eta_\mu\eta_\nu])^2$	δ_1	0	$\frac{1}{2e^2}$	0	0	0
$\text{Tr}\,([(\tilde{\eta}-\eta)_\mu, (\tilde{\eta}-\eta)_\nu])^2$	δ_2	0	0	0	0	0
$\text{Tr}\,([(\tilde{\omega}-\omega)_\mu, (\tilde{\omega}-\omega)_\nu])^2$	γ	0	0	0	0	0
VMD		yes	no	yes	yes	yes
Universality		yes	yes	yes	yes	yes
KSFR		yes, for $N = 2M(\alpha - 1)^2$	yes	yes, for $N = 2M$	yes, for $N = 2M$	yes, for $m_A^2 = 2m_\varrho^2$

3.2. The second order Lagrangian

In order to make some definite phenomenological predictions from the PCAC, it is desirable to have the Lagrangian with the lowest possible number of derivatives when expressed in terms of physical fields (to quarantee smooth momentum dependence of amplitudes). The Lagrangian L^{VMD} given by the formula (30) (with $\beta = \delta_1 = \delta_2 = \gamma = 0$) is quartic in derivatives. The condition for L^{VMD} to be quadratic, determines uniquely the function $S(\sigma)$:

$$S(\sigma) = \frac{\varepsilon}{1-\varepsilon\sigma}, \quad \varepsilon = 1 - \left(\frac{m_A g_\varrho}{g_A m_\varrho}\right)^2. \tag{32}$$

We obtain also the following conditions on the parameters of the Lagrangian:

$$d^{(+)} = 0 = \alpha, \quad 1-\varepsilon^2 = 2cg_\varrho^2, \quad d^{(-)}+c(1-2cg_\varrho^2) = 0, \tag{33}$$

so Weinberg's relation (see Eq. (29)) takes the standard form:

$$f_\pi^2 = \left(\frac{m_\varrho}{g_\varrho}\right)^2 \left(1 - \left(\frac{m_\varrho g_A}{g_\varrho m_A}\right)^2\right). \tag{34}$$

In such a way we arrive at the following Lagrangian (in more conventional notation it is presented in the Appendix)

$$L_2^{\mathrm{VMD}} = \frac{1}{2g_A^2} \mathrm{Tr}\,(\tilde{D}_\mu \tilde{\eta}_\nu - \tilde{D}_\nu \tilde{\eta}_\mu)^2$$

$$+ \frac{1}{2g_\varrho^2} \mathrm{Tr}\,[F_{\mu\nu}(\tilde{\omega}) + (1-\varepsilon^2)\,[\tilde{\eta}_\mu, \tilde{\eta}_\nu]]^2 - \left(\frac{m_A}{g_A}\right)^2 \mathrm{Tr}\,\tilde{\eta}_\mu \tilde{\eta}^\mu$$

$$- \left(\frac{m_\varrho}{g_\varrho}\right)^2 \mathrm{Tr}\,[(\tilde{\eta}_\mu - \eta_\mu)^2 + (\tilde{\omega}_\mu - \omega_\mu)^2 - \eta_\mu^2], \tag{35}$$

where the forms η_μ, ω_μ, $\tilde{\eta}_\mu$ and $\tilde{\omega}_\mu$ are given by the Eqs (38) with $S(\sigma)$ defined Eq. (32). The KSFR condition [17] reads: $\varepsilon = -1$. The Lagrangian (35) was constructed many years ago by Ogievetsky and Zupnik [13] who discussed also widely its phenomenological consequences. This Lagrangian is the most general chiral invariant one of the second order in derivatives fulfilling the VMD condition. However, being of the second order in derivative it does not allow stable solitonic solution. For this it is desirable to examine the more general Lagrangian given in previous Section.

3.3. The Lagrangians without axial mesons

For the completness of our discussion we consider now the question of eliminating of the axial-vector mesons. Although such a procedure is contradictory to the general idea of the effective Lagrangians (since we believe, that all the states lying below some energy scale should be included), it is frequently considered in the literature (Refs [3, 4, 14]). The elimination should be performed in a gauge covariant way and the corresponding condition should involve the axial-vector quantity. The simplest possible condition is:

$$\tilde{\eta}_\mu = 0, \tag{36}$$

or equivalently in the terms of the fields $A^{L(R)}$

$$A_\mu^L = UA_\mu^R U^\dagger + U\partial_\mu U^\dagger.$$

We can solve it explicitly for the axial fields:

$$\vec{A}_\mu = -\frac{1}{f_\pi \sigma}(\partial_\mu \vec{\pi} - g_\varrho \vec{\varrho}_\mu \times \vec{\pi}).$$

