Vol. B19 (1988) ACTA PHYSICA POLONICA No 3

NON-LINEAR QUARK FIELD MODELS OF EXTENDED HADRONS*

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(*Received January 4, 1988*)

A class of Dirac field models with non-linear terms in the form of fractional powers of the scalar field invariants is discussed. The resulting equations contain a simple mechanism which implies that the solutions have compact supports and form what may be called soft or hard bags. The usual MIT bag is obtained as a limiting case (hardest bag). In the case of interacting coloured and flavoured quark fields introduction of some terms weakly breaking the $SU_c(3)$ symmetry provides a simple and rigorous mechanism of quark confinement. In several physically interesting cases this symmetry breaking may even not show up, so this mechanism may be called "hidden symmetry breaking".

PACS numbers: 12.40.Aa

1. *Introduction*

The problem of quark confinement within hadrons is still rather far from being completely solved and well understood. In order to explain apparent non-existence of free quarks, the physicists have proposed several models starting from simple oriented strings, rather strange potentials keeping the quarks together in white $q\bar{q}$ or qqq states, then even more ad hoc constructed bag models, up to the most sophisticated field theory in the form of QCD. Unfortunately, even QCD — in spite of several partial successes and its mathematical beauty — does not provide a completely satisfactory solution of this problem but rather suggests various possibilities for the mechanism of confinement.

Though QCD is constructed in a similar manner as QED or the standard model of electroweak interactions, the physical situation of hadrons is quite different. In QCD we are dealing with very strong interactions which make the otherwise very effective perturbation expansions in general rather useless. Next, the fundamental particles, i.e. quarks and gluons, never appear as isolated free particles and thus do not allow the construction

* Work supported in part by the Research Program CPBP 01.03.

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of asymptotic or scattering states. There is already enough experimental evidence for the existence of coloured and flavoured spin 1/2 quarks with fractional electric and baryonic charges predicted already by the first naive quark model that was based mainly on simple group theoretical arguments. However, the evidence for the existence of the octet of coloured but electrically neutral gluons is still weak. E.g. no glueballs or other exotic states predicted by QCD have been established. The same applies to Higgsons which are introduced in QCD to provide the mechanism of spontaneous symmetry breaking and production of non-vanishing masses.

The complex and still unclear situation sketched above justifies other attempts at solving the problem of quark confinement. Thus, in the present paper we assume that the necessary strong interactions between quarks can be described without involving gluon fields by suitable terms that are non-linear in the quark fields alone. Next — we observe that the very concept of confinement contains two necessary ingredients: localization and inseparability of the respective fundamental fields. In the model considered in this paper localization means that the quark fields are different from zero only in small regions of space occupied by extended hadrons with radii of the order of 10^{-13} cm. Inseparability means that none of the quark fields can appear isolated from all the others but only certain combinations of quark fields appear in nature in the form of "white" hadrons. It follows that in particular the quark fields corresponding to different colours are inseparable.

Of course one can impose a priori a general phenomenological condition that all observable hadrons be white or $SU_c(3)$ scalars. However, from the point of view of theory such a condition is fully legitimate when applied to the initial states. The emergence of only white objects (hadrons) in the final state should then result from the equations of motion and cannot be imposed as an additional requirement for the allowed final states. In general, exact $SU_c(3)$ invariance imposed on the equations of motion and the choice of only white hadrons in the initial state implies certainly that the final state must be also white as a whole. This, however, does not exclude emergence in the final state of several well separated coloured objects whose colours are so superimposed that the whole system remains white. Thus it is feasible that not only localization but also inseparability of colours require suitable dynamics involving even some symmetry breaking.

In this paper we shall study the attractive possibility of describing the structure of extended hadrons in terms of solitons, i.e. non-dispersive, shape preserving wave packets which are solutions of suitable non-linear field equations (NLFE). Obviously, the main aim consists in finding a suitable system of coupled NLFE which for finite values of the charges Q and energies E have only soliton like solutions satisfying definite stability, localizability and inseparability conditions.

