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SYMMETRY BREAKING IN THE LATTICE GAUGE THEORY IN INFINITE DIMENSIONS*

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It is argued that the gauge symmetry can be broken spontaneously in the infinite dimension limit.

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The mean-field approximation provides a useful tool in analyzing various lattice models. It is also known or believed to become exact in some limiting cases as for example the long-range [1], many component [2] or high-density [3] limits. The gauge models pose special problems. For example, the many-component limit of the gauge Potts model can be accurately described by the mean-field theory only after some gauge-fixing [4] or by allowing slightly larger class of trial measures [5]. The reason is that usual choice of trial measures in the form of the product of independent measures for each degree of freedom explicitly breaks the gauge invariance. Such breakdown manifests itself often in the violation of the Elitzur theorem [6] stating that the gauge system cannot magnetize spontaneously, whereas meanfield approximation allows the spontaneous magnetization. In spite of this confusing state of matter the mean-field method gives surprisingly good results in the case of gauge theories [7] and it was strongly advocated by Drouffe et al. [8] who proposed also the method of circumventing the above difficulty. Roughly speaking this method consists in representing the mean-field approximation as a saddle-point approximation and summing over all degenerate saddles resulting from the gauge symmetry. This procedure restores the gauge symmetry and the validity of the Elitzur theorem is granted. On the other hand, it must be remembered that the mean-field theory can be exact only after some limit has been taken; it is not clear whether the Elitzur theorem survives this limit.

The crucial point in the proof of this theorem [6] is the assumption that the local gauge transformation in one site affects only finite number of degrees of freedom. This is obviously

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not the case in the limit of infinite number of internal degrees of freedom: "large N" or the limit of infinite dimension. In fact, in the former case it was shown [9] that the gauge symmetry can be spontaneously broken. We argue below that the same holds true in the later case. We consider the d-dimensional hypercubic lattice Λ consisting of N sites i, Nd links, $N\binom{d}{2}$ plaquettes p etc.; the periodic boundary conditions are imposed. We define the Z_2 gauge theory as usual: σ_l is the spin variable sitting on the link l and taking the values ± 1; the gauge transformation reads $\sigma_l \to \varrho_i \varrho_l \varrho_j$, where the sites i, j are the endpoints of l and $\varrho_i, \varrho_j = \pm 1$. We define the free energy per one degree of freedom as

$$F_d = \lim_{N\to\infty} -\frac{1}{Nd\beta} \log Z, \tag{1}$$

$$Z = \sum_{\sigma_l = \pm 1} \exp(-\beta H_0), \quad \beta = (kT)^{-1},$$

here H_0 is some gauge invariant hamiltonian, i.e. the function of plaquette variables, $\sigma_P = \prod_{l\in\partial P} \sigma_l$ and the free energy in the limit $d \to \infty$

$$F_\infty = \lim_{d\to\infty} F_d. \tag{2}$$

For the above limit to exist the coupling appearing in H_0 must be scaled appropriately. Typically if any link couples with the strength J to $\sim d$ neighbours, we must replace J by J/d etc. Let us consider the fixed set of all gauge equivalent configurations. It is completely determined by choosing the set of frustrated plaquettes ($\sigma_P = -1$) and can be characterized as follows: choose the Cayley tree T in Λ and fix the gauge by $\tilde{\sigma}_l = 1$ for any $l \in T$; then solve for other links l determining finally the representative configuration $\{\tilde{\sigma}_l\}$. Any other configuration under consideration can be written as

$$\sigma_l = \varrho_i \tilde{\sigma}_l \varrho_j. \tag{3}$$

To break the gauge symmetry we couple each σ_l to the external magnetic field h. The hamiltonian becomes

$$H = H_0 - h\sum_l \sigma_l; \quad h > 0. \tag{4}$$

Notice that h need not be scaled[1]. Let $\langle\sigma_l\rangle_{h,N,d}$ denote the expectation value of σ_l. Elitzur theorem states that

$$\lim_{h\to 0}\lim_{N\to\infty} \langle\sigma_l\rangle_{h,N,d} = 0 \tag{5}$$

for any d. We argue below that at low temperatures

$$\lim_{h\to 0}\lim_{d\to\infty}\lim_{N\to\infty} \langle\sigma_l\rangle_{h,N,d} \neq 0. \tag{6}$$

[1] For $h < 0$ we choose all $\tilde{\sigma}_l = -1$, and the conclusions remain unchanged.

Consider first the simplified model. H_0 is such that there is an infinite energy gap between the ground state $\{\sigma_P = 1 \text{ for all } P\}$ and the first excited state.

