SUPERPARTICLES*

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A superparticle is defined as an object classically described by the position four-vector and a Dirac bispinor, both treated as one-dimensional fields dependent on proper time. When quantized, they become Bose and Fermi quantum-mechanical operators, respectively. Dirac constraint method is used to propose two options for the first-quantization wave equation. Two one-dimensional broken supersymmetries, both connected with the proper time, are introduced to define dichotomic charge states of the superparticle. Then, spin-1/2 states of such a superparticle form four charge doublets that may be interpreted as four lepton generations. The lower bound for mass of the fourth-generation charged lepton is 5246 MeV.

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1. *Introduction*

We will call a *superparticle* a hypothetical physical object described classically by the position four-vector $x^\mu(\tau)$, $\mu = 0, 1, 2, 3$, and a Dirac bispinor $\psi_\alpha(\tau)$, $\alpha = 1, 2, 3, 4$, both dependent on the proper time τ. In general, such an object is not (and need not be) supersymmetrical in four dimensions[1]. Instead, we shall introduce two one-dimensional broken supersymmetries connected with the proper time. They will be responsible for the existence of dichotomic charge states of our superparticle.

On the level of classical mechanics, we can specify the dynamics of the one-dimensional fields $x^\mu(\tau)$ and $\psi_\alpha(\tau)$ by postulating the action $\mathscr{A} = \int_{\tau_1}^{\tau_2} d\tau \mathscr{L}$, where $\mathscr{L}$ is an invariant lagrangian. To this end we make use of Dirac's method of homogeneous velocities [1]. Restricting

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[1] Such a four-dimensional superparticle may be called *phenomenological,* in contrast to the supersymmetrical superparticle discussed recently by W. Siegel, *Class. Quantum Grav.* **2**, L95 (1985); *Nucl. Phys.* **B263**, 93 (1986) (cf. also L. Brink, J. H. Schwarz, *Phys. Lett.* **100B**, 310 (1981); H. Terao, S. Uehara, *Z. Phys.* **C30**, 647 (1986)), that may be viewed as a construction representing the pointlike, zero-mode approximation to the ten-dimensional supersymmetrical superstring.

ourselves to the case of a superparticle interacting with an external electromagnetic field $A_\mu(x)$, our simple choice is

$$\mathscr{L} = -[m+M\bar{\psi}\psi+\tfrac{1}{2}F(\bar{\psi}\psi)^2+\tfrac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2]\sqrt{\dot{x}^2}-e\dot{x}\cdot A+i\bar{\psi}\frac{d}{d\tau}\psi, \tag{1}$$

where $\dot{f}(\tau) = df(\tau)/d\tau$ and $\gamma = (\gamma^\mu) = (\beta, \beta\vec{\alpha})$, while $g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$. Here, m, M, F and Λ are mass-dimensional nonnegative constants. The progress in this model may consist in establishing relations between these mass scales. Note that $\bar{\psi}(\tau)\gamma^\mu\psi(\tau)$ with $\bar{\psi}(\tau) = \psi^+(\tau)\beta$ is a four-vector whose all components are real. The lagrangian (1) is real up to the full derivative $-\frac{1}{2}id(\bar{\psi}\psi)/d\tau$ that vanishes when inserted into the Hamilton's principle $\delta\mathscr{A} = 0$ with fixed ends. Here, τ may be arbitrarily reparametrized leaving the action invariant, so it is not necessary to have $\dot{x}^2(\tau) = 1$.

From Eq. (1) we get the following canonical momenta conjugate with $x^\mu(\tau)$ and $\psi_\alpha(\tau)$:

$$p_\mu = -\frac{\partial\mathscr{L}}{\partial\dot{x}^\mu}$$

$$= [m+M\bar{\psi}\psi+\tfrac{1}{2}F(\bar{\psi}\psi)^2+\tfrac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2]\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}+eA_\mu \tag{2}$$

and

$$\pi_\alpha = \frac{\partial\mathscr{L}}{\partial\dot{\psi}_\alpha} = i\bar{\psi}_\alpha. \tag{3}$$

The Euler-Lagrange equations are

$$\frac{d}{d\tau}\left\{[m+M\bar{\psi}\psi+\tfrac{1}{2}F(\bar{\psi}\psi)^2+\tfrac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2]\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}\right\} = eF_{\mu\nu}\dot{x}^\nu \tag{4}$$

(or $\dot{p}_\mu = e\dot{x}^\nu\partial_\mu A_\nu$) and

$$\left\{i\frac{d}{d\tau}-[M+F\bar{\psi}\psi+\tfrac{1}{4}\Lambda(\bar{\psi}\gamma\psi)\cdot\gamma]\sqrt{\dot{x}^2}\right\}\psi = 0, \tag{5}$$

where $F_{\mu\nu}(x) = \partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)$. Note that Eq. (5) implies

$$\frac{d}{d\tau}(\bar{\psi}\psi) = 0, \quad \frac{d}{d\tau}(\bar{\psi}\gamma\psi)^2 = 0, \tag{6}$$

so that the effective mass in Eqs (1), (2) and (4) is τ-independent.