After imposing the constraint (36) the Lagrangian (30) takes the form:

$$L_{\tilde{\eta}=0} = \frac{1}{2g_\varrho^2} \operatorname{Tr}(F_{\mu\nu}(\tilde{\omega}))^2 - \left(\frac{m_\varrho}{g_\varrho}\right)^2 \operatorname{Tr}[(\tilde{\omega}_\mu - \omega_\mu)^2 + (1-\beta)\eta_\mu^2]$$

$$+\delta \operatorname{Tr}([\eta_\mu, \eta_\nu])^2 + \gamma \operatorname{Tr}([(\tilde{\omega}-\omega)_\mu, (\tilde{\omega}-\omega)_\nu])^2 + \dots \tag{37}$$

Note also, that Weinberg's sum rule (29) should be replaced by

$$f_\pi^2 = \left(\frac{m_\varrho}{g_\varrho}\right)^2 (1-\beta). \tag{38}$$

We remind that in order to obtain the Lagrangian fulfilling VMD one has to put $\beta = \delta = \gamma = 0$. From the Eq. (24) it follows that $\varrho_\mu = \frac{1}{g_\varrho} V_\mu$. Then the above Lagrangian coincides with the one given by Weinberg [15]. It is of the same form as the Lagrangian proposed by Bando et al. [3] and Kaymakcalan and Schechter [4]. However, their choice of physical field differs from that of Weinberg and our (they identify the ϱ_μ field with $i\tilde{\omega}_\mu$). Consequently the VMD is spoiled by the terms of higher orders in fields.

Let us now discuss the KSFR relation. As in the case of VMD condition the validity of KSFR relation depends crucially on the choice of the physical ϱ_μ field. If we take $\varrho_\mu = \frac{1}{g_\varrho} V_\mu$ then

$$g_{\varrho\pi\pi} = -g_\varrho(1-\beta)^{-1}. \tag{39}$$

The Eqs (38) and (39) imply then that the KSFR relation is fulfilled if $\beta = -1$. Consequently, VMD and KSFR relation cannot hold simultaneously. On the other hand the choice $\varrho_\mu = i\tilde{\omega}_\mu$ implies

$$g_{\varrho\pi\pi} = -\tfrac{1}{2} g_\varrho(1-\beta)^{-1} \tag{40}$$

instead of Eq. (39) and the KSFR relation holds for $\beta = \frac{1}{2}$. The summary of results relevant to this section is given in Table II.

TABLE II

The comparison of different Lagrangians (using the original notation) containing vector mesons only. Each invariant enters into Lagrangian with appropriate coefficient shown in the Table

Lagrangian	$\mathrm{Tr}\,(F_{\mu\nu}(\tilde{\omega}))^2$	$\mathrm{Tr}\,(\tilde{\omega}_\mu-\omega_\mu)^2$	$\mathrm{Tr}\,(\eta_\mu\eta^\mu)$	$\mathrm{Tr}\,([\eta_\mu, \eta_\nu])^2$	$\mathrm{Tr}\,[(\tilde{\omega}-\omega)_\mu, (\tilde{\omega}-\omega)_\nu]^2$	rho field ϱ_μ	VMD	KSFR	Universality
L Eq. (37)	$\frac{1}{2g_\varrho^2}$	$-\left(\frac{m_\varrho}{g_\varrho}\right)^2$	$(\beta-1)\left(\frac{m_\varrho}{g_\varrho}\right)^2$	δ	γ	$\frac{1}{g_\varrho}V_\mu$	for $\beta = 0$	for $\beta = -1$	for $\beta = 0$
						$\frac{i}{g_\varrho}\tilde{\omega}_\mu$	no	for $\beta = \frac{1}{2}$	for $\beta = \frac{1}{2}$
L^{VMD} Eq. (37)	$\frac{1}{2g_\varrho^2}$	$-\left(\frac{m_\varrho}{g_\varrho}\right)^2$	$-\left(\frac{m_\varrho}{g_\varrho}\right)^2$	0	0	$\frac{1}{g_\varrho}V_\mu$	yes	no	yes
Ref. [4]	$\frac{1}{2g^2}$	$\frac{2m_0^2-B}{g^2}$	$\frac{2m_0+B}{g^2}$	0	0	$\frac{i}{g}\tilde{\omega}_\mu$	no	yes, for $B = -\frac{2}{3}m_0^2$	yes, for $B = -\frac{2}{3}m_0^2$
Ref. [3]	$\frac{1}{2g^2}$	$-af_\pi^2$	$-f_\pi^2$	0	0	$i\tilde{\omega}_\mu$	no	yes, for $a = 2$	yes, for $a = 2$
Ref. [18]	$\frac{1}{2g^2}$	$-af_\pi^2$	$-f_\pi^2$	$\frac{1}{2e^2}$	0	$i\tilde{\omega}_\mu$	no	yes, for $a = 2$	yes, for $a = 2$
Ref. [19]	$\frac{1}{2g^2}$	$-af_\pi^2$	$-f_\pi^2$	$\frac{1}{2e_-^2}$	$\frac{1}{2e_+^2}$	$i\tilde{\omega}_\mu$	no	yes, for $a = 2$	yes, for $a = 2$

