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THERMAL MODIFIED THOMAS-FERMI APPROXIMATION WITH THE SKYRME INTERACTION FOR THE $^{208}Pb + ^{208}Pb$ SYSTEM

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A generalization of the modified Thomas-Fermi (MTF) approximation to finite temperatures is used to calculate the optical potential for the $^{208}Pb + ^{208}Pb$ system using the energy density formalism derived from different effective forces of Skyrme type. The nuclear optical potential becomes more attractive when the temperature is increased. Pockets are also predicted in the total potential (Nuclear + Coulomb) whose depths are dependent on both the type of effective force and the temperature.

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1. *Introduction*

Several attempts have been made to derive the nucleus-nucleus potential from an effective two body interaction using e.g. the proximity approximation [1] and the double folding model [2–4]. For spherical nuclei although the double folding predicts qualitative features of the elastic scattering data it requires a renormalization of the strength of the real potential to about one half of its value at the strong absorption radius to get quantitative agreement [5]. To account for such discrepancy one may add the effect of exchange forces due to antisymmetrization and the saturation effects.

Another method to calculate the nucleus-nucleus potential is by using a Hamiltonian energy density derived from density dependent effective interactions [6–8]. This method takes into account the effect of saturation of nuclear forces and the exchange effects due to antisymmetrization [9, 10]. In the present work we use Skyrme force SK III as an effective interaction which has been successfully applied to many problems in nuclear structure calculations. The usual Skyrme-forces like SK III have a too large compressibility K. This value lies between 300 and 400 MeV. In reality we know from the monopole breathing mode that the incompressibility is around 200 MeV. Therefore, besides SK III, we used an extended version of it SK E which is constructed to cover more features of nuclear structure [11]. Besides the original Skyrme type of force the last one contains in addition a momentum dependent three-body term in order to describe wider features of the excited states of nuclei and nuclear matter properties. In the present work we show the effect of this momentum dependent three-body term on the ion-ion potential.

In heavy ion collisions part of the relative kinetic energy between the two nuclei is transformed in excitations of the nuclei. This means that the nuclei are getting hot. By this the interaction between the two nuclei is modified. The modification is a change of the kinetic energy density of the nucleus in the two nuclei and also a change of the mass distribution of the two nuclei. In addition one expects also a change of the effective interaction. The interaction potential between two heavy ions has revealed the existence of dissipative phenomena [12–16], e.g. fusion and deep inelastic reactions. In the deep inelastic process the two scattered nuclei emerge after the reaction with a total kinetic energy smaller than the initial one whereas in the fusion process the system remains trapped into a pocket of the potential energy surface. The purpose of the present work is twofold. First of all the ion-ion interaction potential is calculated for two types of Skyrme force namely SK III force and SK E force, then we compare between the optical potential obtained by the famous SK III force and its extended version with a momentum dependent three-body force. The second aim is to investigate in an approximate way the effects of the temperature on the interaction potential between two heavy ions for both types of force. The method is very simple and it reproduces similar results to those obtained by other authors. This method is based on a Taylor expansion for the total energy of the considered system to second order of powers of T^2 [17].

For heavy ions using the energy density formalism at finite temperature and the sudden approximation it has been found that the real part of the optical potential change when the temperature of two ions increases [18]. Here we used a generalization to finite temperature of the modified Thomas-Fermi approximation [17] adopting different effective forces of the Skyrme type to study the temperature dependence of the ion-ion potential. Specifically we have chosen to study the $^{208}Pb + {}^{208}Pb$ system. In the next Section we briefly describe the method of calculation and in Section 3 the results are presented.

