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RENORMALIZATION OF GAUGE FIELD THEORIES IN RIEMANN-CARTAN SPACE-TIME. I. ABELIAN MODELS

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We study the structure of divergences of the one-loop effective action for two Abelian models with non-minimal torsion coupling. Some previous results are corrected and problems of non-minimal interactions are discussed.

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1. Introduction

Riemann-Cartan geometry with curvature and torsion arises naturally within the framework of the Poincaré gauge theory of gravity. The simplest example is given by the Einstein-Cartan theory [1] in which the coupling of spin and torsion is realised in a degenerate algebraic manner. More general models are based on the Yang-Mills type Lagrangians, quadratic both in torsion and curvature [2]. Classical dynamics of the Poincaré gravitational fields is at present intensively studied (some bibliography can be found in [2, 3]). Among other topics one should mention investigations in cosmology, in particular of the singularity problem, search for exact solutions of the generalised gravitational equations, study of motion of classical objects etc. Eventually the role of classical torsion is becoming more and more clear. However, one can say very little about quantum dynamics of the Poincaré gauge theory of gravity. In fact there exist only a few works in which the quantisation of the gravitational gauge field is considered. Renormalisation problems are discussed in [3–5], while in [6] an attempt is made to understand the space-time torsion as a quantum collective phenomenon arising from the interactions of quantised spinor matter. The problem of unitarity for gravity theories with dynamical torsion is treated on the tree level

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in Refs. [7]. Partially, the lack of progress here is connected with enormous technical difficulties arising in quantum models with torsion. However, the main reason seems to be the absence of quantum version of the Einsteinian general relativity, related with the lack of general understanding how the gravity should be quantised. Hence it is reasonable to study at first non-selfconsistent problems in which only matter fields are quantised while the geometrical gravitational background is treated classically. Previously this class of problems has been thoroughly discussed for the case of purely Riemannian geometry without torsion. The obtained results (for review see the books [8]) provided important progress in the theory of black holes, astrophysics and cosmology, as well as in the quantum theory of fields.

The case of the Riemann-Cartan geometry is currently under investigation. The problem of particle production in space-times with torsion has been considered in Ref. [9] for fermions and in Ref. [10] for bosons. In the latter case the anisotropisation effect in cosmological models with torsion and scalar particles is predicted. Vacuum polarisation effects and renormalisation problems are discussed in [11]. Recently, some general calculational methods for covariantly quantised fields in the Riemann-Cartan space-time have been developed [12–13]. These techniques enable one to compute gravitational anomalies and counterterms for arbitrary material field in space with torsion.

Present work is devoted to the discussion of renormalisation of gauge fields theories in the Riemann-Cartan space-time. As it is well known the free scalar field does not interact minimally with torsion. The case of the free spinor fields has been considered recently in [12, 14]. Hence, the main attention will be paid to the interacting field models. In this paper we discuss the Abelian models, while the case of general renormalisable non-Abelian gauge theory will be described in the forthcoming second part.

It is well known that in arbitrary spin field theory the consistency conditions severely restrict the minimal interaction of torsion with matter [14–15]. Moreover, the gauge fields which are described by connections in a principal bundle, also cannot interact with torsion directly. Hence, it becomes particularly interesting to study models with non-minimal torsion interaction. We consider two Abelian gauge models suggested recently in the literature, with non-minimal torsion coupling. The structure of divergences of the one--loop effective action is obtained and some arising problems are discussed.

2. *Preliminaries*

In this section we give our notations and conventions and briefly outline the algorithm which we use for calculation of one-loop divergences of the effective action.

The Riemann-Cartan space-time U_4 is assumed to be the four-dimensional smooth compact manifold without the boundary, which is supplied with a pseudo-Riemannian metric $g_{\mu\nu}$ with the signature $(+1, -1, -1, -1)$ and with a (world) affine connection $\tilde{\Gamma}^{\alpha}_{\beta\mu}$. The latter is compatible with the metric, i.e. $\tilde{\nabla}_{\mu} g_{\alpha\beta} = 0$, but is in general non-symmetric. The skew-symmetric part is the torsion

$$Q^{\alpha}_{\cdot\mu\nu} = \tilde{\Gamma}^{\alpha}_{[\mu\nu]}. \tag{1}$$

It is always possible to split the Riemann-Cartan connection into (torsion independent) Riemannian connection and the contortion tensor,

$$\tilde{\Gamma}^{\alpha}_{\beta\mu} = \Gamma^{\alpha}_{\beta\mu} + Q^{\alpha}_{\cdot\beta\mu} + Q_{\beta\mu\cdot}^{\ \ \ \alpha} + Q_{\mu\beta\cdot\cdot}^{\ \ \ \alpha}. \tag{2}$$

Here $\Gamma^{\alpha}_{\beta\mu}$ is the standard Christoffel symbol, constructed from the metric $g_{\mu\nu}$. By a tilde we denote throughout the Riemann-Cartan geometrical quantities (e.g. connection, covariant derivatives, curvature etc.) constructed from $\tilde{\Gamma}^{\alpha}_{\beta\mu}$, while the usual notation without additional marks is used for the Riemannian objects.