Since the lagrangian (1) is a first-order homogeneous function of $\dot{x}^\mu(\tau)$ and $\dot{\psi}_\alpha(\tau)$, the invariant hamiltonian

$$\mathscr{H} = -p_\mu\dot{x}^\mu+\pi_\alpha\dot{\psi}_\alpha-\mathscr{L} \tag{7}$$

vanishes in the weak sense. On the other hand, Eqs (1), (2), (3) and (7) lead to

$$\begin{aligned}\mathscr{H} &= \frac{\sqrt{\dot{x}^2}}{m+M\bar{\psi}\psi+\frac{1}{2}F(\bar{\psi}\psi)^2+\frac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2}\{-(p-eA)^2 \\ &\quad +[m+M\bar{\psi}\psi+\frac{1}{2}F(\bar{\psi}\psi)^2+\frac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2]^2\} \\ &= -\dot{x}\cdot(p-eA)+\sqrt{\dot{x}^2}\,[m+M\bar{\psi}\psi+\frac{1}{2}F(\bar{\psi}\psi)^2+\frac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2].\end{aligned} \tag{8}$$

Thus, we get the constraint

$$-(p-eA)^2+[m+M\bar{\psi}\psi+\frac{1}{2}F(\bar{\psi}\psi)^2+\frac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2]^2 = 0 \tag{9}$$

or

$$\begin{aligned}\mathscr{H} &\equiv -\dot{x}\cdot(p-eA) \\ &\quad +\sqrt{\dot{x}^2}\,[m+M\bar{\psi}\psi+\frac{1}{2}F(\bar{\psi}\psi)^2+\frac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2] = 0.\end{aligned} \tag{10}$$

Here, $\dot{x}^2(\tau)$ is arbitrary. One may choose $\dot{x}^2(\tau) = 1$.

2. Quantum superparticle

On the level of quantum mechanics (or the first-quantization level), the dynamical variables $x^\mu(\tau)$, $p_\mu(\tau)$ and $\psi_\alpha(\tau)$, $\bar{\psi}_\alpha(\tau)$ become in the Schrödinger picture the τ-independent operators x^μ, p_μ and ψ_α, $\bar{\psi}_\alpha$ satisfying the following commutation and anticommutation relations:

$$[x_\mu, p_\nu] = -ig_{\mu\nu}, \quad [x_\mu, x_\nu] = 0 = [p_\mu, p_\nu] \tag{11}$$

and

$$\{\psi_\alpha, \bar{\psi}_\beta\} = \delta_{\alpha\beta}, \quad \{\psi_\alpha, \psi_\beta\} = 0 = \{\bar{\psi}_\alpha, \bar{\psi}_\beta\} \tag{12}$$

(note that $\{\psi_\alpha, \psi_\beta^+\} = \beta_{\alpha\beta}$). In this picture, the state vector $\Psi(\tau)$ fulfills the state equation

$$i\frac{d}{d\tau}\Psi(\tau) = \mathscr{H}\Psi(\tau) \tag{13}$$

but, in fact, it is τ-independent, being subject to the "Klein-Gordon-type" constraint (cf. Eq. (9))

$$\{-(p-eA)^2+:[m+M\bar{\psi}\psi+\frac{1}{2}F(\bar{\psi}\psi)^2+\frac{1}{8}\Lambda(\bar{\psi}\gamma\psi)^2]^2:\}\Psi = 0 \tag{14}$$

or, alternatively, the "Dirac-type" constraint (cf. Eq. (10) with $\sqrt{\dot{x}^2(\tau)} = 1$)

$$\mathscr{H}\Psi \equiv [-\bar{\psi}\gamma\psi\cdot(p-eA)+m+M\bar{\psi}\psi+\frac{1}{2}F:(\bar{\psi}\psi)^2:+\frac{1}{8}\Lambda:(\bar{\psi}\gamma\psi)^2:]\Psi = 0. \tag{15}$$

Here, in the latter case, we *postulated*[2] that the dynamical variables $\dot{x}^\mu(\tau)$ became the operators $\bar{\psi}\gamma^\mu\psi$. Then, we get consistently

$$i\dot{x}^\mu \equiv [x^\mu, \mathscr{H}] = i\bar{\psi}\gamma^\mu\psi, \tag{16}$$

because $p_\mu = i\partial_\mu$ when the position representation is used (where $\Psi = \Psi(x)$). Similarly, we obtain

$$i\dot{p}_\mu \equiv [p_\mu, \mathscr{H}] = ie\bar{\psi}\gamma^\nu\psi\partial_\mu A_\nu \tag{17}$$

and

$$i\dot{\psi} \equiv [\psi, \mathscr{H}] = [-\gamma\cdot(p-eA)+M+F\bar{\psi}\psi+\tfrac{1}{4}\,\Lambda(\bar{\psi}\gamma\psi)\cdot\gamma]\psi, \tag{18}$$

where, of course, x^μ, p_μ and ψ_α are quantum dynamical variables in the Schrödinger picture (in the Heisenberg picture they become τ-dependent and then the dot denotes the real derivative $d/d\tau$). Note that the form of Eq. (18) differs from classical Eq. (5) (based on the lagrangian (1)) due to our "Dirac-type" postulate leading to the constraint (15). The forms of Eq. (17) and classical Eq. (4) are identical if we invoke Eq. (16).

