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SECOND BORN APPROXIMATION IN ELASTIC-ELECTRON SCATTERING FROM NUCLEAR STATIC ELECTRO-MAGNETIC MULTIPOLES

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Second Born approximation corrections to electron scattering by nuclei with arbitrary spin are considered. Explicit integral expressions for the charge, magnetic dipole and interference differential cross sections are obtained. Magnetic and interference relative corrections are then investigated in the case of backward electron scattering using shell model form factors for nuclear targets ^{9}Be, ^{10}B, and ^{14}N. To understand exponential growth of these corrections with square of the electron energy K_0^2, the case of electron scattering by ^{6}Li is considered using monopole model charge form factor with power-law asymptotics.

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1. Introduction

Elastic and inelastic electron-nuclei scattering is a useful tool for studying nuclear structure. Usually, for light nuclei the first Born approximation (FBA) well describes electron scattering. Improved precision of the experimental data available enables one to take into account different corrections to FBA. One of the very important among them is the second Born approximation (SBA). These corrections in the case of Coulomb scattering were computed by Dalitz [1], and for extended nuclei they were investigated by several authors [2–5]. In all of these works the static and dispersive corrections were studied for spinless nuclei only, in order to simplify the problem as well as to search for these corrections in the most sensitive range of q^2 near the diffraction minimum or to explain the nature of this minimum [6].

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In the present study, the case of electron scattering by nuclei with arbitrary spin is considered. The formulas are obtained for second Born correction which contain pure electric, pure magnetic and interference terms. The last is absent in the FBA, and was first considered in our earlier paper [7].

In the case when the nucleus possesses charge and dipole magnetic moment only we give an explicit expressions for all of these three types of corrections through appropriate nuclear form factors. The formulas obtained are very simple and can be used in straightforward way to fit the experimental data.

2. *Differential cross section*

Neglecting excitation and recoil of the nucleus in the intermediate state, we write the differential cross section of elastic electron-nucleus scattering as the sum of the first and second Born approximations:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{(1)}}{d\Omega} + \frac{d\sigma^{(2)}}{d\Omega}$$

$$= \frac{1}{16\pi^2\eta^2}\,\frac{1}{2I+1}\sum_{M_iM_f}\left\{|T_{fi}^{(1)}|^2 + \sum_{M_n} 2\,\mathrm{Re}\,(T_{fi}^{(1)}T_{fi}^{(2)*})\right\}. \tag{1}$$

Here, $\eta = 1+2(K_0/m_A)\sin^2(\theta/2)$ is kinematic recoil factor, K_0 is energy of the incident electron, m_A is nuclear mass, θ is electron scattering angle; I is nuclear spin with M_i, M_f and M_n — its projections in the initial, final, and intermediate states. The FBA and SBA amplitudes are given by the following expressions:

$$T_{fi}^{(1)} = -\left(\frac{4\pi Z\alpha}{q^2}\right)\bar{u}(K')\hat{a}u(K), \tag{2}$$

$$T_{fi}^{(2)} = -i(4\pi Z\alpha)^2\int\frac{d^4\kappa}{(2\pi)^4}\,\frac{E^{\mu\nu}(\kappa)}{\kappa^2-m_e^2+i0}\,\frac{H_{\mu\nu}(\vec{q}_2,\vec{q}_1;\kappa_0)}{(q_1^2+i0)(q_2^2+i0)}. \tag{3}$$

Here, the electronic tensor $E^{\mu\nu}(\kappa)$ is given by:

$$E^{\mu\nu}(\kappa) = \bar{u}(K')\gamma^\mu(\hat{\kappa}+m_e)\gamma^\nu u(K);$$

and the hadronic tensor $H^{\mu\nu}(\vec{q}_2,\vec{q}_1;\kappa_0)$:

$$H^{\mu\nu}(\vec{q}_2,\vec{q}_1;\kappa_0) = \frac{a_2^\mu b_1^\nu}{K_0'-\kappa_0+i0} + \frac{a_1^\nu b_2^\mu}{\kappa_0-K_0+i0};$$

$$\hat{a} = a_\mu\gamma^\mu;$$

$$a^\mu(\vec{q}) = \langle IM_f;\vec{p}'|J^\mu(0)|IM_i;\vec{p}\rangle;$$

$$a_{1,2}^\mu(\vec{q}_{1,2}) = \langle IM_f;\vec{p}'|J^\mu(0)|IM_n;\vec{p}'-\vec{q}_{1,2}\rangle;$$

$$b_{1,2}^\mu(\vec{q}_{1,2}) = \langle IM_n;\vec{p}+\vec{q}_{1,2}|J^\mu(0)|IM_i;\vec{p}\rangle.$$