2. *Theory*

The real part of the ion-ion potential in the sudden approximation as a function of the separation distance R between the centers of the two colliding nuclei is written as

$$V(R) = \int [H(\varrho, \tau) - H(\varrho_1, \tau_1) - H(\varrho_2, \tau_2)] d\vec{r}, \tag{1}$$

where ϱ_i and τ_i $(i = 1, 2)$ are, respectively, the density distributions and the kinetic energy densities of the two separated nuclei. ϱ and τ are the same quantities for the composite system. $H(\varrho, \tau)$ is the energy density functional of the composite system and $H(\varrho_1, \tau_1)$ and $H(\varrho_2, \tau_2)$ are the energy density functionals of the two separated nuclei. In the sudden approximation the density of the composite system $\varrho = \varrho_1 + \varrho_2$. Hence, using SK III force [19] the energy density of a nucleus may be expressed in terms of the neutron and proton densities ϱ_n and ϱ_p, their gradients $\vec{\nabla}\varrho_n$ and $\vec{\nabla}\varrho_p$, and the kinetic energy densities τ_n and τ_p [20], viz:

$$H = \frac{\hbar^2}{2m}(\tau_n+\tau_p)+\tfrac{1}{2}t_0[(1+\tfrac{1}{2}x_0)\varrho^2-(x_0+\tfrac{1}{2})(\varrho_n^2+\varrho_p^2)]$$
$$+\tfrac{1}{4}(t_1+t_2)\varrho(\tau_n+\tau_p)+\tfrac{1}{8}(t_2-t_1)(\varrho_n\tau_n+\varrho_p\tau_p)$$
$$+\tfrac{1}{16}(t_2-3t_1)\varrho\nabla^2\varrho+\tfrac{1}{32}(3t_1+t_2)(\varrho_n\nabla^2\varrho_n+\varrho_p\nabla^2\varrho_p)+\tfrac{1}{4}t_3\varrho_n\varrho_p\varrho. \quad (2)$$

The density of the ^{208}Pb nucleus is represented by the Fermi-type distribution

$$\varrho(r) = \varrho_0\Big/\left[1+\exp\left(\frac{r-R_0}{a}\right)\right], \quad (3)$$

where the parameters are taken from Stancu and Brink [7]. For the proposed extension of the Skyrme force, SK E, an extra term is added to the above Hamiltonian density [11], i.e.

$$H_{W_1} = \tfrac{1}{24}t_4[-\varrho_n\varrho_p\nabla^2\varrho-\tfrac{1}{2}\varrho_p^2\nabla^2\varrho_n-\tfrac{1}{2}\varrho_n^2\nabla^2\varrho_p+2\tau\varrho_n\varrho_p$$
$$+\tau_n\varrho_p^2+\tau_p\varrho_n^2+\tfrac{1}{2}\varrho\vec{\nabla}\varrho_p\cdot\vec{\nabla}\varrho_n+\tfrac{1}{2}\varrho_p(\vec{\nabla}\varrho_n)^2+\tfrac{1}{2}\varrho_n(\vec{\nabla}\varrho_p)^2]. \quad (4)$$

The parameters used in the present work for SK III, and two types of SK E, namely, SK E1 and SK E2 forces are listed in Table I.

TABLE I

The Skyrme force parameters

	t_0 MeV fm^3	t_1 MeV fm^5	t_2 MeV fm^5	t_3 MeV fm^6	t_4 MeV fm^8	x_0	Incompressibility K (MeV)
SK III	−1128.75	395.00	−95.00	14000.00	0	0.450	356
SK E1	−1272.76	806.08	−30.40	15065.68	−11727.51	0.158	230
SK E2	−1299.30	802.41	−67.89	19558.96	−15808.79	0.270	200

For the kinetic energy density instead of using the usual TF functional

$$\tau_q = \tfrac{3}{5}(3\pi^2)^{2/3}\varrho_q^{5/3} \qquad q = n, p \quad (5)$$

we used the modified TF functional [21] which is written as

$$\tau_q = \alpha(A_q)\varrho_q^{5/3}+\beta(\vec{\nabla}\varrho_q)^2/\varrho_q, \quad (6)$$

with

$$\alpha(A_q) = \alpha(\infty)\tanh\sqrt{5/3}\,(A_q-2)^{1/5}, \qquad \alpha(\infty) = \tfrac{3}{5}(3\pi^2)^{2/3},$$
$$\beta = \tfrac{1}{9}\left(\frac{m^*}{m}+\tfrac{1}{4}\right),$$

where m^*/m is the value of the effective mass at the saturation point of symmetric nuclear matter. Equation (1) defines the interaction potential between two ions at zero temperature.

