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REDUCTION OF COUPLINGS*

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Relations amongst coupling parameters consistent with the renormalization group are governed by a set of ordinary differential equations — the "reduction equations". Those are derived and then solved for a number of examples. They play a crucial role for constructing theories with β-functions vanishing to all orders. In the standard model they imply constraints on the Higgs and top quark mass and the mixing angles.

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1. *The notion of reduction, reduction equations* [1] [2] [3]

The aim of theoretical particle physics is the consistent description of particles i.e. of their properties and of their interactions. Its predictive power depends on the number of necessary free parameters. In the framework of quantum field theory the latter are given by coupling constants and masses which are — in renormalized perturbation theory — the number of interaction vertices of dimension less than or equal to four and the number of elementary fields, respectively. Restrictions on their number are usually obtained by imposing symmetries which relate different couplings or masses. The consistency problem is in this case the question whether the symmetry survives quantization or not: the study of anomalies for a given symmetry group. The answer to the question whether this is the only possibility of consistent relations amongst couplings is the subject proper of these lectures.

Suppose a model is given by its field content A, ψ, v_μ (spin 0, $\frac{1}{2}$, 1) and a set of power counting renormalizable vertices with couplings $\lambda_0, \lambda_1, \ldots, \lambda_n$. Then it is trivially possible to make the couplings λ_i to functions of a "fundamental" coupling g — which could be one of the λ's or a function thereof —

$$\lambda_i = \lambda_i(g) \quad i = 0, 1, \ldots, n \tag{1}$$

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namely at *one* normalization point. The non-trivial part is to check whether the same relations hold at any other normalization point. To find this out [1] one may either study directly effective couplings or proceed via the renormalization group equations — what we shall do here. For simplicity we discuss — up to a comment — only massless theories. We assume that for the models before and after reduction renormalization group equations hold

$$(\mu\partial_\mu + \sum_i \beta_{\lambda_i}\partial_{\lambda_i} - \gamma\mathcal{N})\Gamma(\lambda_0, \ldots, \lambda_n) = 0, \tag{2}$$

$$(\mu\partial_\mu + \beta_g\partial_g - \gamma^*\mathcal{N})\Gamma^*(g) = 0. \tag{3}$$

(Γ is the generating functional for vertex functions.) The transition from one theory to the other is provided by the definition

$$\Gamma^*(g) = \Gamma(\lambda_0(g), \ldots, \lambda_n(g)). \tag{4}$$

Using the chain rule and testing on suitable vertex functions one finds

$$\gamma^*(g) = \gamma(\lambda_0(g), \ldots, \lambda_n(g)), \tag{5}$$

$$\beta_g \frac{d\lambda_i}{dg} = \beta_{\lambda_i}, \quad i = 0, 1, \ldots, n. \tag{6}$$

The equations (6) will be called "reduction equations". They are not only necessary but also sufficient for having relations (1) (see [1]). An important special case is

$$g = \lambda_0 \Rightarrow \beta_g = \beta_{g0}, \tag{7}$$

$$\beta_g \frac{d\lambda_i}{dg} = \beta_{\lambda_i}, \quad i = 1, \ldots, n, \tag{8}$$

where the model lends itself to call one coupling the "primary" one. Let us comment on the massive case. The above derivation is valid as well — mutatis mutandis. But only if suitable off-shell normalization conditions allow for mass-independent β-functions will the latter define effective couplings amongst which (1) corresponds to the elimination of the scale variable. I.e. only under this additional hypothesis it is guaranteed that the relation amongst the couplings is normalization point independent.

Before proceeding to examples let us note that solving the reduction equations is non--trivial since the β-functions obtained in perturbation theory vanish with the couplings: the differential equations become singular at precisely that point where we ought to solve them. Let us also classify somewhat the possible solutions. A solution containing $n+1$ integration constants represents the general solution where no reduction has taken place: The original couplings have been traded for the integration constants. If we demand that the λ_i vanish for vanishing g i.e. we search for asymptotic solutions around zero, we shall have put a non-trivial restriction. The strongest postulate is to ask for

$$\lambda_i(g) = g(\varrho_i^{(0)} + \varrho_i^{(1)}g + \ldots) \tag{9}$$

i.e. power series expansion. If such a solution exists we have a perturbative model in one coupling constant and we shall speak of "complete" reduction. In practice solutions are obtained by starting with the strongest postulate (9) and searching then for more general ones by using stability theory.

