ON THE ONSET OF INFLATION*

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(Received February 25, 1987)

We consider a simple model of transition between a radiation dominated isotropic Universe and the de Sitter Universe. We show that the expansion rate within a few e-folding times approaches a constant value. The temperature of radiation decreases more slowly and it approaches the Hawking temperature after about 10 e-folding times. Hawking radiation does not influence the dynamics of the inflation.

PACS numbers: 98.80.Bp

The inflationary model of the early evolution of the Universe proposed by Alan Guth [1] and later improved by Linde [2], and Albrecht and Steinhardt [3] seems to be able to solve several basic problems of the standard Big Bang model. In the new inflationary model it is assumed that at the very early stages of evolution the Universe was filled in with radiation and relativistic particles, and it was expanding. At a later stage the Universe cools down to the temperature of the spontaneous symmetry breaking of the Grand Unified Theory and it adiabatically supercools in a false vacuum state. During this stage of evolution the mean energy content of the Universe is dominated by vacuum energy and the Universe expands exponentially. The Higgs field trapped in a false vacuum state slowly "rolls down" toward the true vacuum state. The final transition to the true vacuum state is very rapid and during this process the Universe is reheated to a temperature comparable to the temperature of the spontaenous symmetry breaking. Further evolution follows the standard Big Bang scenario.

The new inflationary model has been recently criticized by Brandenberger and Kahn [4], who point out that the standard procedure of using the zero temperature effective

* This research was supported in part by the Polish Interdisciplinary Research Project CPBP 01.03.

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potential is inconsistent. Mazenko et al. [5] argue that prior to the onset of inflation large thermal fluctuations will produce inhomogeneities in the scalar field. Spatial domains will form already at the critical temperature and the inflation will be suppressed.

In the present paper we would like to investigate the transition between the radiation dominated and the vacuum energy dominated epoch of the evolution of the Universe. In particular we want to find out how soon after the onset of inflation one should take into account the Hawking radiation.

We assume that initially the Universe is uniformly filled in with radiation and vacuum energy (cosmological constant). At early stages of evolution, close to the initial singularity effects of vacuum energy are negligible and the expansion rate is determined by energy density of radiation. For simplicity we consider the flat ($k = 0$) Friedman-Robertson-Walker model, so

$$ds^2 = dt^2 - R^2(t)(dr^2 + r^2 d\Omega^2), \tag{1}$$

and

$$R^2(t) = R_0^2 \sinh 2Ht, \tag{2}$$

where $d\Omega^2$ is the line element of the unit sphere, $R_0^2 = 2(g\pi^2/(30\varrho_v))^{1/2}(90S/(4g\pi^2))^{2/3}$, $H = (8\pi G\varrho_v/3)^{1/2}$, g is the effective number of spin states of relativistic particles, S is the entropy of relativistic particles, and ϱ_v the vacuum energy density.

Close to the initial singularity ($2Ht \ll 1$) $R(t) \sim t^{1/2}$, and the expansion rate is determined by the radiation energy, at later times for $2Ht \gg 1$, $R(t) \sim e^{Ht}$, the expansion rate is determined by the vacuum energy (cosmological constant). The expansion rate (Hubble constant) is given by $H(t) = H \operatorname{ctgh} 2Ht$.

We assume that the Universe cools down adiabatically, so $T \sim R^{-1}$, and the radiation energy density $\varrho_r \sim R^{-4}$. Explicitly we have $\varrho_r = \varrho_v \sinh^{-2} 2Ht$. The radiation energy density becomes equal to the vacuum energy density when $\sinh 2Ht = 1$, ($Ht \cong 0.44$). Temperature of radiation at that moment we call the critical temperature.

At later stages of evolution ($Ht \gg 1$) our model can be approximated by the de Sitter universe. This becomes apparent when we introduce new coordinates

$$\tilde{r} = 1/2 R_0 r e^{Ht},$$
$$\tilde{t} = -1/(2H) \ln (e^{-2Ht} - 1/4 R_0^2 r^2 H^2). \tag{3}$$

The metric expressed in the new coordinates is

$$ds^2 = (1-\tilde{r}^2H^2)d\tilde{t}^2 - d\tilde{r}^2/(1-\tilde{r}^2H^2) - \tilde{r}^2 d\Omega^2$$
$$+ e^{-4Ht}(\tilde{r}^2H^2d\tilde{t}^2 - 2H\tilde{r}/(1-\tilde{r}^2H^2)d\tilde{t}d\tilde{r} + d\tilde{r}^2/(1-\tilde{r}^2H^2)^2 + \tilde{r}^2 d\Omega^2). \tag{4}$$

The coordinates $\tilde{t}$, $\tilde{r}$ asymptotically, for $Ht \gg 1$, become static de Sitter coordinates.