The constraint (14) or, alternatively, (15) is the proposed wave equation for our superparticle. Here, the wave function $\Psi(x)$ can be represented in the "intrinsic Fock space" [2] whose basic vectors

$$\begin{aligned}
&\langle 0|,\\
&\langle\alpha| = \langle 0|\psi_\alpha,\\
&\langle\alpha_1\alpha_2| = \frac{1}{\sqrt{2!}}\langle 0|\psi_{\alpha_1}\psi_{\alpha_2},\\
&'\langle\alpha| = \varepsilon_{\alpha\alpha_1\alpha_2\alpha_3}\frac{1}{\sqrt{3!}}\langle 0|\psi_{\alpha_1}\psi_{\alpha_2}\psi_{\alpha_3},\\
&'\langle 0| = \varepsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\frac{1}{\sqrt{4!}}\langle 0|\psi_{\alpha_1}\psi_{\alpha_2}\psi_{\alpha_3}\psi_{\alpha_4}
\end{aligned} \tag{19}$$

correspond to the possible consecutive Fermi excitations $n \equiv \bar{\psi}\psi = 0, 1, 2, 3, 4$ resulting into spins $s = 0, 1/2, 0, 1/2, 0$, respectively (spin 1 turns out to be excluded by the antisymmetry of $\langle\alpha_1\,\alpha_2|$ in the bispinor indices α_1 and α_2). Here, $\langle 0|\bar{\psi}_\alpha = 0$, so the vector $\langle 0|$

[2] The classical formalism for our superparticle (especially for that of the Dirac-type where $\dot{x}^\mu(\tau) = \bar{\psi}(\tau)\gamma^\mu\psi(\tau)$) requires Grassmannian bispinor coordinates $\psi_\alpha(\tau)$ in order to properly introduce Poisson brackets involving spin degrees of freedom. Notice that in the classical formalism for the usual Dirac particle (thus *without* intrinsic spin excitations) Grassmannian pseudovector and pseudoscalar coordinates (rather than bispinor ones) are needed to go over, after the first quantization, into the Dirac matrices $\gamma_5\gamma^\mu$ and γ_5 being in this case spin quantum coordinates, cf. F. A. Berezin, M. S. Marinov, *Ann. Phys.* (N.Y.) **104**, 336 (1977) (I am indebted to Iwo Białynicki-Birula for calling my attention to this pioneering paper). Thus, in some sense, one may adopt Dirac's point of view that spin 1/2 has no classical analogy. However, there exists for our superparticle a level of the zeroth quantization, where $\psi_\alpha(\tau)$ is a number-valued wave function for *one* spin excitation (within a superparticle) and γ^μ are spin quantum coordinates (cf. Appendix).

is the vacuum bra with respect to Fermi excitations created by the operators ψ_α when acting on $\langle 0|$ (note that $\langle\alpha|\beta\rangle = \beta_{\alpha\beta}$ and hence $\sum_{\alpha,\beta} |\alpha\rangle\beta_{\alpha\beta}\langle\beta|$ is the unit operator in the "intrinsic Fock subspace" of one-excitation states $n = 1$).

Restricting ourselves to the case of the "Dirac-type" wave equation (15) and making use of the representation (19), we reduce this equation to the set of three nontrivial component wave equations corresponding, respectively, to $n = 1, 2, 3$ and so $s = 1/2, 0, 1/2$:

$$[-\gamma\cdot(p-eA)+m+M]\Psi^{(1)} = 0, \tag{20}$$

$$[-(\gamma_1+\gamma_2)\cdot(p-eA)+m+2M+F+\tfrac{1}{8}\Lambda(\gamma_1+\gamma_2)^2-\Lambda]\Psi^{(2)} = 0, \tag{21}$$

$$[\gamma^{\mathrm{T}}\cdot(p-eA)+m+3M+3F-\Lambda]\Psi^{(3)} = 0. \tag{22}$$

(Note that for $n = 0$ and $n = 4$ the kinetic term $\bar{\psi}\gamma\psi\cdot p$ in Eq. (15) vanishes.) In Eq. (21), γ_1^μ and γ_2^μ are two commuting sets of Dirac matrices, so that $\beta^\mu = \frac{1}{2}(\gamma_1^\mu+\gamma_2^\mu)$ are the Duffin-Kemmer-Petiau matrices [3] (but spin 1 does not appear because of the antisymmetry of $\Psi^{(2)}$ in the Dirac indices α_1 and α_2). In Eq. (22), $\gamma^{\mu\mathrm{T}}$ are the transposed Dirac matrices: $\gamma^{\mu\mathrm{T}} = -C^{-1}\gamma^\mu C$, where $C^+ = C^{-1}$ and $C^{\mathrm{T}} = -C$. Notice that there are *no transitions* between two spin-1/2 states $\Psi^{(1)}$ and $\Psi^{(3)}$, since $[\bar{\psi}\psi, \mathscr{H}] = 0$ and thus $n = \bar{\psi}\psi$ is conserved. This conclusion is true for any gauge interaction if introduced into Eq. (15).