2. *Examples*

2.1. One Dirac spinor, one pseudoscalar field [1]

The simplest and perhaps most striking example is given by the massless model of one Dirac spinor ψ and one pseudoscalar field B in renormalizable interaction:

$$\mathscr{L}_{\text{int}} = g\bar{\psi}B\gamma_5\psi - \frac{\lambda}{4!}B^4. \tag{10}$$

The β-functions are given to one loop order by

$$\beta_{g^2} = \frac{1}{16\pi^2}(5g^4 + \ldots), \tag{11}$$

$$\beta_\lambda = \frac{1}{16\pi^2}(\tfrac{3}{2}\lambda^2 + 4\lambda g^2 - 24g^4 + \ldots). \tag{12}$$

We try to solve the reduction equation

$$\beta_{g^2}\frac{d\lambda}{dg^2} = \beta_\lambda, \tag{13}$$

with the ansatz $\lambda = g^2(\varrho^{(0)} + \varrho^{(1)}g^2 + \ldots)$ and find as the only positive solution for small g

$$\lambda = \tfrac{1}{3}(1+\sqrt{145})g^2 + \varrho^{(1)}g^4 + \ldots . \tag{14}$$

The coefficients $\varrho^{(n)}$ are all uniquely determined. Hence we have reduced the above two--coupling parameter model to a perturbatively renormalizable one of one coupling. No symmtery is known for this system, hence symmetries need not to be the only tool for restricting couplings consistently. The general solution is given by

$$\lambda = g^2(\tfrac{1}{3}(1+\sqrt{145}) + \varrho^{(1)}g^2 + \varrho^{(2)}g^4 + c_1 g^{2\xi} + \ldots), \tag{15}$$

with $\xi = \frac{1}{5}\sqrt{145}$, c_1 arbitrary and all higher order terms — namely even powers of g and fractional powers formed with ξ are uniquely determined. This shows how the second coupling parameter enters the system: it vanishes asymptotically as $g^{2\xi}$ goes faster to zero than g^2.

The general analysis for theories with two couplings and β-functions like the above has been presented in [2] and [4]. A case with different type of β-functions is provided in the next example.

2.2. Two scalar fields [5]

The most general renormalizable interaction of two massless scalar fields is given by

$$\mathcal{L}_{\text{int}} = -\frac{1}{4!}(\lambda_0 A_1^4 + \lambda_4 A_2^4 + 6\lambda_1 A_1^2 A_2^2 + 4\lambda_1 A_1^3 A_2 + 4\lambda_3 A_1 A_2^3). \tag{16}$$

Here no coupling plays a prominent role hence the form (6) of the reduction equations is appropriate. They can be solved most easily by using $O(2)$-covariance and by regrouping $\mathcal{L}_{\text{int}}$ into representations of it. By suitable redefinition of the couplings one arrives at three solutions

(*i*)
$$\mathcal{L}_{\text{int}} = -\frac{g}{4!}(A_1^2 + A_2^2)^2, \tag{17}$$

(*ii*)
$$\mathcal{L}_{\text{int}} = -\frac{1}{4!}(\lambda_0' A_1'^4 + \lambda_4' A_2'^4), \tag{18}$$

(*iii*)
$$\mathcal{L}_{\text{int}} = -\frac{1}{4!}\lambda_0' A_1'^4, \quad A_2' \text{ free field}, \tag{19}$$

where

$$A' = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} A \tag{20}$$

and for

(*ii*)
$$\cos^2\varphi = \tfrac{1}{2} \pm \sqrt{\frac{1-\varrho_1}{4(1+\varrho_1)}}, \quad 0 \leqslant \varrho_1 \leqslant 1, \tag{21}$$

(*iii*)
$$\tan\varphi = \varrho_2 \text{ arbitrary}. \tag{22}$$

Thus the only theory with "maximal" interaction of all fields is the 0(2)-symmetric one. Reduction has produced a symmetry as solution.