In the above expressions the following notations are used: $K = (K_0, \vec{K})$ and $p = (p_0, \vec{p})$ ($K' = (K'_0, \vec{K}')$ and $p' = (p'_0, \vec{p}')$) are the 4 momenta of electron and nucleus in the initial (final) states; $q = K-K' = p'-p$ is the 4-momentum transfered to the nucleus; $\kappa = (\kappa_0, \vec{\kappa})$ is the 4-momentum of the electron in the intermediate state; $q_1 = K-\kappa$, $q_2 = \kappa-K'$, $q = q_1+q_2$; Ze is the electric charge of the nucleus; $\alpha = 1/137$. For current matrix elements the following multipole expansions are used ($\vec{n} = \vec{q}/|\vec{q}|$):

$$a^0(\vec{q}) = \sum_{lm} a_{lm}(|\vec{q}|)Y_{lm}(\vec{n}),$$

$$\vec{a}(\vec{q}) = \sum_{\lambda lm} b_{lm}^{(\lambda)}(|\vec{q}|)\vec{Y}_{lm}^{(\lambda)}(\vec{n}). \tag{4}$$

The current conservation allowes one to exclude the longitudinal multipoles $b_{lm}^{(-1)}$, and with the account of T-invariance the nonzero expansion coefficients are given by the following expressions:

$$a_{lm}(|\vec{q}|) = \frac{4\pi i^l}{(2l+1)!!}\langle IM_{\mathrm{f}}|\hat{Q}_{lm}|IM_{\mathrm{i}}\rangle$$

for even $l \leqslant 2I$ and

$$b_{lm}^{(0)}(|\vec{q}|) = \frac{4\pi i^{l-1}}{(2l+1)!!}\langle IM_{\mathrm{f}}|\hat{M}_{lm}^{(0)}|IM_{\mathrm{i}}\rangle$$

for odd $l \leqslant 2I$.

After appropriate summation and averaging over the spin projections in (1), the SBA cross section can be represented as the sum of electric (E), magnetic (M) and interference (EM) terms:

$$d\sigma^{(2)} = d\sigma_{\mathrm{E}}^{(2)} + d\sigma_{\mathrm{M}}^{(2)} + d\sigma_{\mathrm{EM}}^{(2)}. \tag{5}$$

Taking into account only the charge and magnetic dipole moment of the nucleus we get the following formulas for each of the three terms in (5):

$$\frac{d\sigma^{(2)}_{(i=\mathrm{E,M,EM})}}{d\Omega} = \sigma_{\mathrm{Mott}}\left(\frac{Z\alpha}{\pi^2}\right) 2\,\mathrm{Re}\, J_i(K_0, \theta), \tag{6}$$

where

$$J_{\mathrm{E}}(K_0, \theta) = \frac{K_0}{2}\,\mathrm{tg}^2\left(\frac{\theta}{2}\right)\int \frac{d^3\kappa}{\vec{\kappa}^2 - K_0^2 - i0}\,\frac{\vec{P}^2 + 2(\vec{P}\vec{\kappa})}{(\vec{q}_1^2+\lambda^2)(\vec{q}_2^2+\lambda^2)}\,F_{\mathrm{C}}(\vec{q}_1^2)F_{\mathrm{C}}(\vec{q}_2^2)F_{\mathrm{C}}(\vec{q}^2), \tag{6a}$$

$$J_{\mathrm{M}}(K_0, \theta) = \left(\frac{I+1}{6I^2}\right)\left(\frac{\mu_I}{Ze}\right)^3 \mathrm{tg}^2\left(\frac{\theta}{2}\right)\int \frac{d^3\kappa}{\vec{\kappa}^2 - K_0^2 - i0}\,\frac{F_{\mathrm{M1}}(\vec{q}_1^2)F_{\mathrm{M1}}(\vec{q}_2^2)F_{\mathrm{M1}}(\vec{q}^2)}{\vec{q}_1^2\vec{q}_2^2}$$