In the case of a free superparticle where $A_\mu(x) = 0$, we obtain from Eqs (20)–(22) the Klein-Gordon equations $(p^2-m^{(n)2})\Psi^{(n)} = 0$, $n = 1, 2, 3$, with the masses

$$m^{(1)} = m+M, \tag{23}$$

$$m^{(2)2} = (\tfrac{1}{2}m+M+\tfrac{1}{2}F-\tfrac{1}{4}\Lambda)(\tfrac{1}{2}m+M+\tfrac{1}{2}F+\tfrac{1}{2}\Lambda), \tag{24}$$

$$m^{(3)} = |m+3M+3F-\Lambda|. \tag{25}$$

w $\frac{1}{2}m+M+\frac{1}{2}F \geqslant \frac{1}{4}\Lambda$, we get the real $m^{(2)}$. Otherwise, $m^{(2)}$ is imaginary and, then, the tate $n = 2$, $s = 0$ cannot physically exist (e.g., if $m \to 0$, $M = 0$ and $F = 0$ but $\Lambda > 0$, lse have such a case).

3. *Superparticle with dichotomic charge states*

Now, we would like to extend the model of our superparticle by introducing dichotomic charge states of such an object. To this end we engage one-dimensional Fermi annihilation and creation operators,

$$\hat{a} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{a}^+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{26}$$

satisfying the anticommutation relations

$$\{\hat{a}, \hat{a}^+\} = 1, \quad \{\hat{a}, \hat{a}\} = 0 = \{\hat{a}^+, \hat{a}^+\}. \tag{27}$$

Then, we define a new wave equation for our superparticle by making in the wave equation (15) the following substitution (for a comment cf. the passage preceding Eqs (48) and (49)):

$$\begin{aligned} e \to \hat{e} &= e\hat{N}\otimes\hat{I}, \\ \Lambda \to \hat{\Lambda} &= \Lambda\hat{N}\otimes\hat{I}, \\ M \to \hat{M} &= M\hat{N}\otimes\hat{N}, \\ F \to \hat{F} &= F\hat{N}\otimes\hat{N}. \end{aligned} \tag{28}$$

Here,

$$\hat{N} = \hat{a}^{+}\hat{a} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{29}$$

is the Fermi occupation number operator with two eigenvalues $N = 0, 1$ corresponding to the eigenbras $\langle u| = (1, 0)$ and $\langle d| = (0, 1)$, respectively ($\langle u|$ plays here the role of vacuum bra with respect to the Fermi excitation created by $\hat{a}$ when acting on $\langle u| : \langle u|\hat{a} = \langle d|$). In this way we introduce four-component wave function with two up-down degrees of freedom,

$$\Psi = \begin{pmatrix} \Psi_{uu} \\ \Psi_{du} \\ \Psi_{ud} \\ \Psi_{dd} \end{pmatrix}, \tag{30}$$

satisfying our new wave equation. This equation can be split into the following set of four wave equations for four components of $\Psi(x)$:

$$\begin{aligned} (-\bar{\psi}\gamma\psi \cdot p + m)\Psi_{uu} &= 0, \\ [-\bar{\psi}\gamma\psi \cdot (p - eA) + m + \tfrac{1}{8}\Lambda:(\bar{\psi}\gamma\psi)^2:]\Psi_{du} &= 0, \\ (-\bar{\psi}\gamma\psi \cdot p + m)\Psi_{ud} &= 0, \\ [-\bar{\psi}\gamma\psi \cdot (p - eA) + m + M\bar{\psi}\psi + \tfrac{1}{2}F:(\bar{\psi}\psi)^2: + \tfrac{1}{8}\Lambda:(\bar{\psi}\gamma\psi)^2:]\Psi_{dd} &= 0. \end{aligned} \tag{31}$$

In particular, the dd wave equation is identical with Eq. (15). Other Eqs (31) can be also identified with Eq. (15) for specific formal choices of e, Λ, M and F.

In the representation (19), each of the wave equations (31) reduces to a set of three nontrivial components corresponding to $n = 1, 2, 3$ and thus $s = 1/2, 0, 1/2$. These component wave equations can be easily obtained from Eqs (20)–(22) by formally specifying e, Λ, M and F. In this way we get four spin-1/2 charge doublets (all with charges 0 and e):

$$\begin{pmatrix} \Psi^{(1)}_{uu} \\ \Psi^{(1)}_{du} \end{pmatrix}, \quad \begin{pmatrix} \Psi^{(3)}_{uu} \\ \Psi^{(3)}_{du} \end{pmatrix}, \quad \begin{pmatrix} \Psi^{(1)}_{ud} \\ \Psi^{(1)}_{dd} \end{pmatrix}, \quad \begin{pmatrix} \Psi^{(3)}_{ud} \\ \Psi^{(3)}_{dd} \end{pmatrix}. \tag{32}$$

