ON APPLICATION OF EXOTIC COMMUTATOR METHOD TO BARYONS*

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The Exotic Commutator Method is applied to baryons in SU(3) and SU(4) symmetries. It is demonstrated that the maximal possible constraints on the masses and mixings follow from some finite set of exotic commutators. An enlargement of this set does not give additional constraints. A possible significance of such stable solutions is briefly discussed.

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1. Introduction

The Exotic Commutator Method (ECM) [1] has been introduced as a method of non-perturbative breaking of a flavour symmetry in the group theoretical approach. In application to mesons, besides of reproducing the known mass formulae, it is capable of describing some subtle properties of the mass spectrum [2–5]. For example, the weak violation of ideality of 2^{++} nonet can be explained with this method by an admixture of a glueball being an essential ingredient of the $f_2(1720)$ meson [3, 4].

The experience in investigation of SU(3) and SU(4) meson multiplets allows one for the following statements:

(*i*) The maximal possible ECM constraints on masses and mixings follow from some finite set of exotic commutators. An enlargement of this set (even to infinity) does not give additional constraints. We shall refer to thisproperty of ECM as the stability.

(*ii*) For irreducible multiplets the stable result is the full mass degeneracy. To avoid the mass degeneracy one must introduce the reducible representation.

(*iii*) For reducible representation (nonet of SU(3) and 16-plet of SU(4) symmetry) the stable result is an ideally mixed multiplet.

So, the ideal mixing of mesons can be regarded as a particular manifestation of the stability.

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We take now no account of the heuristic meaning of the algebraic (ECM) conditions defining the ideal mixing. But we would like to recall the pragmatic significance of the ideality for meson spectroscopy. Having formulated the conditions on ideal mixing one can investigate the origins of possible violations of ideality; this opens a possible way for investigation of the mass spectrum independently of the quark model. On the other hand the basis of the ideal states is very convenient in the quark model. In connection with OZI rule it allows one to explain why some decays are suppressed. Such suppression is often regarded as a definition of ideal mixing.

Unlike to the meson case, in the baryon one there are no such favoured states. There exists a concept of baryonic "ideal mixing" [6], but it is reserved for rather special situation (a suppression of some BBM vertices if one pair of baryon quarks is orbitally excited [7]).

The role of the favoured states (corresponding to the ideal ones for mesons) might be played by the baryonic stable solutions of ECM. In this sense we put the baryons and the mesons on the same grounds.

The aim of the present investigation is verification of the stability of ECM solutions foı baryons. We apply ECM to the lowest baryon multiplets appearing in SU(3) and SU(4) symmetries. We are also interested in checking the efficiency of ECM to predict intermultiplet mixings, especially the ones between nonsinglet multiplets (e.g. $20' \oplus 4$ in SU(4)). It is proved that the results are stable and the full degeneracy occurs neither for reducible nor for irreducible multiplets.

2. *The Exotic Commutator Method (ECM)*

Let us recall the essential points of ECM. We postulate vanishing of the following set of commutators:

$$[T_a, \dot{T}_b]_{\text{exot}} = 0,$$
$$[T_a, \ddot{T}_b]_{\text{exot}} = 0,$$
$$\dots\dots\dots\dots, \tag{2.1}$$

where $\{T_a\}$ are generators of the flavour group, $\{\dot{T}_a, \ddot{T}_a, \dots\}$ are their time derivatives and (a, b) are exotic combinations of indices. The sum rules are obtained in the $p \to \infty$ limit. Then the following substitutions may be done:

$$\dot{T}_a \to [m^2, T_a], \quad \ddot{T}_a \to [m^2, [m^2, T_a]], \text{ etc.},$$

where m^2 is operator of the mass squared. Only one particle external and intermediate states are used in calculation of the matrix elements of the commutators. All the states are taken from the multiplet (irreducible or reducible) of particles under consideration. The sum rules are of the form

$$\langle\alpha| [T_a, [m^2, T_b]] |\beta\rangle_{1-\text{part}} = 0,$$
$$\langle\alpha| [T_a, [m^2, [m^2, T_b]]] |\beta\rangle_{1-\text{part}} = 0,$$
$$\dots\dots\dots\dots\dots\dots\dots\dots \tag{2.2}$$

So, to receive the explicit form of the sum rules, only the matrix elements of T_a's must be calculated. The form of Eqs (2.2) leads to the mass formulae quadratic in masses for baryons as well as for the mesons. As in the case of meson multiplets [1] we follow step by step, each time discussing the effect of inclusion of the next constraint from Eqs (2.2). For the multiplets, we discuss, no more than three first commutators must be regarded.