To extend this calculation for higher temperatures, we will follow the same method of Barranco and Treiner [17]. Hence each term in Eq. (1) will have a correction to be added to it, i.e.

$$\int H^T(\varrho, \tau)d\vec{r} = \int H(\varrho, \tau)d\vec{r} + aT^2, \tag{7}$$

where $H^T(\varrho, \tau)$ is the energy density functional at temperature T and

$$a = \frac{\pi^2}{4} \sum_q \frac{\alpha(A_q)}{\alpha(\infty)} \int \varrho_q(r) \frac{2m_q^*}{\hbar^2 K_{F_q}^2} d\vec{r},$$

$$K_{F_q} = [3\pi^2 \varrho_q(r)]^{1/3}, \tag{8}$$

and the potential at finite but small temperatures would be defined as

$$V(R, T) = \int [H^T(\varrho, \tau) - H_1^T(\varrho_1, \tau_1) - H_2^T(\varrho_2, \tau_2)]d\vec{r}. \tag{9}$$

R is defined previously as the vector joining the centers of both nuclei.

The main approximations here for the nuclear part of the ion-ion potential are as follows:

(*i*) We have used the energy density formalism and the modified Thomas-Fermi expression for the kinetic energy density. The MTF method allows one to obtain densities with a realistic diffuseness.

(*ii*) We used two effective interactions of the Skyrme type SK III and SK E which are assumed to be temperature independent as a first approximation.

The Coulomb potential between the two ^{208}Pb nuclei is calculated using the double folding model. The six dimensional integral has been transformed to a term which contains one dimensional integrals using the procedure presented by Rhoades-Brown et al., [4].

3. *Results and discussion*

To illustrate the temperature dependence of the $^{208}Pb + ^{208}Pb$ optical potential we have considered two types of the Skyrme force SK III and a modified version of it with two sets of parameters SK E1 and SK E2. The reason for taking the last two sets is that besides their good reproduction of the excited states of finite nuclei they also produce good values for the compressibility and effective mass in nuclear matter. The calculations are performed at four different temperatures: $T = 0, 1, 2, 3$ MeV. The results are presented in Figs 1–7.

One can observe that the optical potential becomes more and more attractive when the temperature of the nuclei increases. This is in agreement with previous calculations [18, 22]. The convergence of the expansion in Eq. (7) has been tested and then it was found that the second term is generally less than 1% for $T = 1$; 4% for $T = 2$ and 9% for $T = 3$