$$\{(\vec{v}_1[\vec{q}_1\times[\vec{q}_1\times\vec{q}_2]]) + (\vec{v}_2[\vec{q}_2\times[\vec{q}_1\times\vec{q}_2]]) - (\vec{q}_1[\vec{P}\times\vec{q}])(\vec{q}_2[\vec{P}\times\vec{q}])\}, \tag{6b}$$

$$J_{\mathrm{EM}}(K_0, \theta) = \left(\frac{I+1}{3I}\right)\left(\frac{\mu_I}{Ze}\right)^2 K_0\,\mathrm{tg}^2\left(\frac{\theta}{2}\right)\int \frac{d^3\kappa}{\vec{\kappa}^2 - K_0^2 - i0}\,\frac{1}{(\vec{q}_1^2+\lambda^2)(\vec{q}_2^2+\lambda^2)}$$

$$\{F_C(\vec{q}^2)F_{M1}(\vec{q}_1^2)F_{M1}(\vec{q}_2^2)\varphi_0+F_C(\vec{q}_1^2)F_{M1}(\vec{q}^2)F_{M1}(\vec{q}_2^2)\varphi_1$$
$$+F_C(\vec{q}_2^2)F_{M1}(\vec{q}_1^2)F_{M1}(\vec{q}^2)\varphi_2\}. \tag{6c}$$

Here, F_C and F_{M1} — charge and magnetic dipole form factors of the nucleus, μ_I — its magnetic dipole moment; $\vec{P} = \vec{K}+\vec{K}'$, λ — screening parameter (the problem of Coulomb singularity factorization when $\lambda \to 0$ have been discussed, for example, in [1, 4]). In (6b) and (6c) we have introduced also

$$\vec{v}_1 = \vec{q}^2\vec{\kappa}-2(\vec{K}\vec{q}_1)\vec{K}', \quad \vec{v}_2 = \vec{q}^2\vec{\kappa}+2(\vec{K}'\vec{q}_2)\vec{K},$$

$$\varphi_0 = \vec{P}^2(\vec{\kappa}^2-K_0^2)+\left(\frac{\vec{P}^2}{2}-(\vec{P}\vec{\kappa})\right)\left(\frac{\vec{P}^2}{2}-(\vec{P}\vec{\kappa})+2\vec{\kappa}^2+2K_0^2\right),$$

$$\varphi_{1,2} = \vec{q}^2(\vec{\kappa}^2-K_0^2)+(\vec{q}\vec{q}_{2,1})\,[4K_0^2+(\vec{q}\vec{q}_{2,1})].$$

We must stress here that expression (6b) is free of sharp singularities at points $q_1^2 = 0$ or $q_2^2 = 0$ mainly due to well known relation between M1-reduced matrix element and corresponding form factor $\langle I \,||\, \hat{M}_1^{(0)} \,||\, I\rangle \propto qF_{M1}(q^2)$. This fact leads to noticeable suppression of the pure dipole magnetic contribution $d\sigma_M^{(2)}$ in comparison to pure Coulomb $d\sigma_E^{(2)}$ or interference $d\sigma_{EM}^{(2)}$ contributions. This suppression must be much greater in the cases of higher nuclear multipole moments Q2 or M3, so in what follows we may neglect them altogether.

3. *Electron scattering by light nuclei*

The formulas obtained can be applied to investigation of SBA contribution using both methods of numerical integration [7] and analytical computation in the framework of definite model for nuclear form factors [5, 8].

First we consider shell model nuclear form factors with a harmonic oscillator potential [8]

$$F_C(\vec{q}^2) = (1-a_C\vec{q}^2)e^{-\frac{b\vec{q}^2}{2}}, \quad F_{M1}(\vec{q}^2) = (1-a_M\vec{q}^2)e^{-\frac{b\vec{q}^2}{2}}, \tag{7}$$

which are commonly used in analysis of the electron scattering data for target nuclei with mass numbers $4 \leqslant A \leqslant 16$ [9]. Inserting expressions (7) into formulas (6a)–(6b) one can show [8, 10] that SBA to FBA relative contribution grows rapidly as $\exp(b\vec{q}^2/4)$.