2.3. Frame theory for $N = 2$ SYM [6]

We choose a multiplet A^a of real scalar, B^a of pseudoscalar, ψ^a of Dirac spinor and v_μ^a of vector fields transforming under the adjoint representation of, say, SU(2). Imposing for simplicity R-invariance we arrive at the following invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(D_\mu A^a)^2 + \tfrac{1}{2}(D_\mu B^a)^2$$
$$- i\bar{\psi}^a \gamma D \psi^a - i\sqrt{\lambda_1}\,\varepsilon^{abc}\bar{\psi}^a(A^b + \gamma_5 B^b)\psi^c - \tfrac{1}{4}\lambda_2(A^2+B^2)^2 + \tfrac{1}{4}\lambda_3(A^a A^b + B^a B^b)^2. \tag{23}$$

It has the field content of $N = 2$ pure SYM-theory in components and permits a non-supersymmetric embedding of it. Amongst the power series solutions of the reduction equations (8) we find indeed that one which is in the tree approximation $N = 2$ symmetric

$$(i) \qquad \varrho_1^{(0)} = \varrho_2^{(0)} = \varrho_3^{(0)} = 1, \tag{24}$$

but also one which does *not* have this symmetry:

$$(ii) \qquad \varrho_1^{(0)} = 1 \quad \varrho_2^{(0)} = \frac{9}{\sqrt{105}}, \quad \varrho_3^{(0)} = \frac{7}{\sqrt{105}}. \tag{25}$$

Both power series are uniquely determined to all orders. There exist still two other solutions, but they lead to negative classical potential.

Amongst the general solution there is only one for which $\lambda_i \to 0$ with $g \to 0$. It has the form

$$\lambda_1^{(a)} = g^2 + \sum_{m=4}^{\infty} g_{1m} d^m,$$

$$\lambda_2^{(a)} = g^2 + ag^3 + \sum_{m=4}^{\infty} d_{2m} g^m,$$

$$\lambda_3^{(a)} = g^2 + 3ag^3 + \sum_{m=4}^{\infty} d_{3m} g^m. \tag{26}$$

a is arbitrary, but all d_{im} are uniquely fixed. This solution is interesting because it represents hard breaking of $N = 2$ beginning with the odd power g^3 — compatible with renormalization.

2.4. Frame theory for $N = 4$ SYM [6]

We provide the field content of $N = 4$ SYM by the following set: 3 scalars A_i, 3 pseudoscalars B_i, 4 Majorana spinors ψ_K, 1 vector v_μ. All transform according to the adjoint representation of the gauge group — for simplicity — SU(2). In order to eliminate some couplings we require also a SU(2) × SU(2) rigid invariance, this results into the following Lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(D_\mu A_i^a)^2 + \tfrac{1}{2}(D_\mu B_i^a)^2 - \frac{i}{2}\bar{\psi}_K^a \not{D} \psi_K^a \\ &- \tfrac{1}{2}\sqrt{\lambda_1}\, \varepsilon^{abc} \bar{\psi}_K^a (\alpha_{KL}^i A_i^b + \gamma_5 \beta_{KL}^i B_i^b) \psi_L^c \\ &- \tfrac{1}{4}\lambda_2 (A_i^a A_i^a + B_i^a B_i^a)^2 + \tfrac{1}{4}\lambda_2 (A_i^a A_i^b + B_i^a B_i^b)^2. \end{aligned} \tag{27}$$

The reduction equations (8) yield two permissible unique power series solutions with lowest order coefficients

$$(i) \qquad \varrho_1^{(0)} = \varrho_2^{(0)} = \varrho_3^{(0)} = 1,$$

$$(ii) \qquad \varrho_1^{(0)} = 1, \quad \varrho_2^{(0)} = 0{,}7579, \ldots, \quad \varrho_3^{(0)} = 0{,}2523 \ldots \tag{28}$$

The former corresponds to the $N = 4$ SYM-theory, the latter does not have any known symmetry yielding such a relation amongst couplings in the tree approximation. It is to be noticed that due to the reduction equations its β-functions vanish too in the one-loop order. Hence it has also conformal symmetry in one loop.

3. *An application: finiteness conditions*

Supersymmetric gauge theories have interesting structural properties in addition to those coming from supersymmetry and gauge invariance. In particular their super-*conformal* covariance leads in a natural way to relations amongst anomalies which otherwise seem to be unrelated: namely those of a γ_5-symmetry (the so-called R-invariance) and the dilatations (see [7] for a systematic exposition). Since anomalies of γ_5-symmetries satisfy sometimes nonrenormalization theorems one might wonder if those should not imply consequences for the dilatation anomalies too. This is in fact true [8] and we shall now present the results of combining this structural analysis with the concept of reduction of couplings.