2. Dirac solitons on compact supports

We shall start from a possible solution of the problem of localization for a single Dirac spinor field. Consider a Lagrangian density of the form

$$\mathscr{L} = -i\bar{\psi}\tilde{\partial}\psi - \mu\bar{\psi}\psi - b\kappa^{a-1}\bar{\psi}\psi \tag{1}$$

with

$$\kappa = |\bar{\psi}\psi|, \quad 0 < b, \quad 0 < a < 1, \quad \mu = \pm|\mu|. \tag{1'}$$

The corresponding NLDE

$$-i\partial\!\!\!/\psi = W(\kappa)\psi \quad \text{with} \quad W = \mu + ab\kappa^{a-1} \tag{2}$$

contain a function $W(\kappa)$ that can be interpreted either as the field dependent effective mass or, equivalently, as an effective scalar potential of forces. It has the peculiar property of tending to $+\infty$ whenever one approaches the surface $\kappa = 0$. This produces strong forces repelling the field from such surfaces. This implies also some sort of the true "horror vacui", i.e. flight from the vacuum where $\psi = 0$ and formation of non-dispersive lumps of field matter.

NLFE with such a form of the effective mass have been first proposed and studied by the author some time ago [1, 2] for both the Dirac and Klein-Gordon eqs. (NLKGE) in the ordinary $3+1$ dimensional Minkowski space. Several physicists investigated the properties of such types of NLDE and NLKGE finding some explicit solutions for NLKGE in $1+1$ dimensions and performing interesting numerical calculations [3–8]. Of particular interest is the paper by Morris [3] who gave a rigorous proof that all solutions of such NLKGE corresponding to finite values of Q and E are nondispersive. Unfortunately, no similar proof has been given for the respective NLDE.

If the solution ψ of (2) is supposed to describe an extended particle of definite spin in a rest frame, it must transform irreducibly under rotations. Because of the non-linearity of the Eqs. (2) this condition can be satisfied only for $j = 1/2$. Moreover, since we are most interested in finding shape preserving solitons we shall restrict our search to stationary solutions. Thus the Dirac spinor that satisfies both these conditions must have the following general form

$$\psi(\vec{r}, t) = \begin{Bmatrix} -ig(r)\chi \\ \dfrac{\vec{\sigma}\vec{r}}{r} f(r)\chi \end{Bmatrix} \exp -i\omega t \tag{3}$$

where

$$\chi = \begin{pmatrix} c \\ d \end{pmatrix}, \quad |c|^2 + |d|^2 = 1.$$

The radial functions g and f satisfy the Eqs.:

$$g' + (\omega+\mu)f + ab|g^2 - f^2|^{a-1}f = 0,$$

$$f' + \frac{2}{r}f - (\omega-\mu)g + ab|g^2 - f^2|^{a-1}g = 0. \tag{4}$$

Both the numerical calculations as well as the physical discussions become much simpler after performing a scaling transformation proposed by Mathieu and Sally [9]:

$$G(R) = \left(\frac{\omega-\mu}{b}\right)^{1/(2-2a)} g(r), \quad F(R) = \left(\frac{\omega-\mu}{b}\right)^{1/(2-2a)} f(r) \tag{5}$$

with

$$R = (\omega-\mu)r, \quad \gamma = \frac{\omega+\mu}{\omega-\mu}, \quad 0 < \omega \neq \mu. \tag{5'}$$

The rescaled functions depend only on the dimensionless variable R and satisfy somewhat simpler eqs.

$$G'+\gamma F + a|G^2-F^2|^{a-1}F = 0,$$

$$F'+2/RF-G+a|G^2-F^2|^{a-1}G = 0. \tag{6}$$

Instead of four parameters a, b, μ, ω which appear in (4) the Eqs. (6) contain only two parameters: a and γ. Therefore, whenever necessary we shall write $G_{\gamma a}$, $F_{\gamma a}$. It is to be noted that the dependence on b of the original functions g, f is contained in the scaling factor, and that for a given model determined by the values of a, b and $\mu \neq 0$ the choice of the value of $\gamma \neq 1$ fixes the value of ω in a unique manner. For models with $\mu = 0$, i.e. $\gamma = 1$, the frequency is arbitrary, and hence in this case the dependence of g and f on frequency is also described explicitly by the scaling factor.