When $m \to 0$ and $3M + 3F \geqslant \Lambda$, their masses are (cf. Eqs (23) and (25))

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \Lambda \end{pmatrix}, \quad \begin{pmatrix} 0 \\ M \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 3M + 3F - \Lambda \end{pmatrix}, \tag{33}$$

respectively. There exist also one spin-0 neutral state $\Psi^{(2)}_{uu}$ with mass 0 (spin-0 charged state $\Psi^{(2)}_{du}$ develops an imaginary mass) and, when $M+\frac{1}{2}F \geqslant \frac{1}{4}\Lambda$, one spin-0 charge doublet

$$\begin{pmatrix} \Psi^{(2)}_{ud} \\ \Psi^{(2)}_{dd} \end{pmatrix} \tag{34}$$

with masses (cf. Eq. (24))

$$\begin{pmatrix} 0 \\ (M+\frac{1}{2}F-\frac{1}{4}\Lambda)^{1/2}(M+\frac{1}{2}F+\frac{1}{2}\Lambda)^{1/2} \end{pmatrix} \tag{35}$$

(and charges 0 and e).

We can conclude that our superparticle with two up-down degrees of freedom and so with two pairs of dichotomic charge states described by Eqs (31), may be interpreted as an approximate model for three known lepton doublets (ν_e, e^-), (ν_μ, μ^-) and (ν_τ, τ^-), predicting one extra lepton doublet which we will call (ν_ω, ω^-). Then, $m_e = 0$, $m_\mu = \Lambda$ and $m_\tau = M$, implying $m_\omega = 3M+3F-\Lambda = 3F+5246$ MeV > 5246 MeV (here, $0 = m \ll \Lambda \ll M$ and, presumably, $M \ll F$). In addition, there should appear one neutral scalar with mass 0 and one charge doublet of scalars with masses 0 and $(M+\frac{1}{2}F-\frac{1}{4}\Lambda)^{1/2}(M+\frac{1}{2}F+\frac{1}{2}\Lambda)^{1/2} > 1797$ MeV (and charges 0 and $e = -|e|$). To summarise, in the case of spin 1/2, each of two pairs of dichotomic charge states is split into two charge doublets $n = 1$ and $n = 3$ due to the Fermi intrinsic excitations created by ψ_α. It results into four spin-1/2 charge doublets (all with charges 0 and e).

Note that the new Fermi operators given by Eq. (26) lead to the Pauli matrices

$$\hat{\tau}_1 = \hat{a}+\hat{a}^+, \quad \hat{\tau}_2 = \frac{1}{i}(\hat{a}-\hat{a}^+), \quad \hat{\tau}_3 = \hat{I}-2\hat{a}^+\hat{a} \tag{36}$$

and thus to the familiar up and down projection operators

$$\tfrac{1}{2}(\hat{I}+\hat{\tau}_3) = \hat{a}\hat{a}^+, \quad \tfrac{1}{2}(\hat{I}-\hat{\tau}_3) = \hat{a}^+\hat{a}. \tag{37}$$

The conventional weak isospin is here given by the operator

$$\hat{\hat{T}} = (\tfrac{1}{2}\hat{\hat{\tau}}\otimes\hat{I})\tfrac{1}{2}(1+\gamma_5), \tag{38}$$

when acting on the component wave functions $\Psi^{(n)}(x)$ with $n = 1, 3$ and thus $s = 1/2$.

One may try to consider the term $M\bar{\psi}\psi+\frac{1}{2}F:(\bar{\psi}\psi)^2:$ in the wave equation (15) or the dd wave equation (31) as a part of a "neat" operator like, for instance,

$$M:\bar{\psi}e^{\lambda\bar{\psi}\psi}\psi: = M[\bar{\psi}\psi+\lambda:(\bar{\psi}\psi)^2:+\frac{1}{2!}\lambda^2:(\bar{\psi}\psi)^3:+\frac{1}{3!}\lambda^3:(\bar{\psi}\psi)^4:]$$

$$= M\bar{\psi}\psi(1+\lambda)^{\bar{\psi}\psi-1}, \tag{39}$$

where $n \equiv \bar{\psi}\psi = 0, 1, 2, 3, 4$ when acting on basic vectors in the representation (19). Here, $\lambda > 0$ is a number. For this particular example, $m_\tau = M$ always, but $m_\omega = 3\times(1+\lambda)^2 M-\Lambda > 5246$ MeV depends on λ. If e.g. $\lambda = 1$ or 2 one gets $m_\omega = 21.30$ GeV or 48.06 GeV, respectively[3]. If $\lambda = 1.31$, one obtains the relation

$$\frac{m_\omega - m_\tau}{m_\tau - m_\mu} \simeq \frac{m_\tau - m_\mu}{m_\mu - m_e} \simeq 16 \tag{40}$$

and then $m_\omega = 28.5$ GeV.