The general expression for the partition function which can be written in the form

$$Z = \sum_{\{\tilde{\sigma}_l\}} \exp\left[-\beta H_0(\tilde{\sigma})\right] \sum_{\{\varrho_i\}} \exp\left[\beta h \sum_l \tilde{\sigma}_l \varrho_i \varrho_j\right] \tag{7}$$

reduces to one term corresponding, according to our prescription, to $\tilde{\sigma}_l = 1$ for all l. The mean value reduces to the correlation function of the Ising model with the coupling h and with no external field

$$\langle \sigma_{l_0} \rangle_{h,N,d} = Z^{-1} \exp\left[-\beta H_0(1)\right] \sum_{\{\varrho_i\}} \exp\left[\beta h \sum_l \varrho_i \varrho_j\right] \varrho_{i_0} \varrho_{j_0} = \langle \varrho_{i_0} \varrho_{j_0} \rangle^{\text{Ising}}_{h,N,d}. \tag{8}$$

This will vanish if we let $h \to 0$ after taking $N \to \infty$. Let us however take the $N \to \infty$ and then $d \to \infty$. It is well known [10] that if we take the limit $d \to \infty$ for the Ising model we should scale β or h by d^{-1}. In other words, in the case under consideration the limit $d \to \infty$ corresponds to the infinite coupling or zero temperature. But then $\langle \varrho_{i_0} \varrho_{j_0} \rangle \to 1$ (this is proven rigorously in the Appendix). Taking finally the limit $h \to 0$ we get

$$\lim_{h \to 0} \lim_{d \to \infty} \lim_{N \to \infty} \langle \sigma_{l_0} \rangle_{h,N,d} = 1. \tag{9}$$

Although this result concerns a very simple model it is not quite trivial because as we stressed above the Elitzur theorem is valid for any gauge-invariant hamiltonian. So we have shown that in the limiting case $d \to \infty$ the validity of this theorem does depend on the choice of the gauge-invariant hamiltonian. Moreover, it is easy to see that Elitzur theorem is true for any gauge-invariant set of configurations not necessarily all. Let us consider now the conventional Z_2 gauge theory. The hamiltonian reads

$$H_0 = -\sum_P \sigma_P, \qquad \sigma_P = \prod_{i \in \partial P} \sigma_i. \tag{10}$$

We have (taking $l_0 \in T$)

$$\langle \sigma_{l_0} \rangle_{h,N,d} = Z^{-1} \sum_{\{\tilde{\sigma}_l\}} \exp\left[-\beta H_0(\tilde{\sigma}_l)\right] \sum_{\{\varrho_i\}} \exp\left[\beta h \sum_l \tilde{\sigma}_l \varrho_i \varrho_j\right] \varrho_{i_0} \varrho_{j_0}. \tag{11}$$

For any fixed gauge-invariant configuration $\{\tilde{\sigma}_l\}$ we are now dealing with the spin system described by $\sum_{\{\varrho_i\}} \exp\left[\beta h \sum_l \tilde{\sigma}_l \varrho_i \varrho_j\right]$; it is now no longer purely ferromagnetic and there are some anti ferromagnetic couplings corresponding to $\tilde{\sigma}_l = -1$. However, as the temperature lowers, we expect the configurations with small fraction of frustrated plaquettes to dominate. With our conventions concerning the choice of the representative configuration $\{\tilde{\sigma}_l\}$ this corresponds to the small fraction of links with $\tilde{\sigma}_l = -1$. The corresponding spin system becomes then ferromagnetic and one can expect that again after taking the limit $d \to \infty$ the correlation function points to the positive value independent of h. If this scenario works, the infinite dimensional gauge system magnetizes spontaneously at low temperatures contradicting the Elitzur theorem. Let us note that the above results do not contradict those of Drouffe et al. [8]. To carry through their argument concerning the summation over the degenerate saddles (which is obviously invalid in the case of global symmetries) one has to introduce small symmetry breaking field and essentially repeat the Elitzur's argument. Also the contribution from the degenerate saddles is of the order $\frac{1}{d}$ [8].

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APPENDIX

We prove here that if $\langle\sigma_a\sigma_b\rangle_{\Lambda,d,J}$ denotes the correlation function for the Ising model on d-dimensional hypercubic lattice Λ with the coupling J then

$$\lim_{d\to\infty}\lim_{\Lambda\to\infty}\langle\sigma_a\sigma_b\rangle_{\Lambda,d,J} = 1. \tag{12}$$

Let l be the link with the endpoints a, b and P the set of 2 $(d-1)$ plaquettes having l as a bordering link. Consider the hamiltonian H' defined as follows

$$H' = -\sum_l J_l\sigma_i\sigma_j, \tag{13}$$

where

$$J_l = \begin{cases} J \text{ if } l \text{ belongs to the border of some plaquette from } P \\ 0 \text{ otherwise.}\end{cases}$$

One can easily calculate the correlation function for this model hamiltonian

$$\langle\sigma_a\sigma_b\rangle' = \frac{\left[1-\left(\frac{1-x^3}{1+x^3}\right)^{2(d-1)}\right]+x\left[1+\left(\frac{1-x^3}{1+x^3}\right)^{2(d-1)}\right]}{\left[1+\left(\frac{1-x^3}{1+x^3}\right)^{2(d-1)}\right]+x\left[1-\left(\frac{1-x^3}{1+x^3}\right)^{2(d-1)}\right]}, \tag{14}$$

where $x = \operatorname{th}\beta J$. Now, it follows from the correlation inequalities that

$$\langle\sigma_a\sigma_b\rangle_{\Lambda,d,J} \geqslant \langle\sigma_a\sigma_b\rangle'$$

and

$$\lim_{\Lambda\to\infty}\langle\sigma_a\sigma_b\rangle_{\Lambda,d,J} \geqslant \langle\sigma_a\sigma_b\rangle'.$$

Using the Eq. (14) and taking the limit $d\to\infty$ we get (12).

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