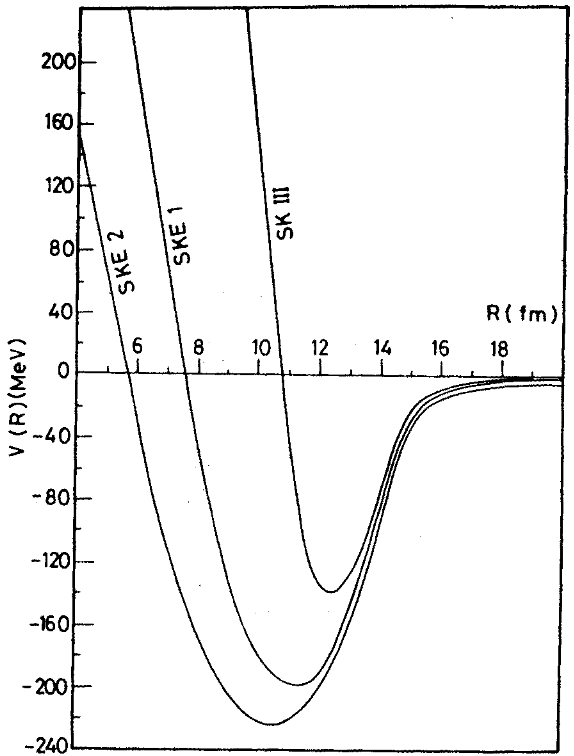

Fig. 1

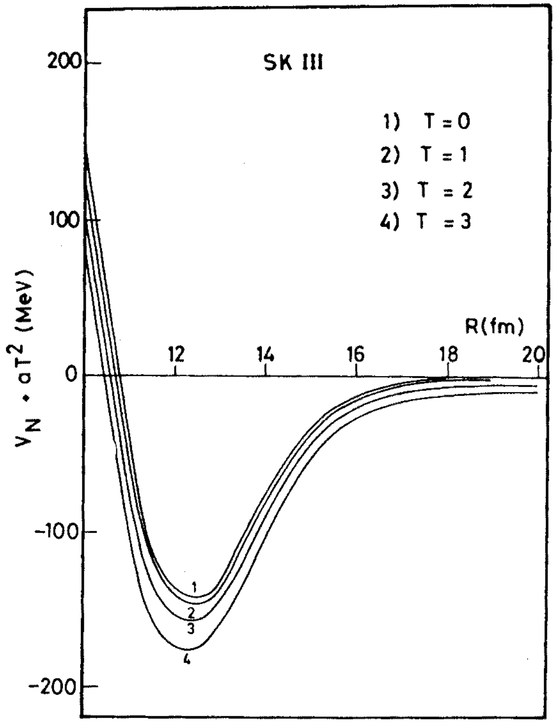

Fig. 2

Fig. 1. The nuclear ion-ion potential for $^{208}Pb + ^{208}Pb$ system as a function of the distance R between the two ions at zero temperature for different Skyrme forces

Fig. 2. Ion-ion nuclear potential as a function of the distance R at different temperatures for the Skyrme force SK III