This feature is connected directly to Gaussian nature of the form factors in harmonic oscillator model, and restricts validity of the SBA corrections to incident electron energy not higher than several hundreds MeV.

Fig. 1 serves as fair illustration to this point. We consider here backward electron scattering, because of the FBA cross section in this case is caused exclusively by magnetic effects for electron energies $K_0 \gtrsim 50$ MeV. The SBA contribution is given now by only magnetic and interference corrections for which compact analytical expressions can be obtained [10].

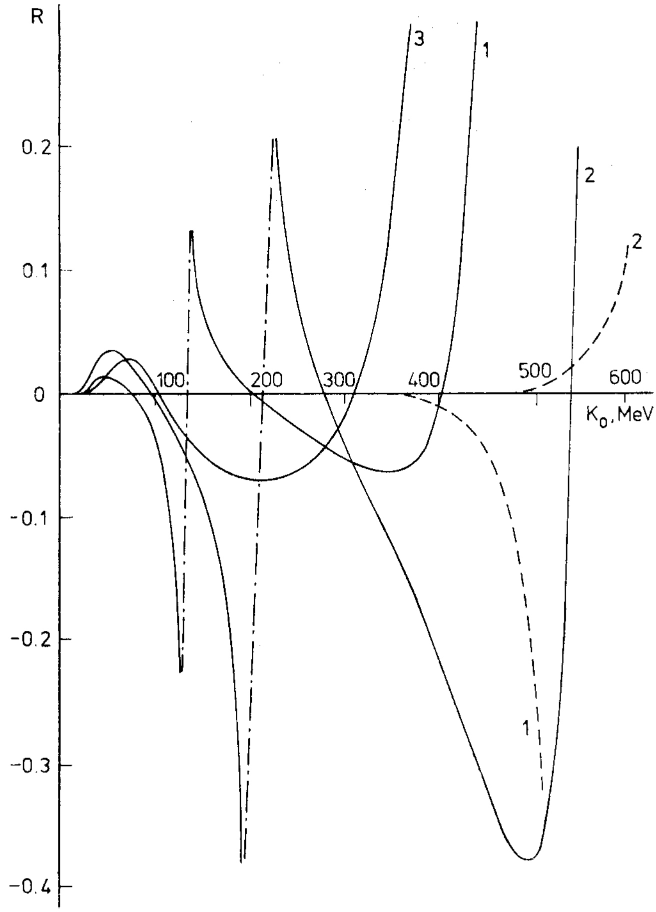

Fig. 1. Energy dependences of the relative SBA corrections R_{EM} (solid curves) and R_{M} (dashed curves) in backward electron scattering for shell model form factors: *1* — ^{9}Be, *2* — ^{10}B and *3* — ^{14}N. Dash-dot lines show the region of diffraction zeroes of the form factors F_{M1}

The energy dependences of the ratios

$$R_{\mathrm{M,EM}} = d\sigma^{(2)}_{\mathrm{M,EM}}/d\sigma^{(1)}$$

plotted in Fig. 1 were computed using following parameters of the form factors involved:

$$b = \frac{a_0^2}{2}\,\frac{A-1}{A} + \frac{r_{\mathrm{p}}^2}{3}, \qquad a_{\mathrm{C}} = \frac{Z-2}{6Z}\,a_0^2 = \tfrac{1}{6}\left(1-\frac{2}{Z}\right)a_0^2,$$

$$a_{\mathrm{M}} = (1-R_1)\,\frac{a_0^2}{6} = \tfrac{1}{6}(1-R_1)a_0^2. \tag{8}$$

Here, a_0 is oscillator parameter, $r_p^2 = 0.427\ \mathrm{fm}^2$ is proton rms radius. The nuclei considered are characterized by:

^{9}Be: $Z = 4, \quad I = 3/2, \quad a_0 = 1.67\ \mathrm{fm}, \quad \kappa_m = -1.177, \quad R_1 = -0.38,$

$^{10}B: Z = 5, \quad I = 3, \quad a_0 = 1.42 \text{ fm}, \quad \kappa_m = 1.8, \quad R_1 = 0.32,$

$^{14}N: Z = 7, \quad I = 1, \quad a_0 = 1.61 \text{ fm}, \quad \kappa_m = 0.404, \quad R_1 = 2.48.$

One can see that main contribution to SBA corrections arises from interference term, which oscillates and increases rapidly in absolute value with electron energy.