Our starting point is a general $N = 1$ SYM theory with simple gauge group G. It has the classical action

$$\Gamma = \mathrm{Tr}\int dS \frac{-1}{128g^2} F^\alpha F_\alpha + \tfrac{1}{16}\int dV \sum_R \bar{A}_R e^{\Phi_i T_R{}^i} A^R + \int D\, SU(A) + \int d\bar{S}\bar{U}(\bar{A}), \tag{29}$$

with $F^\alpha = \bar{D}\bar{D}(e^{-\Phi}D^\alpha e^{\Phi})$, $\Phi \equiv \Phi_i \tau^i$, R: irred. repres., U: superpotential

$$U = \lambda_{(rst)} A^r A^s A^t, \tag{30}$$

$r \equiv (R, \varrho)$ where R labels the irrep. and ϱ is the index in the representation space (see [7] for more details on notation). The β-functions of such theories read

$$\beta_g = b_0 g^3 + O(\hbar^2), \tag{31}$$

$$\beta_{\lambda_{rst}} = \lambda_{rsu}\gamma^{(1)u}{}_t + \lambda_{stu}\gamma^{(1)u}{}_r + \lambda_{tru}\gamma^{(1)u}{}_s + O(\hbar^2), \tag{32}$$

and our aim is to find necessary and sufficient conditions operating at one loop for them to vanish to all orders. We shall study first the necessary conditions.
Obviously

$$b_0 = 128r = 128(-3C_2(G) + \sum_R T(R)) = 0 \tag{33}$$

is necessary.

$$\begin{aligned} T(R)\delta^{ij} &= Tr(T_R^i T_R^j) && R\text{: irrep.} \\ T(R)\delta^{ij} &= C_2(G)\delta^{ij} = f^{ikl} f^{jkl} && R\text{: adjoint repr.} \end{aligned} \tag{34}$$

It requires a non-trivial proof [9] to show that also

$$\gamma^{(1)r}{}_s = \delta^\varrho{}_\sigma \gamma^{(1)R}{}_S = \bar{\lambda}^{ruv}\lambda_{suv} - \alpha g^2 \delta^r{}_s C_2(R) = 0 \tag{35}$$

(α: numerical constant) is necessary. But $\gamma^{(1)} = 0$ implies $\lambda_{rst} = \lambda_{rst}(g)$, hence reduction of couplings has to take place:

$$\beta_g \frac{d\lambda_{rst}}{dg} = \beta_{\lambda_{rst}}, \quad \beta_g \frac{d\bar{\lambda}_{rst}}{dg} = \beta_{\bar{\lambda}_{rst}}. \tag{36}$$

The last necessary condition is less obvious. Consider the counting operator

$$\mathcal{N}^S{}_R = \sum_{\varrho} \int A^{S\varrho} \frac{\delta}{\delta A^{R\varrho}} = \sum_{\varrho,\sigma} \delta^{\varrho}{}_{\sigma} \mathcal{N}^s{}_r = \sum_{\varrho,\sigma} \delta^{\varrho}{}_{\sigma} A^s \frac{\delta}{\delta A^r} \tag{37}$$

appearing in the Callan-Symanzik equation. Assume that a linear combination of them annihilates the superpotential

$$\text{for} \quad \mathcal{N}_{a0} = \sum_{R,S} e^R_{aS} \mathcal{N}^S{}_R: \quad \mathcal{N}_{a0} U = 0. \tag{38}$$

Then — by non-trivial proof [9]

$$r_{a0} = \sum_R e^R{}_R T(R) = 0. \tag{39}$$

Let us now turn to the sufficient conditions. Its key ingredient is the theorem alluded to above. For the anomaly coefficients of some axial currents and the β-functions of the theory holds the following equation:

$$r = \left(\frac{1}{128 g^3} + r_g\right) \beta_g + r_{\lambda\ldots} \beta_{\lambda\ldots} - \sum_a \gamma_{0a} r_{0a} + \gamma_k t_{ka} r_{0a}. \tag{40}$$