Explicit solutions of (4) have been found by the author [1] for arbitrary $\mu < 0$, and $1/2 < a < 1$ but only for definite $\omega = -\mu > 0$ and definite $b = b(\mu, a)$. Expressed in terms of G and F they have the form

$$G_{0a}(R) = C_{0a}(1-R^2/R_{0a}^2)^{a/(2-2a)},$$

$$F_{0a}(R) = R/R_{0a}G_{0a}(R) \tag{7}$$

for $R < R_{0a}$ and vanish for $R > R_{0a}$. Here

$$R_{0a} = \frac{3-2a}{1-a}, \quad C_{0a} = (3-2a)^{1/(2-2a)}. \tag{7'}$$

This means that the Dirac field described by this solution is different from zero only within a sphere of radius

$$r_{0a}(\mu) = \frac{3-2a}{2\mu(1-a)} \tag{7''}$$

and thus has a compact support. For the whole indicated range of a the radial functions vanish for $r \to r_{0a}$ and thus can be matched continuously with the vacuum solution $G = F = 0$. However, for models with $a < 2/3$ the first derivatives become infinite at the surface of the sphere.

Numerical solutions presented in [9] for $\gamma = 1, 10, 100$ and several fractional values of a show a similar behaviour. They all have compact supports specified by definite radii $r_{\gamma a}(\mu)$ at which they vanish. The value of a at which the sharp drop of the radial functions occurs, depends on γ but the general properties remain the same. The models for which the first derivatives at the surface are finite (including zero) will be called soft bags, and those with a sharp drop at the surface will be called hard bags. For a given value of γ the bag is the harder the smaller is the value of a. Furthermore, the quoted numerical solutions show that for a fixed value of γ

$$\lim_{a \to 1} r_{\gamma a}(\mu) = +\infty \tag{8}$$

as expected, because $a = 1$ means a linear DE with dispersing wave packets as the only possible finite energy solutions.

Since the functions G, F depend on a smaller number of parameters the discussion of physically interesting properties of the solutions of (4) is simpler. Of particular interest is the fact that in the limit $a \to 0$ one obtains the MIT bag (as the hardest bag). Although the radius of the bag turns out to be finite even in this extreme case, the radial functions become discontinuous. However, $\bar{\psi}\psi$ is, in all cases considered, continuous and non--negative for $r \leqslant r_{\gamma a}$. Generalizing these results we shall assume that all finite energy solutions of (4) have compact supports and have non-negative values of $\bar{\psi}\psi$.

The relations to the MIT bag model can easily be seen from the form of Q and E valid for stationary solutions

$$Q = \int (g^2+f^2) d^3r, \qquad E = \omega Q + b(1-a) \int (g^2-f^2)^a d^3r. \tag{9}$$

For $a = 0$ one obtains

$$E = Q\omega + bV_{\gamma 0}, \tag{10}$$

where $V_{\gamma 0}$ is the volume of the bag characterized by the value of γ and μ. This volume energy is the most characteristic part of the MIT bag. In this limit the coupling constant b acquires the physical meaning of some pressure. A more detailed discussion of this limiting case for different values of μ and γ and in particular the possible energy spectra will be presented in another paper.