4. *Concluding remarks*

In this paper we discuss the general framework for constructing the mesonic effective Lagrangians. Our emphasis was on general setting rather than on particular applications. Because of frequently contradictory statements appearing in the literature we have tried to clarify the situation in a systematic way.

Finally let us note that the generalization of the above formalism to the case of SU(N) group is in principle straightforward. All the equations and relations concerning Cartan's forms remain unchanged provided they are expressed in terms of U's and ξ's (which become the $N \times N$ unitary matrices). The only difference is that because of the more complicated form of the reduction of the tensor product of the adjoint representations two additional terms appear in the Lagrangian (30):

$$(\mathrm{Tr}\, \tilde{\eta}_\mu \tilde{\eta}^\mu)^2, \qquad \mathrm{Tr}\, (\tilde{\eta}_\mu \tilde{\eta}_\nu)\, \mathrm{Tr}\, (\tilde{\eta}^\mu \tilde{\eta}^\nu).$$

There are, however, some subtleties concerning the hiercharchy of symmetry breaking and VMD. The discussion of these problems will be given in next paper.

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APPENDIX

1. Below we list some formulae useful for the comparison of different Lagrangians

$$2\xi\tilde{\eta}_\mu\xi^\dagger \equiv (\nabla_\mu U)U^\dagger = -U\partial_\mu U^\dagger - UA^{\mathrm{R}}_\mu U^\dagger + A^{\mathrm{L}}_\mu,$$

$$2\xi\eta_\mu\xi^\dagger = -U\partial_\mu U^\dagger,$$

$$2(\xi\tilde{\omega}_\mu\xi^\dagger + \xi\partial_\mu\xi^\dagger) = U\partial_\mu U^\dagger + UA^{\mathrm{R}}_\mu U^\dagger + A^{\mathrm{L}}_\mu,$$

$$2(\xi\omega_\mu\xi^\dagger + \xi\partial_\mu\xi^\dagger) = U\partial_\mu U^\dagger,$$

$$2\xi(\tilde{D}_\mu\tilde{\eta}_\nu - \tilde{D}_\nu\tilde{\eta}_\mu)\xi^\dagger = F^{\mathrm{L}}_{\mu\nu} - UF^{\mathrm{R}}_{\mu\nu}U^\dagger,$$

$$2\xi(F_{\mu\nu}(\tilde{\omega}) + [\tilde{\eta}_\mu, \tilde{\eta}_\nu])\xi^\dagger = F^{\mathrm{L}}_{\mu\nu} + UF^{\mathrm{R}}_{\mu\nu}U^\dagger,$$

where $F^{\mathrm{L(R)}}_{\mu\nu} \equiv F_{\mu\nu}(A^{\mathrm{L(R)}})$.

2. The general Lagrangian (30) expressed in terms of the matrix U reads

$$L = \tfrac{1}{8}\left(\frac{1}{g_\varrho^2} + \frac{1}{g_{\mathrm{A}}^2}\right)\mathrm{Tr}\, [(F^{\mathrm{L}}_{\mu\nu})^2 + (F^{\mathrm{R}}_{\mu\nu})^2] + \tfrac{1}{4}\left(\frac{1}{g_\varrho^2} - \frac{1}{g_{\mathrm{A}}^2}\right)\mathrm{Tr}\, F^{\mathrm{L}}_{\mu\nu}UF^{\mathrm{R}\mu\nu}U^\dagger$$