On our space-time manifold let us consider a massless scalar field satisfying the Klein-Gordon equation. On an initial $t = \text{const.}$ ($Ht \ll 1$) hypersurface we introduce a basis $f_{\omega lm}$ (ω denotes the energy quantum number, l, m are angular momentum quantum numbers). Positive frequency is defined with respect to the initial time coordinate. At a late time ($Ht \gg 0.44$) space like hypersurface we define another basis $g_{\omega lm}$ of positive frequency solutions. The two bases are related by a Bogoliubov transformation

$$g_i = \sum_j (\alpha_{ij} f_j + \beta_{ij} \bar{f}_j), \tag{5}$$

($\bar{f}_j$ is a complex conjugate of f_j) where i, j represent all quantum numbers. If

$$|\alpha_{\omega lm}|^2 = \exp(T_{\rm H}\omega)\, |\beta_{\omega lm}|^2 \tag{6}$$

for all ω, then the inertial observer detects black body radiation at temperature $T_{\rm H}$. $T_{\rm H}$ is called the Hawking temperature.

To determine the Hawking temperature we follow the procedure used by Brandenberger and Kahn [4]. Their calculation can be easily modified and adapted to our model. Therefore without going into details of the calculation let us state the final result

$$k_{\rm B} T_{\rm H}(t) = hH(t)/(2\pi) = hH/(2\pi)\,\mathrm{ctgh}\, 2Ht, \tag{7}$$

where $k_{\rm B}$ is the Boltzmann constant, h the Planck constant, and this formula is valid for $Ht > 0.44$.

It is now interesting to see how the Hawking temperature compares with the radiation temperature. In Table I we present our results. In our calculations we assumed that $T_c = 10^{14}$ GeV.

We notice that for a long time after the onset of inflation temperature of radiation is much higher than the Hawking temperature and only at $Ht \cong 10$ temperature of radiation becomes equal to the Hawking temperature.

The new inflationary model is based on a Coleman-Weinberg potential. For the SU(5) Georgi-Glashow model the finite temperature one loop effective potential is

$$V_{\rm eff}(\phi, T) = B\phi^4(\ln(\phi^2/\sigma^2) - 1/2) + 1/2 B\sigma^4 + CT^2\phi^2, \tag{8}$$

where $B = 5625g^4/(512\pi^2)$, $C = 75g^2/16$, and g is the gauge coupling constant. The SU(5) unified gauge group is spontaneously broken at critical temperature

$$T_c = (B/(2C))^{1/2}\sigma, \tag{9}$$

where σ is a parameter determining the unification energy, typically of the order of 10^{14}–10^{15} GeV.

TABLE I

Ht	ϱ_r/ϱ_v	$H(t)/H-1$	T_r/T_c	T_r/T_H
0.44	1	$\sqrt{2}-1$	1	3.7×10^4
1	0.07	0.04	0.5	2.5×10^4
5	8.2×10^{-9}	4×10^{-9}	10^{-2}	5.3×10^2
10	1.7×10^{-17}	8.5×10^{-18}	6.4×10^{-5}	3.4
15	3.5×10^{-26}	1.7×10^{-26}	4.0×10^{-7}	2.1×10^{-2}
20	7.2×10^{-35}	3.6×10^{-35}	2.9×10^{-9}	1.5×10^{-4}

In the new inflationary scenario the temperature corrections were usually omitted. Let us now calculate corrections to the vacuum energy density due to the Hawking radiation. The potential (8) is normalized so that $V(0, 0) = 1/2B\sigma^4$, and $V(\sigma, 0) = 0$, so $\Delta V(T = 0) = V(0, 0) - V(\sigma, 0) = 1/2B\sigma^4$. When $T = T_H$ we have in the first order in $(T_H/T_c)^2$, $\Delta V(T = T_H) = V(0, T_H) - V(\phi_{min}, T_H) = 1/2B\sigma^4(1-(T_H/T_c)^2)$, here $\phi_{min} = \sigma(1-1/8(T_H/T_c)^2)$ is the value of ϕ at the global minimum of the potential for $T = T_H$.

In our model $T_c = 10^{14}$ GeV, $T_H = 1.9 \times 10^9$ GeV, so $T_H/T_c = 2 \times 10^{-5}$ and the thermal corrections to the effective potential during the de Sitter epoch could be neglected. As it is well known thermal fluctuations during the de Sitter epoch play an important role in generation of density perturbations.

It is interesting to notice that during the transition between the radiation dominated epoch and the de Sitter epoch the expansion rate very quickly approaches a constant value characterizing the de Sitter epoch. Temperature of radiation is approaching the Hawking temperature at a much slower pace. Thermal fluctuations present at the onset of inflation might speed up the transition to the true vacuum state substantially shortening the duration of de Sitter epoch. Our model is not sufficiently realistic and we cannot discuss this problem.

Recently Albrecht, Brandenberger and Matzner [6], and Matzner [7] numerically analyzed the onset of inflation taking into account local inhomogeneities in the scalar field. It turns out that thermal fluctuations produce similar corrections as local inhomogeneities. The numerical analysis shows that for a wide range of parameters thermal fluctuations do not prevent inflation.

We conclude therefore that thermal fluctuations do not play an important role in dynamics of inflationary models.

This work was initiated at the Pulkovo Observatory and finished at the Center for Theoretical Physics of the University of Texas at Austin. One of us (M.D.) wants to thank Professor Yu. N. Gnedin and Professor J. A. Wheeler for hospitality.

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