Concluding our discussion of the NLDE of the form (2) we may say that they provide a rather satisfactory answer to the problem of localization. Choosing different values of the parameters a, b and μ one obtains a great variety of field models of extended objects. The fact that in the limit of vanishing a one can have the MIT bag model seems to be very encouraging, because this ad hoc constructed model had scored several successes but definitely lacked a more profound field theoretical derivation. Though from the computational point of view the hard bag models with $a = 0$ may be simpler that those with $a \neq 0$, they do not have the important advantages of true field theoretical models with continuous solutions. Thus it seems that the soft bags may be more useful for the description of extended but well localized non-dispersive hadrons and hadron-hadron interactions.

3. *Inseparability of colours*

Let us now investigate the second necessary condition of quark confinement, i.e. the requirement of inseparability of 3 quark colours. We shall make a tentative assumption that both the internal structure of hadrons as well as their interactions can be described by a suitable set of classical NLDE for the fundamental quark fields

$$\psi_{cf}(x), \quad c = 1, 2, 3; \quad f = 1, \ldots, 6, \tag{11}$$

where c and f are respectively colour and flavour indices. In principle some sets of NLDE involving only quark fields may emerge at least as approximations of QCD after suitable elimination of gluonic fields. It is also a priori not excluded that the coloured vector currents constructed from the quark fields alone may simulate the presence of an octet of quasi--gluons.

As the starting point let us take the Lagrangian of the strong interactions in the form

$$\mathscr{L}_0 = \sum_f \sum_c (-i\bar{\psi}_{cf}\gamma^\alpha \partial_\alpha \psi_{cf} - \mu_f \bar{\psi}_{cf}\psi_{cf} - U(\kappa)\bar{\psi}_{cf}\psi_{cf}) \tag{12}$$

with

$$\kappa = |\sum_f \sum_c \bar{\psi}_{cf}\psi_{cf}|$$

that is invariant with respect to $SU_c(3)$ and 6 Abelian $U_f(1)$ groups which guarantee conservation of each flavour. Inspired by the results concerning the problem of localization for one Dirac field we take tentatively $U(\kappa)$ of the form

$$U(\kappa) = b\kappa^{a-1}. \tag{13}$$

The resulting eqs. of motion

$$D_f \psi_{cf} = 0; \quad D_f = i\gamma_\alpha \partial^\alpha + \mu_f + ab\kappa^{a-1} \tag{14}$$

imply 9×6 conservation laws for the vector currents

$$j^\alpha_{c'cf} = \bar{\psi}_{c'f}\gamma^\alpha \psi_{cf}, \quad \partial_\alpha j^\alpha_{c'cf} = 0. \tag{15}$$

As a direct consequence of (15) we obtain the respective conservation laws for the observable white currents

$$\partial_\alpha j^\alpha_B = 0, \quad \partial_\alpha j^\alpha_e = 0, \quad \partial_\alpha j^\alpha_f = 0, \tag{16}$$

where

$$j^\alpha_f = \sum_c j^\alpha_{ccf}, \quad j^\alpha_B = \sum_f j^\alpha_f, \quad j^\alpha_e = \sum_f e_f j^\alpha_f.$$

The remaining conserved octets of coloured currents are redundant from the experimental point of view because they have been never observed as isolated objects. Coloured currents may appear within hadrons but then there is no reason for them to be conserved.

In the considered class of models, involving only quark fields, the rigorous $SU_c(3)$ symmetry allows for creation in the hadron-hadron collision of several coloured isolated objects provided that the final state remains white as a whole. For example, suppose that only one quark field, say ψ_{11}, is different from zero in some region of space and time, all the other fields with $c \neq 1, f \neq 1$ being equal to zero. Then the set of NLDE (14) reduces to one NLDE of the type (2) discussed in the previous section. This means that well localized, isolated, non-dispersive solutions representing one quark or antiquark carrying a single colour can appear in the final state.

This undesirable result follows from the assumed rigorous $SU_c(3)$ symmetry and cannot be removed by choosing another form of the non-linearity. Therefore, we must conclude that at least for the models that involve solely quark fields the only way out of this difficulty is to break the rigorous colour symmetry.