The coefficients r_g, r_λ, γ_{0a} are at least of order $\hbar$, r is given above and is precisely of one--loop order due to a non-renormalization theorem; r_{0a} starts with the one-loop contribution given above. It is precisely of one-loop order also, if its associated operators $\mathcal{N}_{0a}$ commute with the Callan-Symanzik operator

$$[\mathcal{N}_{0a}, \mathrm{CS}] = 0 \tag{41}$$

and if they are *all* operators satisfying

$$\mathcal{N}_{0a} U(A)_{|\lambda\ldots = \lambda\ldots(g)} = 0. \tag{42}$$

Here $\lambda\ldots(g)$ is any specific solution of (36). It is clear that $\beta_g = \beta_\lambda = \beta_{\bar{\lambda}} = 0$ to all orders if (33), (39), (41), (42) hold. For then (40) becomes homogeneous in β_g with invertible coefficient and vanishing l.h.s. — The simplest example is provided by the $N = 1$ supersymmetric embedding of the $N = 4$ SYM theory, others are discussed in [9]. The necessary and sufficient conditions for vanishing β-functions presented here constitute an easy recipe for finding such models.

4. *Conclusions*

The examples discussed in Sect. 2 and the use of reduction of couplings made in Sect. 3 for finding systematically theories with vanishing β-functions should make it clear that "reduction of couplings" yields new insight into seemingly well-known theories. It generates unexpected perturbatively renormalizable models like symmetries do, namely with less free parameters than the genuine theories. Via the study of stability of solutions it lends itself naturally to the analysis of asymptotic behaviour of effective couplings.

5. *Reduction in the standard model*

The standard model is phenomenologically extremely successful. It describes better than ever all known data. Theoretically too it is consistent and perturbatively well defined (perhaps up to problems with the Landau ghost coming from the U(1) subgroup), but it has unesthetically many free parameters. Hence it appears to be a good testing ground for the idea of reduction. Could it permit relations amongst couplings and thus be consistent with less parameters like the models in lecture I? If so, could we make predictions? Could we find limits for the validity of the standard model?

Let us first make somewhat more precise which model we are going to deal with. We consider the gauge group $SU(3)_c \times SU(2)_L \times U(1)$ (couplings α_s, α_2, α resp.), one Higgs doublet with self-interaction λ and n families of quarks and leptons. Their Yukawa couplings are complex $n \times n$ matrices G which can be decomposed as

$$\begin{aligned} G^{(l)} &= U^{(l)} H_L \quad && \text{l: Leptons} \\ G^{(u)} &= U^{(u)} H_U \quad && \text{u: up-quarks} \\ G^{(d)} &= U^{(d)} H_D \quad && \text{d: down-quarks} \end{aligned} \tag{43}$$

(U unitary, H Hermitean). For this first test of the ideas we simplify somewhat and perform the reduction in the completely massless model, calculate explicitly to one-loop and use these results in the *tree* approximation of the massive model. The tacit assumption made here is, of course, that higher orders (two loops and more) would not change the picture dramatically.

5.1. Reduction of gauge couplings [10]

The β-functions read in the one-loop approximation

$$\begin{aligned} \beta_\alpha &= (\tfrac{1}{10} + \tfrac{4}{3} n)\alpha^2 + \dots \\ \beta_{\alpha_2} &= (-\tfrac{43}{6} + \tfrac{5}{3} n)\alpha_2^2 + \dots \\ \beta_{\alpha_s} &= (-11 + \tfrac{4}{3} n)\alpha_s^2 + \dots . \end{aligned} \tag{44}$$

The abelian coupling is infrared free, the non-abelian ones are UV-free (for $n \leqslant 5$).

Trying now reduction between α_s and α i.e.

$$\alpha_s = \alpha(\alpha^{(0)}+\alpha^{(1)}\alpha+ \dots) \tag{45}$$

we find

$$\alpha^{(0)} = 0, \quad \alpha^{(0)} = -\tfrac{70}{41}\,(n = 3) \tag{46}$$

which is inconsistent (α has to be positive) — this inconsistency going hand in hand with the different asymptotic behaviour of α_s and α. Clearly, for the same reason α_2 cannot be reduced to α. Reducing α_2 to α_s

$$\alpha_2 = \alpha_s(\alpha_2^{(0)}+\alpha_2^{(1)}\alpha_s+ \dots) \tag{47}$$

we find

$$\alpha_2^{(0)} = \tfrac{42}{19}, \quad \alpha_2^{(0)} = 0\,(n = 3) \tag{48}$$

as possibilities. The latter value turns out to be inconsistent in the next order, the former leads to $\alpha_2 > \alpha_s$ which is phenomenologically unacceptable. We conclude that reduction of gauge couplings is impossible *within* the standard model, the Weinberg angle θ_W is indeed a free parameter. It is to be calculable then only in a model having more fields. This shows the sensitivity of the method to the model.