3. *Mass formulae for baryons in $SU(3)_F$*

3.1. Decouplet

From the exotic commutators $[T_a, \dot{T}_b]_{\text{exot}} = 0$ we get the familiar equidistance formula

$$m^2 = m_0^2 + aS, \tag{3.1}$$

where S is strangeness. The commutators with higher derivatives do not disturb the formula.

3.2. Octet

a. Assuming $[T_a, \dot{T}_b]_{\text{exot}} = 0$, we get the GMO mass formula

$$2(\mathrm{N}+\Xi) = \Sigma + 3\Lambda,$$

where the particle symbol denotes its mass squared.
b. If $[T_a, \ddot{T}_b]_{\text{exot}} = 0$ are taken into account too, we obtain

$$\Sigma = \Lambda = \tfrac{1}{2}(\mathrm{N}+\Xi), \tag{3.2}$$

i.e.

$$m^2 = m_0^2 + aS. \tag{3.3}$$

c. Commutators with higher derivatives do not give farther restrictions.

3.3. Octet + singlet

a. From $[T_a, \dot{T}_b]_{\text{exot}} = 0$ we conclude that physical isosinglets Λ and Λ' must obey the inequalities

$$\Lambda \leqslant \Lambda_8 \leqslant \Lambda',$$

where

$$\Lambda_8 = \tfrac{1}{3}[2(\mathrm{N}+\Xi) - \Sigma]$$

is the mass squared of the octet isosinglet.
b. If commutators $[T_a, \ddot{T}_b]_{\text{exot}} = 0$ are taken into account too, we get the Schwinger mass formula for a baryon nonet

$$\Lambda\Lambda' = \Lambda_8(\Lambda + \Lambda' - \Lambda_8) - \tfrac{2}{9}(2\Sigma - \mathrm{N} - \Xi)^2. \tag{3.4}$$

c. If, in addition, we include into consideration the next commutators $[T_a, \dddot{T}_b]_{\text{exot}} = 0$, then we obtain

$$\Lambda = \quad , \quad \Lambda' = \mathrm{N} + \Xi - \Sigma. \tag{3.5}$$

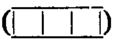
The mixing of the octet ($|\Lambda_8\rangle$) and singlet ($|\Lambda_1\rangle$) states is given by

$$\begin{pmatrix} |\Lambda\rangle \\ |\Lambda'\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \varepsilon \frac{1}{\sqrt{3}} \\ -\varepsilon \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} |\Lambda_8\rangle \\ |\Lambda_1\rangle \end{pmatrix}, \tag{3.6}$$

where $\varepsilon^2 = 1$.

d. Commutators with higher derivatives do not change the formulae (3.5) and, (3.6).

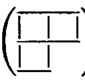
4. *Mass formulae for baryons in $SU(4)_F$*

4.1. 20-plet ()

From the commutators $[T_a, \dot{T}_b]_{\rm exot} = 0$ we get the mass formula with two characteristic intervals

$$m^2 = m_0^2 + aS + bC, \tag{4.1}$$

where C is charm. The commutators with higher derivatives do not disturb the formula.

4.2. 20′-plet ()

a. Sum rules obtained from the commutators $[T_a, \dot{T}_b]_{\rm exot} = 0$ give the following mass restrictions[1]

$$\Xi_c - N = Y - \Sigma,$$

$$\Xi - N = T - \Sigma_c, \tag{4.2}$$

$$2(N+\Xi) = 3\Lambda + \Sigma,$$

$$2(N+\Xi_c) = 3\Lambda_c + \Sigma_c, \tag{4.3}$$

$$A_0 = \tfrac{1}{2}(\Sigma + Y),$$

$$S_0 = \tfrac{1}{6}(4\Xi - 2\Sigma - N + 4\Sigma_c + \Xi_c),$$

$$\alpha = \frac{\sqrt{3}}{4}(\Lambda_c - \Sigma_c), \tag{4.4}$$

where A_0, S_0 and α are the matrix elements of the mass operator

$$\begin{pmatrix} A_0 & \alpha \\ \alpha & S_0 \end{pmatrix}. \tag{4.5}$$

[1] See Appendix, for symbols of the particles.