In order to achieve this goal we shall add to the Lagrangian a symmetry breaking term that has to fulfill two important conditions: all the conservation laws for the white currents must remain valid and the resulting equations of motion should imply inseparability of colours.

Consider the following additional term that breaks the colour symmetry:

$$\mathscr{L}' = A+B, \tag{17}$$

where

$$A = h/4\sum_{f}\sum_{c,c'} (\bar{\psi}_{cf}\psi_{cf}-\bar{\psi}_{c'f}\psi_{c'f})^2,$$

$$B = \sum_{f}\sum_{c\neq c'} d_{cc'}\bar{\psi}_{cf}\psi_{c'f}, \tag{17'}$$

$$d_{12} = d_{23} = d_{31} = id, \quad d_{cc'} = d^*_{c'c}, \quad d > 0.$$

It can easily be seen that $\mathscr{L}'$ violates $SU_c(3)$ symmetry but conserves all flavours. Thus the new field eqs. acquire the form

$$D_f\psi_{1f} = h/2(2\bar{\psi}_{1f}\psi_{1f}-\bar{\psi}_{2f}\psi_{2f}-\bar{\psi}_{3f}\psi_{3f})\psi_{1f}+id(\psi_{2f}-\psi_{3f}). \tag{18}$$

The other two eqs. can be obtained from (18) by cyclic permutation of the colour indices.

We shall prove now that the modified Eqs. (18) imply inseparability of colours. Consider a finite compact region of space and time, e.g.

$$S = \{(x): \vec{r} \in S_\varrho(t); \quad t_0 \leqq t \leqq t_0+\Delta t\},$$

where Δt is a small but non-vanishing interval of time and $S_R(t)$ is the interior of a sphere with radius ϱ whose centre may move.

Definition: The three colours are called separable if there exists a region S where for any flavour only one or two of the coloured quark fields are different from zero, while the remaining coloured fields vanish in S, i.e. if either

I: $\psi_{1i}(x) \neq 0$ for some $x \in S$ and some i

$\psi_{2j}(x) = \psi_{3k}(x) = 0$ for all $x \in S$, and all j and k

or

II: $\psi_{1i}(x) \neq 0, \quad \psi_{2j}(x) \neq 0$ for some $x \in S$ and some i, j

$\psi_{3k}(x) = 0$ for all $x \in S$ and all k.

Conjecture: All finite energy solutions of (18) corresponding to integer values of the total charges of the system have compact supports.

If this is so, one can always find a region S or several disconnected regions of this type which have the property that the quark fields can be different from zero only within these regions.

Consider now the form of our eqs. of motion (18) in these two cases:

Case I:

$$A = h \sum_f (\bar{\psi}_{1f}\psi_{1f})^2, \qquad B = 0,$$

$$D_i\psi_{1i} = 0, \qquad 0 = -dA\psi_{1i}, \qquad 0 = idA\psi_{1i} = 0.$$

Case II:

$$A = h \sum_f |(\bar{\psi}_{1f}\psi_{1f})^2 + (\bar{\psi}_{2f}\psi_{2f})^2 - (\bar{\psi}_{1f}\psi_{1f})(\bar{\psi}_{2f}\psi_{2f})|,$$

$$B = id \sum_f (\bar{\psi}_{1f}\psi_{2f} - \bar{\psi}_{2f}\psi_{1f}),$$

$$D_i\psi_{1i} = h/2(2\bar{\psi}_{1i}\psi_{1i} - \bar{\psi}_{2i}\psi_{2i})\psi_{1i} + id\psi_{2i},$$

$$D_i\psi_{2i} = h/2(2\bar{\psi}_{2i}\psi_{2i} - \bar{\psi}_{1i}\psi_{1i})\psi_{2i} - id\psi_{1i},$$

$$0 = id(\psi_{1i} - \psi_{2i}).$$

It can easily be seen that in both cases our assumption that one or two colours can be separated from the remaining ones leads to contradiction. It follows that our modified equations (18) indeed imply rigorously inseparability of colours provided that our conjecture that all solutions corresponding to finite energies and finite and integer charges have compact supports is correct. We do not claim that the proposed way of achieving localization and inseparability of colours is the only possible or the best one. The main aim of this paper was to show on an explicit example that quark confinement can be described by a set of classical NLDE without involving gluons and quantization of fields.