Of course, it is still conceivable that reduction is possible in the matter sector. There we then have to represent the gauge couplings by their solutions of the evolution equations with initial values chosen as to fit the data. I.e. we take

$$\alpha_2 = \tfrac{42}{19}\,\frac{\alpha_s}{1+c_2\alpha_s}, \quad \alpha = \tfrac{70}{41}\,\frac{\alpha_s}{c\alpha_s-1} \tag{49}$$

and fix c_2, c such that at $t \triangleq M_W$

$$\alpha_s = 0.1, \quad \alpha_s = 0.037, \quad \alpha = 0.016. \tag{50}$$

5.2. Reduction of matter couplings [10, 11]

The above form $\alpha_2 = \alpha_2(\alpha_s)$, $\alpha = \alpha(\alpha_s)$ has been chosen since a closer analysis shows that matter couplings (for $n \geqslant 3$) can neither be reduced to α_2 nor to α. Hence in accordance with numerical magnitudes we shall put α_2 and α equal to zero, perform reduction of the matter couplings to α_s and thereafter take into account non-vanishing α_2, α by a perturbation procedure.

The Hermitean factors H_1 in (43) can be diagonalized

$$H_U = U_A h_U U_A^+,$$

$$H_D = U_B h_D U_B^+, \tag{51}$$

and the diagonalizing unitary matrices give rise to the Kobayashi-Maskawa matrix

$$\mathrm{KM} = U_B^+ U_A. \tag{52}$$

The reduction equations read ($n = 3$)

$$-14\alpha_s^2 \frac{dH_L^2}{d\alpha_s} = 3H_L^4+3H_L^2\,\mathrm{Tr}\,(\ldots),$$

$$-14\alpha_s^2 \frac{dH_U^2}{d\alpha_s} = 3H_U^4-\tfrac{3}{2}\,\{H_D^2, H_U^2\}+2H_U^2(\mathrm{Tr}\,(\ldots)-8\alpha_s),$$

$$-14\alpha_s^2 \frac{dH_D^2}{d\alpha_s} = 3H_D^4-\tfrac{3}{2}\,\{H_D^2, H_U^2\}+2H_D^2(\mathrm{Tr}\,(\ldots)-8\alpha_s),$$

$$-14\alpha_s^2 \frac{d\lambda}{d\alpha_s} = 6\lambda^2+4\lambda\,\mathrm{Tr}\,(\ldots)-8\,\mathrm{Tr}\,(H_L^4+3H_D^4+3H_U^4),$$

$$\mathrm{Tr}\,(\ldots) \equiv \mathrm{Tr}\,(H_L^2+3H_D^2+3H_U^2). \tag{53}$$

Hence they restrict H_U, H_D and thus constrain in principle the couplings (→ masses) and the mixing angles.

As usual we first look for power series solutions

$$H^2 = \alpha_s(A^{(0)}+A^{(1)}\alpha_s+A^{(2)}\alpha_s^2+\ \ldots) \tag{54}$$

We find

$$H_L^2 = 0, \quad \mathrm{KM} = 1, \tag{55}$$

and as only reasonable solution

$$H_D^2 = 0, \quad H_U^2 = \begin{bmatrix} 0 & & \\ & 0 & \\ & & 2/9 \end{bmatrix} \alpha_s + \ \ldots \tag{56}$$

Taken at face value this means

$$m_l = 0 \qquad \text{l: leptons}$$

$$m_q = 0 \qquad \text{q} \neq \text{top, q: quarks}$$

$$m_t = 90\ \mathrm{GeV}$$

$$m_H = 50\ \mathrm{GeV} \tag{57}$$

and no mixing. Calculating by the above mentioned perturbation procedure weak and electromagnetic corrections while keeping the mixing angles zero [10] one finds

$$m_t = 81\ \mathrm{GeV}, \quad m_H = 63\ \mathrm{GeV}, \tag{58}$$

and the other masses can be fitted.