After diagonalization of this matrix, we obtain the physical masses

$$\begin{pmatrix} A \\ S \end{pmatrix} = \tfrac{1}{3}\left[a_1+a_2+a_3 \pm (a_1^2+a_2^2+a_3^2-a_1a_2-a_1a_3-a_2a_3)^{1/2}\right],$$

where

$$a_1 = \Sigma+\Xi_c-\Sigma_c, \quad a_2 = \Sigma-N+\Sigma_c, \quad a_3 = \Xi-\Sigma+\Sigma_c.$$

b. Taking into account also the commutators $[T_a, \ddot{T}_b]_{\text{exot}} = 0$, we observe that Eqs (4.2) remain unchanged, while the other mass formulae are

$$\Sigma = \Lambda = \tfrac{1}{2}(N+\Xi),$$

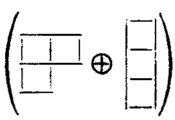
$$\Sigma_c = \Lambda_c = \tfrac{1}{2}(N+\Xi_c),$$

$$A = S = \tfrac{1}{2}(\Sigma+Y). \tag{4.6}$$

So, all the mass formulae can be rewritten in the form

$$m^2 = m_0^2+aS+bC. \tag{4.7}$$

c. The commutators with higher derivatives do not disturb the last equation.

4.3. ($20' \oplus 4$)-plet 

In this case we are faced with qualitatively new situation because, at the same time, there are mixings within different groups of particles: (Λ, Λ'), (Λ_c, Λ_c') and (A, S, S'). The corresponding mass matrices have the following forms:

$$\begin{pmatrix} \Lambda_0 & \lambda \\ \lambda & \Lambda_0' \end{pmatrix}, \quad \begin{pmatrix} \Lambda_{c0}' & \lambda_c \\ \lambda_c & \Lambda_{c0} \end{pmatrix}, \quad \begin{pmatrix} A_0 & \alpha & \beta \\ \alpha & S_0 & \sigma \\ \beta & \sigma & S_0' \end{pmatrix}, \tag{4.8}$$

where Λ_0, Λ_{c0}, A_0 and S_0 are members of 20'-plet, Λ_0', Λ_{c0}' and S_0' are members of 4-plet (see Appendix). Applying ECM we obtain:

a. From $[T_a, \dot{T}_b]_{\text{exot}} = 0$,

— the mass restrictions (4.2–4.4), where we must change $\Lambda \to \Lambda_0$ and $\Lambda_c \to \Lambda_{c0}$;

— the new restrictions:

$$\sigma = \lambda-\tfrac{1}{2}\lambda_c, \quad \beta = -\frac{\sqrt{3}}{2}\lambda_c. \tag{4.9}$$

As in the case of a pure 20'-plet, Eqs (4.2) are not disturbed by commutators with higher derivatives.

Despite of a large number of free parameters (λ, λ_c, Λ_0', Λ_{c0}', S_0') the breaking scheme has some predictive power. For example, having known the physical masses of Λ, Λ', Λ_c, Λ_c' and A, one can calculate the physical masses of S and S'.

b. Taking into account the commutators $[T_a, \ddot{T}_b]_{exot} = 0$, we get

$$\lambda = \varepsilon \frac{\sqrt{2}}{3}(2\Sigma - N - \Xi), \tag{4.10}$$

$$\lambda_c = \varepsilon \frac{\sqrt{2}}{3}(2\Sigma_c - N - \Xi_c), \tag{4.11}$$

($\varepsilon = \pm 1$; we have met with this parameter in the mixing matrix (3.6))

$$\Lambda'_0 = \tfrac{1}{3}(N + \Xi + \Sigma) + \delta,$$

$$\Lambda'_{c0} = \tfrac{1}{3}(N + \Xi_c + \Sigma_c) + \delta,$$

$$S'_0 = \tfrac{1}{3}(\Sigma + \Xi + \Sigma_c + \Xi_c - N) + \delta. \tag{4.12}$$

So, there remain only two free parameters: ε and δ. Diagonalizing the mass matrices (4.8) we obtain

(*i*) the SU(3) Schwinger formulae:

$$\Lambda\Lambda' = \Lambda_0(\Lambda + \Lambda' - \Lambda_0) - \lambda^2,$$

$$\Lambda_c\Lambda'_c = \Lambda_{c0}(\Lambda_c + \Lambda'_c - \Lambda_{c0}) - \lambda_c^2;$$

(*ii*) the "SU(4) Schwinger formulae"

$$ASS' = (A + S + S' - A_0 - S_0)(A_0 S_0 - \alpha^2) + 2\alpha\beta\sigma - A_0\sigma^2 - S_0\beta^2,$$

$$AS + AS' + SS' = (A + S + S' - A_0 - S_0)(A_0 + S_0) + A_0 S_0 - \alpha^2 - \beta^2 - \sigma^2. \tag{4.13}$$

These mass formulae are independent of ε. However, the mixing matrix transforming unphysical states into the physical ones does depend on ε.