4. *One-dimensional supersymmetries*

Now, we should like to emphasize that by the substitution (28) we implicitly introduced to the model of our superparticle two commuting sets of generators satisfying the following superalgebraic relations [4]:

$$\{\hat{Q}, \hat{Q}^+\} = 2\hat{K}, \quad \{\hat{Q}, \hat{Q}\} = 0 = \{\hat{Q}^+, \hat{Q}^+\},$$
$$[\hat{Q}, \hat{K}] = 0 = [\hat{Q}^+, \hat{K}] \tag{41}$$

and

$$\{\hat{R}, \hat{R}^+\} = 2\hat{K}, \quad \{\hat{R}, \hat{R}\} = 0 = \{\hat{R}^+, \hat{R}^+\},$$
$$[\hat{R}, \hat{K}] = 0 = [\hat{R}^+, \hat{K}]. \tag{42}$$

Here, the Fermi generators are

$$\hat{Q} = (\hat{a} \otimes \hat{I})(i\bar{\psi}\gamma\psi \cdot p + m), \quad \hat{Q}^+ = (\hat{a}^+ \otimes \hat{I})(-i\bar{\psi}\gamma\psi \cdot p + m) \tag{43}$$

and

$$\hat{R} = (\hat{I} \otimes \hat{a})(i\bar{\psi}\gamma\psi \cdot p + m), \quad \hat{R}^+ = (\hat{I} \otimes \hat{a}^+)(-i\bar{\psi}\gamma\psi \cdot p + m), \tag{44}$$

while in both sets the Bose generator is

$$\hat{K} = (\hat{I} \otimes \hat{I})\,\tfrac{1}{2}\,[(\bar{\psi}\gamma\psi \cdot p)^2 + m^2]. \tag{45}$$

Since our wave equation (15) after the substitution (28) can be written as $\hat{\mathscr{H}}\Psi = 0$ with

$$\hat{\mathscr{H}} = (\hat{I} \otimes \hat{I})(-\bar{\psi}\gamma\psi \cdot p + m) + (\hat{N} \otimes \hat{N})\,[M\bar{\psi}\psi + \tfrac{1}{2} F{:}(\bar{\psi}\psi)^2{:}]$$
$$+ (\hat{N} \otimes \hat{I})\,[e\bar{\psi}\gamma\psi \cdot A + \tfrac{1}{8} \Lambda{:}(\bar{\psi}\gamma\psi)^2{:}], \tag{46}$$

it displays in the formal limit of $e \to 0$, $\Lambda \to 0$, $M \to 0$ and $F \to 0$ two one-dimensional *supersymmetries* generated by $\hat{Q}$, $\hat{Q}^+$, $\hat{K}$ and $\hat{R}$, $\hat{R}^+$, $\hat{K}$, respectively. It is so, because then

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[3] If requiring for the operator (39) the normalization $\langle n \rangle \equiv \sum_{n=0}^{4} :n \exp(\lambda n): / \sum_{n=0}^{4} :\exp(\lambda n): = 1$ where $n = \bar{\psi}\psi$, one gets $\lambda = 2.618$. In this case $m_\omega = 69.95$ GeV. Here, $\Omega \equiv \sum_{n=0}^{4} :\exp(\lambda n): = (1/\lambda) \times [(1+\lambda)^5 - 1]$ and $\partial\Omega/\partial\lambda \equiv \sum_{n=0}^{4} :n \exp(\lambda n): = (1/\lambda^2)\,[(4\lambda - 1)(1+\lambda)^4 + 1]$ so that $\langle n \rangle \equiv \partial \ln \Omega / \partial\lambda$ $(\lambda > 0)$. Then, the parameter λ is an analogon of $(\mu - \varepsilon)/kT$.

the hamiltonian $\hat{\mathscr{H}} \to (\hat{I} \otimes \hat{I})(-\bar{\psi}\gamma\psi \cdot p+m)$ commutes trivially with all above generators. Of course, both supersymmetries are explicitly broken by those terms in $\hat{\mathscr{H}}$ that depend on e, Λ, M and F. Both are connected with the proper time τ via their common Bose generator $\hat{K}$, since it can be defined by the hamiltonian $\hat{\mathscr{H}} \to (\hat{I} \otimes \hat{I})(-\bar{\psi}\gamma\psi \cdot p+m)$ generating τ-translations. In fact,

$$(\hat{I}\otimes\hat{I})\,\tfrac{1}{2}\,[(\hat{\mathscr{H}}-m)^2+m^2] \to \hat{K} \tag{47}$$

when $e \to 0$, $\Lambda \to 0$, $M \to 0$ and $F \to 0$.