It is interesting to note that if all solutions of (18) have compact supports the presented proof of inseparability of colours is valid for arbitrarily small symmetry breaking terms, i.e. for arbitrarily small but non-zero values of the constants h and d.

The Eqs. (18) have an interesting set of solutions which obviously satisfy the inseparability condition and moreover make the $SU_c(3)$ symmetry breaking terms vanish:

$$\psi_{1f}(x) = \psi_{2f}(x) = \psi_{3f}(x) = \psi_f(x). \tag{19}$$

In this case the Eqs. (18) reduce to the $SU_c(3)$ symmetric form

$$(i\gamma_\alpha\partial^\alpha + \mu_f + ab\kappa^{a-1})\psi_f = 0 \quad \text{with} \quad \kappa = 3|\sum_f \bar{\psi}_f\psi_f|. \tag{20}$$

For any solution deviating from the symmetric form the right hand side of (18) that implies inseparability of colours does not vanish and must be taken into account.

Consider now the general expressions for the charges Q_f and the energy for the symmetric solution (19)

$$Q_f = 3 \int \psi_f^+ \psi_f d^3r, \tag{21}$$

$$E = \int \{i\, 3 \sum_f \psi_f^+ \partial_t \psi_f + b(1-a)\kappa^{a-1} 3 \sum_f \bar{\psi}_f \psi_f\} d^3r. \tag{22}$$

The Eqs. (20) have stationary solutions $\psi_f = \varphi_t(\vec{r}) \exp(-i\omega_f t)$ with frequencies ω_f which in principle may be different for each quark appearing in the considered hadrons. Hence, for stationary solutions one obtains

$$E = \sum_f \omega_f Q_f + b(1-a)3^a \int |\sum_f \bar{\psi}_f \psi_f|^{a-1} \sum_f \bar{\psi}_f \psi_f d^3r. \tag{23}$$

Again we see that in the limit $a \to 0$ one obtains the characteristic volume term of the hard bag. Of course the sum over f is to be taken cum grano salis, i.e. it has to be extended over such flavours that are contained in the considered hadron with the possibility of repetition of the same flavour. In fact in the case of Δ^{++}, Δ^-, Ω^- which contain 3 quarks of the same flavour the set (20) reduces to one NLDE of the form (14) but with a rescaled value of the coupling constant. However, this presents no problem and one can take advantage of the explicit or numerical solutions known for one NLDE of the type (14) which were discussed in the preceding section. Further discussions of the more complex hadrons containing several flavours will be given in another paper. Unfortunately, there is rather no chance for finding explicit solutions of (20) in all cases of interest and one must resort to the use of computers.

4. *Summary*

Let us sum up the attractive features of the class of field models discussed in this paper: 1) They allow for a field theoretical description of both the internal structure of hadrons as well as of hadron-hadron interactions. 2) The solutions have compact supports (soft or hard bags). 3) They include the conventional bag model as a limiting case (hardest bags). 4) For some special subclasses some explicit solutions are known (apart from numerical solutions). 5) The models involve strong forces which keep the fields well localized and have a simple physical interpretation. 6) They satisfy the necessary energetic stability conditions [6]. 7) The set of NLDE for coloured quark fields allows for a rigorous proof of quark confinements if $SU_c(3)$ symmetry is weakly broken. In physically interesting cases this symmetry breaking may even not show up.

We should like to remark that several authors have tried to describe extended particles as soliton solutions of various NLFE. However, it seems that none of these models has so many features that are desirable for the description of hadrons as those discussed in this paper.

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