Eliminating the parameter δ from the traces of the mass operators, we get the farther mass restrictions

$$\Lambda + \Lambda' + \Xi_c = \Lambda_c + \Lambda'_c + \Xi, \tag{4.14}$$

$$\Lambda + \Lambda' + \Xi_c + \Sigma + \Sigma_c = A + S + S' + 2N. \tag{4.15}$$

c. Including the commutators $[T_a, \dddot{T}_b]_{exot} = 0$, we obtain $\delta = 0$. Diagonalizing the mass operators (4.8), we get following physical masses:

$$\Lambda = \Sigma, \quad \Lambda' = N + \Xi - \Sigma,$$

$$\Lambda_c = \Sigma_c, \quad \Lambda'_c = N + \Xi_c - \Sigma_c,$$

$$A = \Sigma + \Xi_c - \Sigma_c,$$

$$S = \Sigma - N + \Sigma_c,$$

$$S' = \Xi - \Sigma + \Sigma_c. \tag{4.16}$$

The physical states Λ and Λ' are given by Eq. (3.6). The same mixing is valid for the states Λ_c and Λ_c'. The physical states A, S and S' are

$$\begin{bmatrix} |A\rangle \\ |S\rangle \\ |S'\rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \varepsilon\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\varepsilon\frac{1}{\sqrt{3}} \\ 0 & \varepsilon\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} |A_0\rangle \\ |S_0\rangle \\ |S_0'\rangle \end{bmatrix}. \tag{4.17}$$

d. Direct calculation shows that the commutators with derivatives of 4-th and 5-th order do not give farther restrictions.

5. *Summary and remarks*

(*i*) For irreducible multiplets (10,8 — in SU(3) and 20,20' — in SU(4)) we get, after one or two steps, the stable mass formulae (i.e. the ones that are not disturbed by commutators with higher derivatives):

$$m^2 = m_0^2 + aS,$$

$$m^2 = m_0^2 + aS + bC.$$

The formulae correspond to a simple quark counting ($m_u = m_d < m_s < m_c$). Unlike the case of irreducible meson multiplets (octet of SU(3), 15-plet of SU(4)) these formulae do not give an overall degeneracy of masses. So, in general, a mixing of representations is not necessary to avoid degeneracy. Nevertheless, some additional mechanism for explanation, for example, Σ–Λ splitting is desired.

(*ii*) Including a singlet, we obtain (for $8\oplus1$ multiplet of SU(3)) the stable mass formulae after three steps (like for the meson nonet).

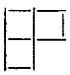
(*iii*) Including the nonsinglet representation 4 (for $20'\oplus4$ multiplet of SU(4)), we obtain the stable mass formulae probably after three steps too. The mixings between related particles of both multiplets may be determined. We hope that ECM will determine also the mixings occurring in the SU(6) × O(3) model (octet-octet, octet-decouplet and so on). The discussion will be given elsewhere.

We do not intent to compare the results with experimental data. The SU(3) mass formulae are well known and have been extensively discussed [8]. The SU(4) mass formulae cannot be verified because of the lack of experimental data on charmed baryons.

The authors are grateful to Dr. P. Kosiński for helpful discussions.

APPENDIX

We accept the following notations for the baryons belonging to 20′ and 4 multiplets of SU(4) symmetry (in brackets: isospin, strangeness and charm).

a. 20′-plet 

N, Λ, Σ, Ξ are baryons belonging to the octet of SU(3) symmetry.

The remaining baryons are indicated as

$$\Sigma_c(T=1,\ S=0,\ C=1),\quad \Lambda_c(T=0,\ S=0,\ C=1),\quad \Xi_c(T=1/2,\ S=0,\ C=2),$$

$$\mathrm{A}(T=1/2,\ S=-1,\ C=1),\quad \mathrm{S}(T=1/2,\ S=-1,\ C=1),$$

$$\mathrm{T}(T=0,\ S=-2,\ C=1),\quad \mathrm{Y}(T=0,\ S=-1,\ C=2).$$

b. 4-plet

$$\Lambda'(T=0,\ S=-1,\ C=0),\quad \Lambda'_c(T=0,\ S=0,\ C=1),$$

$$\mathrm{S}'(T=1/2,\ S=-1,\ C=1).$$

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