$$+\tfrac{1}{4}\left(\frac{m_\varrho^2}{g_\varrho^2} - \frac{m_{\mathrm{A}}^2}{g_{\mathrm{A}}^2}\right)\mathrm{Tr}\, (\nabla_\mu U)U^\dagger(\nabla^\mu U)U^\dagger - \tfrac{1}{2}\frac{m_\varrho^2}{g_\varrho^2}\,\mathrm{Tr}\, [(A^{\mathrm{L}}_\mu)^2 + (A^{\mathrm{R}}_\mu)^2]$$

$$+\tfrac{1}{16}\left(\frac{1}{2g_\varrho^2}-c+d^{(-)}\right)\mathrm{Tr}\,([(\nabla_\mu U)U^\dagger,(\nabla_\nu U)U^\dagger])^2+d^{(+)}\,\mathrm{Tr}\,[(\nabla_\mu U)U^\dagger(\nabla^\mu U)U^\dagger]^2$$

$$+\tfrac{1}{4}\left(c-\frac{1}{2g_\varrho^2}\right)\mathrm{Tr}\,(F^{\mathrm{L}}_{\mu\nu}+UF^{\mathrm{R}}_{\mu\nu}U^+)\,[(\nabla^\mu U)U^\dagger,(\nabla^\nu U)U^\dagger]$$

$$-\tfrac{1}{2}\,\alpha\left(\frac{m_\varrho}{g_\varrho}\right)^2\mathrm{Tr}\,(\partial_\mu U)U^\dagger(\nabla^\mu U)U^\dagger+\tfrac{1}{4}\,\beta\left(\frac{m_\varrho}{g_\varrho}\right)^2\mathrm{Tr}\,(\partial_\mu U)U^\dagger(\partial^\mu U)U^\dagger$$

$$+\tfrac{1}{4}\,\delta_1\,\mathrm{Tr}\,([(\nabla_\mu U)U^\dagger,(\nabla_\nu U)U^\dagger])^2+\tfrac{1}{4}\,\delta_2\,\mathrm{Tr}\,([A^{\mathrm{L}}_\mu-UA^{\mathrm{R}}_\mu U^\dagger,\,A^{\mathrm{L}}_\nu-UA^{\mathrm{R}}_\nu U^\dagger])^2$$

$$+\tfrac{1}{4}\,\gamma\,\mathrm{Tr}\,([A^{\mathrm{L}}_\mu+UA^{\mathrm{R}}_\mu U^\dagger,\,A^{\mathrm{L}}_\nu+UA^{\mathrm{R}}_\nu U^\dagger])^2.$$

The analogous form of the second order Lagrangian (Eq. (35)) is the following

$$L_2^{\mathrm{VMD}}=\tfrac{1}{8}\left(\frac{1}{g_{\mathrm{A}}^2}+\frac{1}{g_\varrho^2}\right)\mathrm{Tr}\,[(F^{\mathrm{L}}_{\mu\nu})^2+(F^{\mathrm{R}}_{\mu\nu})^2]$$

$$+\tfrac{1}{4}\left(\frac{1}{g_\varrho^2}-\frac{1}{g_{\mathrm{A}}^2}\right)\mathrm{Tr}\,F^{\mathrm{L}}_{\mu\nu}UF^{\mathrm{R}\mu\nu}U^\dagger-\tfrac{1}{2}\left(\frac{m_\varrho}{g_\varrho}\right)^2\mathrm{Tr}\,[(A^{\mathrm{L}}_\mu)^2+(A^{\mathrm{R}}_\mu)^2]$$

$$+\tfrac{1}{4}\left(\frac{m_\varrho^2}{g_\varrho^2}-\frac{m_{\mathrm{A}}^2}{g_{\mathrm{A}}^2}\right)\mathrm{Tr}\,(\nabla_\mu U)U^\dagger(\nabla^\mu U)U^\dagger$$

$$+\tfrac{1}{32}\cdot\frac{\varepsilon^4}{g_\varrho^2}\,\mathrm{Tr}\,([(\nabla_\mu U)U^\dagger,(\nabla_\nu U)U^\dagger])^2$$

$$-\tfrac{1}{8}\cdot\frac{\varepsilon^2}{g_\varrho^2}\,\mathrm{Tr}\,(F^{\mathrm{L}}_{\mu\nu}+UF^{\mathrm{R}}_{\mu\nu}U^\dagger)\,[(\nabla^\mu U)U^\dagger,(\nabla^\nu U)U^\dagger],$$

where

$$\varepsilon=1-\left(\frac{m_{\mathrm{A}}g_\varrho}{g_{\mathrm{A}}m_\varrho}\right)^2.$$

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