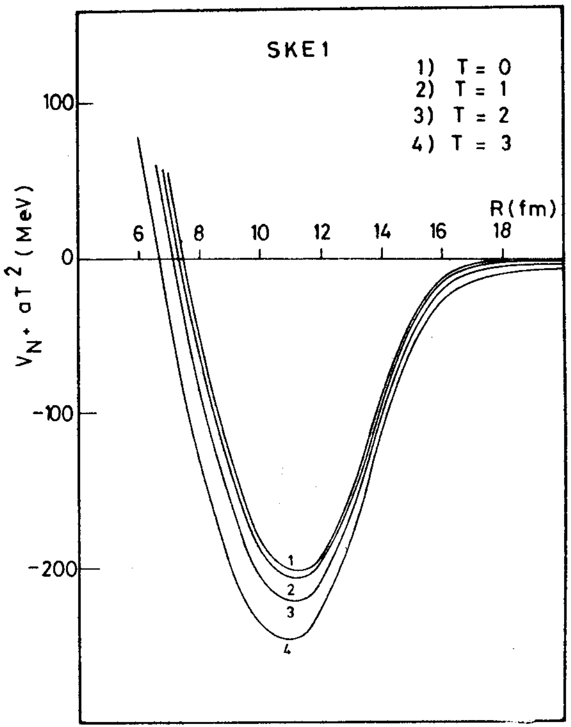

Fig. 3

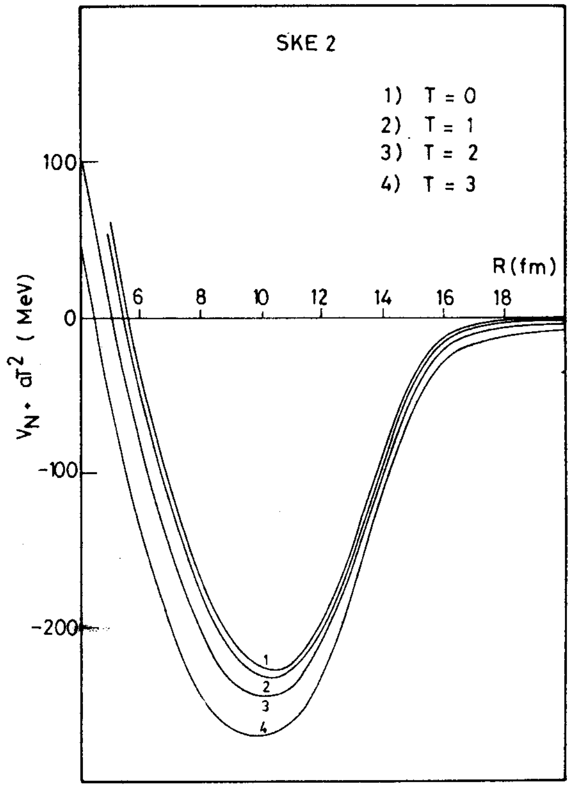

Fig. 4

Fig. 3. Same as Fig. 2 for the Skyrme force SK E1

Fig. 4. Same as Fig. 2 for the Skyrme force SK E2

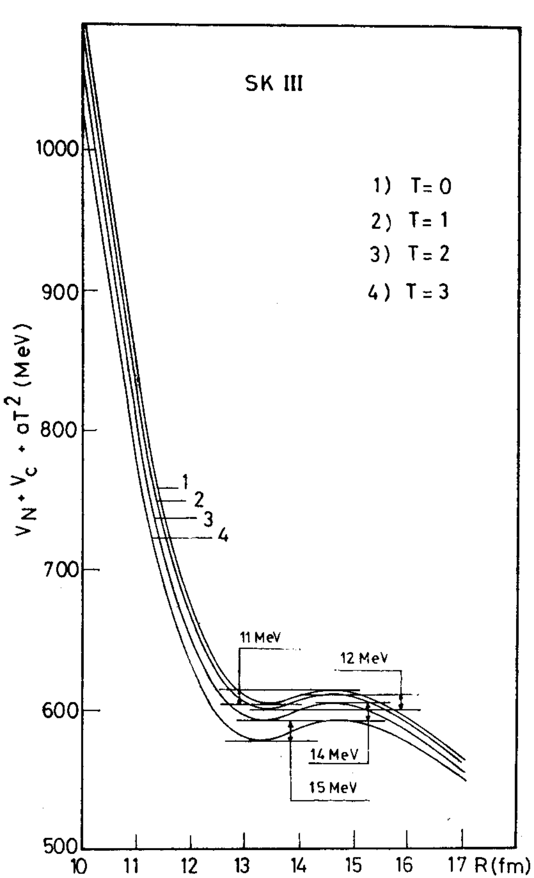

Fig. 5

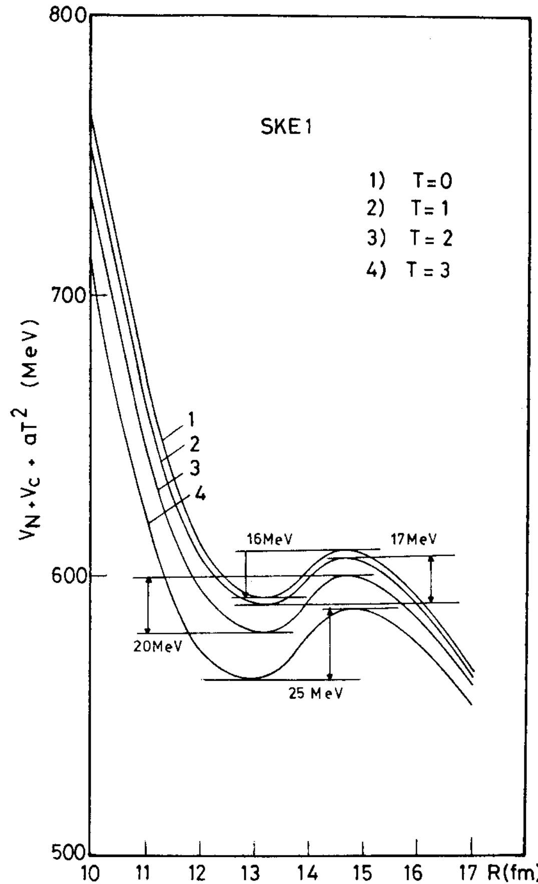

Fig. 6

Fig. 5. Ion-ion (Nuclear + Coulomb) potential as a function of distance R at different temperatures for the Skyrme force SK III