Let us now study the *general* asymptotic solution around (55)–(56) [11]. The stability analysis yields the possible fractional powers of α_s. They turn out to be rational and the solutions have the form

$$H_U^2 = \alpha_s(\tfrac{2}{9} A^{(0)} + A^{(1)}\alpha_s^{1/21} + A^{(2)}\alpha_s^{2/21} + \dots),$$
$$H_D^2 = \alpha_s(B^{(1)}\alpha_s^{1/21} + B^{(2)}\alpha_s^{2/21} + \dots) \tag{59}$$

without logarithms. The general form of the matrices A, B is the following

$$A, B: \left(\begin{array}{cc|c} * & & 0 \\ & & 0 \\ \hline 0 & 0 & * \end{array}\right). \tag{60}$$

Hence no mixing between family 3 and families 1, 2! Arbitrary coefficients are: $a_{11}^{(1)}$, $a_{12}^{(1)}$, $a_{21}^{(1)}$, $a_{22}^{(1)}$, $b_{11}^{(1)}$, $b_{12}^{(1)}$, $b_{21}^{(1)}$, $b_{22}^{(1)}$, $b_{33}^{(2)}$. They are sufficient to fit all other quark masses. The Cabbibo angle is also unconstrained. The above value of the top is changed only by a negligible amount (same for Higgs). We have not yet calculated the weak and electromagnetic corrections to this solution.

The reduction equations (53) permit also the trivial solution $H_L = H_U = H_D = 0$ and we have to check how it is related to the nontrivial one above. The general asymptotic solution around zero turns out to have the form

$$H_U^2 = \alpha_s(A^{(1)}\alpha_s^{1/7} + A^{(2)}\alpha_s^{2/7} + \dots),$$
$$H_D^2 = \alpha_s(B^{(1)}\alpha_s^{1/7} + B^{(2)}\alpha_s^{2/7} + \dots) \tag{61}$$

(again without logarithms). $A^{(1)}$ and $B^{(1)}$ are arbitrary, all other $A^{(i)}$, $B^{(i)}$ $i \geq 2$ are uniquely determined. Hence there are enough free parameters to fit all quark masses and mixing

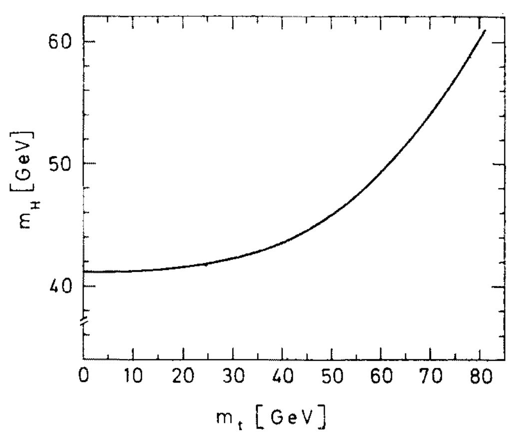

Fig. 1

angles. The Higgs mass is a function essentially of the top mass (see Fig. 1). Since the non--trivial reduction solution provides a limit for the range of the solution around zero there should exist *bounds* for the parameters in $A^{(1)}$, $B^{(1)}$.

5.3. Discussion

Experimentally the top mass can still lie anywhere above 45 GeV, the Higgs mass is essentially unconstrained [12], hence we can play with different assumptions and check their implications. Suppose one finds the top mass to be above 90 GeV. Since we estimate the uncertainty in (58) to be about 10–15% and 90 GeV would be above the upper limit for m_t our calculations would imply that more particles are needed e.g. a fourth generation. If the top is found with $m_t < 90$ GeV, the value of the Higgs mass decides on whether the three family model is appropriate. If it lies on its curve (see Fig. 1) there can be little doubt that other particles — not foreseen in the standard model — have to be very heavy. If it is far away from its curve, again, one needs additional particles nearby in mass. E.g. a fourth generation with $m_{t'} \approx 110$ GeV could easily afford a Higgs mass of 220 GeV.

We conclude that the concept of reduction reveals also in the standard model interesting structures in the solution space of the effective couplings. If its favoured solution, (59), (58), would be realized one could start speculating on the physical mechanism which was its cause.

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