Finally, it is perhaps worthwhile to stress that the substitution (28) displays an evident asymmetry between the first and the second up-down degree of freedom of our superparticle. Thus, *solely* on the aesthetic grounds, one may tentatively speculate that the hamiltonian $\mathscr{H}$ in the wave equation (15) should be supplemented by the term

$$e'\bar{\psi}\gamma\psi \cdot A' + \tfrac{1}{8}\,\Lambda' :(\bar{\psi}\gamma\psi)^2:, \tag{48}$$

where the substitution

$$\begin{aligned} e' &\to \hat{e}' = e'\hat{I}\otimes\hat{N}, \\ \Lambda' &\to \hat{\Lambda}' = \Lambda'\hat{I}\otimes\hat{N} \end{aligned} \tag{49}$$

should be applied, completing the previous substitution (28). Here, $A'_\mu(x)$ is a *new* Abelian neutral gauge field (treated in our considerations as an external field). Due to the substitution (49), the field $A'_\mu(x)$ is coupled only to the down states with respect to the second up-down degree of freedom i.e., to the states $\Psi^{(n)}_{ud}(x)$ and $\Psi^{(n)}_{dd}(x)$, $n = 1, 2, 3$. The term proportional to Λ' contributes only to the states $\Psi^{(n)}_{ud}(x)$ and $\Psi^{(n)}_{dd}(x)$ with $n = 2, 3$. In the case of our lepton interpretation of the spin-1/2 states $\Psi^{(n)}(x)$, $n = 1, 3$, the new field $A'_\mu(x)$ couples to the lepton doublets (ν_τ, τ^-) and (ν_ω, ω^-), while the term with Λ' gives additional masses to (ν_ω, ω^-), resulting into their total masses

$$\begin{pmatrix} \Lambda' \\ 3M+3F-\Lambda-\Lambda' \end{pmatrix} \tag{50}$$

(it is natural to assume that here $M \gg \Lambda'$ and, perhaps, $\Lambda' \simeq \Lambda$). Thus, in the case of the supplement given by Eqs (48) and (49) a new Abelian neutral gauge interaction should appear already for the known leptons of the third generation, ν_τ and τ^-. However, its strength $\alpha' = e'^2/4\pi$ would be a priori unknown (though, perhaps $\alpha' \simeq \alpha$ would be preferred on the aesthetic grounds).

So, in this paper we discussed our superparticle on the level of quantum mechanics. Of course, one may go over to the level of quantum field theory describing all Fock-space configurations of the superparticles interacting with quantum gauge fields. Then, radiative mass corrections appear for the spin-1/2 and spin-0 states considered in this paper. The masses discussed here may play the role of some *effective* masses. It seems to be especially true for the spin-1/2 states where radiative mass corrections are under control of the broken chiral symmetry.

APPENDIX

Is there a zero-quantization level for the superparticle?

As is well known, the quantum field theory may be also considered as the second--quantization level of the general quantum theory. In this case, classical fields (which after quantization become quantum fields) can be identified with the one-particle wave functions of the quantum mechanics representing then the first-quantization level of the general quantum theory.

At this point an intriguing question arises, whether some of the classical dynamical variables ascribed to a particle (which after quantization become its quantum dynamical variables) might be considered as wave functions characterizing a new, very primitive level of the general quantum theory. Such a level could be called the *zero-quantization* level [5], because its relation to the first-quantization level would be analogous to the relation between the first- and second-quantization levels.

In the case of our superparticle, the virtual candidate for such a zero-quantization wave function is the classical one-dimensional field $\psi_\alpha(\tau)$ for which, however, the normalization $\bar{\psi}(\tau)\psi(\tau) = 1$ must be then imposed. Such a zero-quantization wave function $\psi_\alpha(\tau)$ would describe *one* Fermi intrinsic excitation of spin 1/2 (within a superparticle), giving the probability amplitude for measuring the value $\alpha = 1, 2, 3, 4$ of the bispinor index α ascribed to this excitation (and determined by the eigenvalues of the operators $\sigma_3 = \gamma_5\gamma^0\gamma^3$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ or $\beta = \gamma^0$ treated as two zero-quantization commuting observables). Recall that $\alpha = 1, 2, 3, 4$ becomes, on the first-quantization level, the bispinor index of the component wave function $\Psi^{(1)}_\alpha(x) = \langle 0|\psi_\alpha|\Psi(x)\rangle$ where $n \equiv \bar{\psi}\psi = 1$.

If the classical one-dimensional field $\psi_\alpha(\tau)$ could be identified with the *one-excitation* wave function mentioned above, the classical equation of motion for $\psi_\alpha(\tau)$ would be the zero-quantization wave equation[4]. This classical equation of motion for $\psi_\alpha(\tau)$ can be read of from the quantum relation (18):

$$\left(i\frac{d}{d\tau} + \gamma\cdot(p-eA) - M - F(\bar{\psi}\psi) - \tfrac{1}{4}\Lambda(\bar{\psi}\gamma\psi)\cdot\gamma\right)\psi = 0, \tag{A.1}$$

where $p_\mu(\tau)$ and $\psi_\alpha(\tau)$ are classical one-dimensional fields and $A_\mu = A_\mu(x(\tau))$ (in Eq. (A.1) we chose $\dot{x}^2 = 1$). Here, we can consistently put $\bar{\psi}(\tau)\psi(\tau) = 1$ since Eq. (A.1) preserves the norm $\bar{\psi}(\tau)\psi(\tau)$ (note that also $[\bar{\psi}(\tau)\gamma\psi(\tau)]^2$ is preserved and is equal to 1 because of $\dot{x}^\mu(\tau) = \bar{\psi}(\tau)\gamma^\mu\psi(\tau)$). Moreover, the normalization $\bar{\psi}(\tau)\psi(\tau) = 1$ corresponds, on the first-