Fig. 6. Same as Fig. 5 for the Skyrme force SK E1

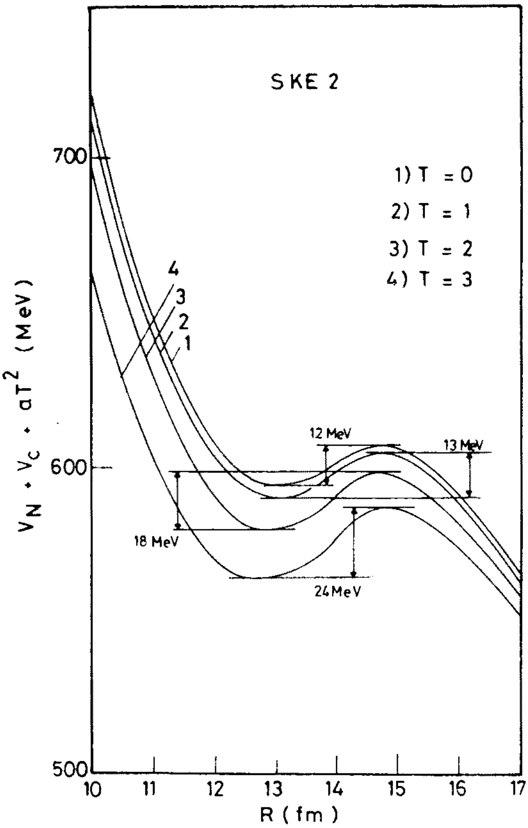

Fig. 7. Same as Fig. 5 for the Skyrme force SK E2

such that a T^4 term is not needed in this calculation. We notice also that the depths of the pockets produced in the total potential vary with the temperature. We also note here that we neglect the temperature dependence of the effective interaction.

In Fig. 1 we compare the nuclear potential for the three different Skyrme potentials SK III, SK E1 and SK E2 at $T = 0$. It is noticeable that the minima are shifted towards smaller R and are getting deeper for the three potentials SK III, SK E1 and SK E2, respectively. Also the potentials are wider for SK E2 than SK E1 and for SK E1 than SK III. The same results are obtained at $T \neq 0$. Figs 2–4 display the nuclear potentials for SK III, SK E1 and SK E2 forces at different temperatures. For each force it is clear that the higher the temperature the deeper the minimum, i.e. more attractive potential. A similar trend has been also observed for lighter nuclei [18]. Also we notice a shift of the positions of the minima towards small R for respectively SK III, SK E1 and SK E2. In Figs 5–7 we present the total potential (Nuclear + Coulomb) for the three forces. Pockets are observed at zero and finite temperatures whose depths change for each force with the temperature as well as with the kind of Skyrme force. The depth becomes larger for higher T values and is generally larger for SK E1 and SK E2 than for SK III.

In conclusion we notice that the extended ansatz for the Skyrme forces reproduces the nuclear compressibility in a better way. So it is interesting to see how this modified forces, SK E1 and SK E2, affects the ion-ion potential, here in the case of the spherical system ^{208}Pb + ^{208}Pb. Whereas, the calculations using the conventional SK III force yield only very flat pockets, the results for SK E1 and SK E2 are in good agreement with the renormalized double folding model [23] predicting deeper minima. The often neglected question of finite temperature effects are discussed and applied again to the same system.

It is a new feature to see that increasing temperature deepens the potential pockets and shifts the position of the minima to smaller distance of center values R. So, for temperature $T = 3$ MeV, the predicted results nearly reach those derived by a phenomenological ansatz [23] combining the liquid drop and the double folding model which is expected to yield good results for interaction processes including overlap of two systems.

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