The relative magnetic correction also oscillates, but its contribution for $K_0 \leqslant 400$ MeV does not exceed 1 %.

Exponential growth of the relative SBA corrections mentioned above is connected with the fact that Gaussian form factors do not satisfy the axioms of the local field theory. Such a growth is absent in the case of form factors with a power-law asymptotics.

To clarify this point let us consider phenomenological monopole model [5] for the charge nuclear form factor

$$F_C(q^2) = \sum_{k=1}^{n} \frac{g_k}{q^2 + b_k^2}. \tag{9}$$

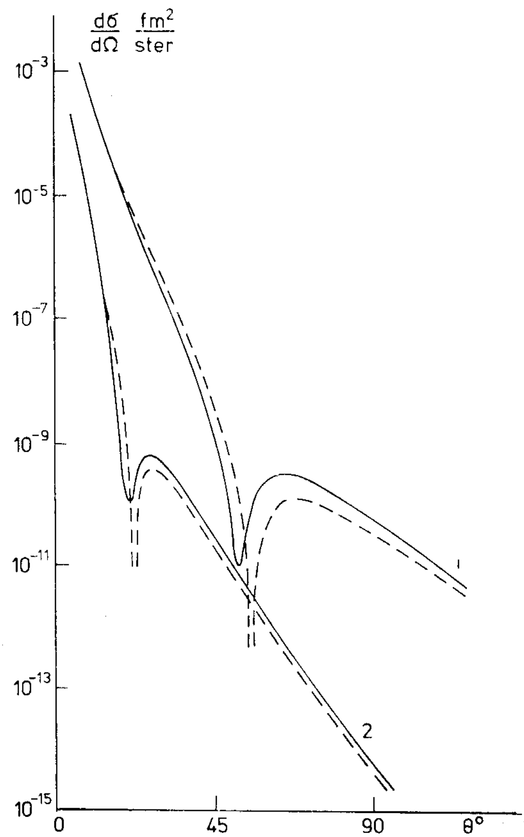

Fig. 2. Angular dependences of differential cross sections for charge electron scattering by ^{6}Li in monopole form factor model. Solid curves — FBA+SBA, dashed curves — FBA; *1* — $K_0 = 500$ MeV, *2* — $K_0 = 800$ MeV

One can show that (9) will have appropriate quark counting power-low asymptotics $F_C(q^2) \approx (1/q^2)^{n-1}$ only if $g_k = C_{k1}g_1 + C_{k2}g_2$, $k = 3, 4, \ldots, n$, number of quarks, $n \geqslant 4$ and

$$C_{kj} = -\prod_{\substack{l \neq k \\ l \leqslant n}} \left(\frac{b_l^2 - b_j^2}{b_l^2 - b_k^2} \right).$$

Evidently, normalization to unity $F_C(0) = 1$ fixes one more parameter, namely g_2, so that for equidistant pole approximation $b_k^2 = b_0^2 + (k-1)h^2$ we have only three free model parameters: b_0^2, h^2, g_1.

Form factor (9) can be fitted to experimental data available on charge electron scattering to fix the values of these parameters. Using data on ^{6}Li for $K_0 = 200$ MeV [11] we get in FBA: $b_0^2 = 2.4\ \mathrm{fm}^{-2}$, $h^2 = 4.2\ \mathrm{fm}^{-2}$, $g_1 = 14.7$ with χ^2 value 11.09.

Straightforward integration of (6a) along with form factor (9) yields a logarithmic expression for pure charge SBA correction. This behaviour is consistent with field theory expectations.

Fig. 2 shows influence of the SBA correction on the differential cross section of the electron charge scattering by ^{6}Li. The calculations were performed in the framework of monopole model and include not only interference of SBA and FBA amplitudes (6a) but also square of the SBA amplitude.

We may conclude now that SBA corrections to elastic electron scattering shift the position of the cross section diffraction minimum in the direction of smaller q^2. This shift results in reducing of the cross section below and in its picking up over the minimum point. Such a deformation of the cross section will change the values of the form factor parameters extracted from experimental data approximately by 10% in magnitude.

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