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[4] Strictly speaking, there is a formal difference between the classical one-dimensional field $\psi_\alpha(\tau)$ that should be a Grassmannian bispinor (cf. Footnote 2) and the zero-quantization wave function $\psi_\alpha(\tau)$ being a *number-valued* bispinor. This difference, however, is perfectly consistent with the fact that the zero-quantization wave function $\psi_\alpha(\tau)$, fulfilling the normalization condition $\bar{\psi}(\tau)\psi(\tau) = 1$, describes (quantally) one intrinsic excitation with spin $(1/2)\vec{\sigma}$, whilst the classical field $\psi_\alpha(\tau)$ should describe (classically) all such excitations with total spin $(1/2)\bar{\psi}(\tau)\vec{\sigma}\psi(\tau)$ which should satisfy the correct spin algebra realized through Poisson brackets.

-quantization level, to the states $\Psi^{(1)}(x)$ where $n \equiv \bar{\psi}\psi = 1$, for which the terms $F\colon (\bar{\psi}\psi)^2\colon$ and $\Lambda\colon (\bar{\psi}\gamma\psi)^2\colon$ vanish effectively (cf. Eq. (20) as well as Eqs (32) and (33)). Thus, in order to describe the classical counterparts of the first-quantization states $\Psi^{(1)}(x)$ where $n = 1$, it is natural to put $F = 0$ and $\Lambda = 0$ in Eq. (A.1), getting in this way the linear first order differential equation

$$i\frac{d}{d\tau}\psi(\tau) = \{-\gamma\cdot[p(\tau)-eA(x(\tau))]+M\}\psi(\tau) \tag{A.2}$$

that can be interpreted as a wave equation for the zero-quantization wave function $\psi_\alpha(\tau)$. Here, $\psi_\alpha(\tau)$ describes *one* Fermi intrinsic excitation of spin 1/2 (within a superparticle in a state $n = 1$) interacting with the classical "field" $p_\mu(\tau)-eA_\mu(x(\tau))$ defined along the world line of the superparticle. Given the phase-space trajectory $x^\mu = x^\mu(\tau)$ and $p_\mu = p_\mu(\tau)$, Eq. (A.2) can be used to calculate $\psi_\alpha(\tau)$. On the other hand, there must be

$$\bar{\psi}(\tau)\gamma^\mu\psi(\tau) = \dot{x}^\mu(\tau), \tag{A.3}$$

where $\dot{x}^2(\tau) = 1$. In particular, for a superparticle at rest $p_\mu(\tau) = g_{\mu 0}\overline{m}$ and $A_\mu(x) = 0$, so then the general solution to Eq. (A.2) is

$$\psi(\tau) = \tfrac{1}{2}(1+\beta)\psi(0)e^{-i(M-\overline{m})\tau}+\tfrac{1}{2}(1-\beta)\psi(0)e^{-i(M+\overline{m})\tau}, \tag{A.4}$$

where $\beta = \gamma^0$. Here, for $\psi_\alpha(\tau)$ within an electron e^-: $M = 0$, $\overline{m} = m_e = m$, and for $\psi_\alpha(\tau)$ within a tauon τ^-: $\overline{m} = m_\tau = m+M$ (cf. Eq. (23)), thus $M-\overline{m} = -m$ in both cases. Since $[\bar{\psi}(\tau)\gamma\psi(\tau)]^2 = 1$, we get from Eq. (A.4) the conditions $\bar{\psi}(0)\beta\psi(0) = 1$ and $\bar{\psi}(0)\vec{\gamma}\psi(0) = 0$ (beside $\bar{\psi}(0)\psi(0) = 1$). Hence, $\frac{1}{2}(1-\beta)\psi(0) = 0$ in Eq. (A.4).

Concluding, in the description of our superparticle there is some room for the zero-quantization level of the general quantum theory.

The following summary may be useful to compare the three levels of quantization for our superparticle (in the Schrödinger picture):

level	quantum coordinates	wave function for one physical object	physical object
zero	γ^μ	$\psi(\tau) = (\psi_\alpha(\tau))$	spin-1/2 excitation
first	$x^\mu,\ \psi = (\psi_\alpha)$	$\Psi(x) = \begin{pmatrix}\Psi_\alpha(x)\\ \Psi_{\alpha_1\alpha_2}(x)\\ \Psi_{\alpha_1\alpha_2\alpha_3}(x)\end{pmatrix}$	superparticle
second	$\Psi(\vec{x}) = \begin{pmatrix}\Psi_\alpha(\vec{x})\\ \Psi_{\alpha_1\alpha_2}(\vec{x})\\ \Psi_{\alpha_1\alpha_2\alpha_3}(\vec{x})\end{pmatrix}$	$\Psi_{\text{field}}([\Psi(\vec{x})], t)$	field of superparticles

where $\Psi_{\text{field}}([\Psi(\vec{x})], t)$ is a normalized functional of all histories of the quantum field of superparticles $\Psi(\vec{x})$ (this wave function can be represented in the (external) Fock space of superparticles).

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