In covariantly quantised theory the fundamental vacuum-to-vacuum amplitude is given by the functional integral

$$Z = \int [dq] e^{iS[g_{\mu\nu}, Q_{\cdot\mu\nu}{}^{\alpha}, \hat{\phi}+q]}, \tag{3}$$

where the classical action S is assumed to be a functional of the background gravitational fields, $g_{\mu\nu}$ and $Q^{\alpha}_{\cdot\mu\nu}$, and material fields $\phi = \hat{\phi}+q$ (we suppress all tensor and spinor indices). The latter, in accordance with the background field method [16], is the sum of classical $\hat{\phi}$ and quantum q parts, and S is expanded in powers of q. If S is invariant under the action of a local gauge group, the integral (3) becomes undetermined. Then one adds to S an appropriate gauge-breaking term, and takes into account the Faddeev-Popov ghost contribution. Then the one-loop effective action

$$\Gamma_{(1)} = -i \ln Z_{(1)}, \tag{4}$$

is determined by the Gaussian integral

$$Z_{(1)} = \int [dq] e^{i\frac{1}{2}q\Delta q} (\det \Delta_{\mathrm{gh}}), \tag{5}$$

where $\Delta = \delta^2(S+S_{\mathrm{GB}})/\delta q^2$ is the operator for small disturbances [16] and Δ_{gh} is the usual Faddeev-Popov ghost operator.

For the real *bosonic* fields q the integral (5) gives

$$\Gamma_{(1)} = \frac{i}{2}\{\ln \det \Delta - 2 \ln \det \Delta_{\mathrm{gh}}\}. \tag{6}$$

The purely fermionic case is also well known.

However, as concerns the general *boson-fermion* systems, only recently [17] the correct method of computation has been proposed. In brief, the main idea is very simple. Let the matter fields q be arranged into a column, the upper part of which are boson variables, while the rest — fermion ones. Then the operator Δ takes the form of supermatrix

$$\Delta = \begin{pmatrix} \Delta_{(\mathrm{B})} & 2K \\ 2L & 2\Delta_{(\mathrm{F})} \end{pmatrix}, \tag{7}$$

diagonal elements of which are respectively the boson-boson operator $\Delta_{(\mathrm{B})}$ and the fermion-fermion operator $\Delta_{(\mathrm{F})}$. Usually the former is the second-order differential operator (Laplace or D'Alembert-like one) while the latter is the first-order (Dirac) operator. The integral (5) for (7) can be easily calculated with the help of the usual Beresin rules, to give

$$Z_{(1)} = [\det (\Delta_{(B)} - 2K\Delta_{(F)}^{-1}L)]^{-1/2}(\det \Delta_{(F)}) (\det \Delta_{gh}). \tag{8}$$

Since it is rather difficult (though in general not impossible [18]) to deal with the first determinant which depends on the fermion propagator $\Delta_{(F)}^{-1}$, one proceeds as follows [17]. With the help of conjugate fermion operator $\Delta_{(F)}^*$ one defines the fermionic determinant to be

$$\det \Delta_{(F)} = (\det \Delta_{(F)}\Delta_{(F)}^*)^{1/2},$$

and then, combining the first two factors in (8), one recognises in them the superdeterminant of the supermatrix

$$\Delta_{(S)} = \begin{pmatrix} \Delta_{(B)} & 2K\Delta_{(F)}^* \\ L & \Delta_{(F)}\Delta_{(F)}^* \end{pmatrix}, \tag{9}$$

which is now the second-order differential operator both in boson and fermion sectors. Hence,

$$Z_{(1)} = (\mathrm{Sdet}\, \Delta_{(S)})^{-1/2}(\det \Delta_{gh}), \tag{10}$$

and the structure of divergences of the effective action (4) is determined by infinities of determinant (9),

$$\Gamma_{(1)} = \frac{i}{2} \{\ln \mathrm{Sdet}\, \Delta_{(S)} - 2 \ln \det \Delta_{gh}\}. \tag{11}$$

It can be shown [17] that divergences of (11) may be obtained by means of the heat kernel method [19], if one succeeds in finding supermatrices $\mathscr{D}_\mu$ and X, such that the former is linear in first (covariant) derivatives, the latter does not contain derivatives, and

$$\Delta_{(S)} = -(\mathscr{D}_\mu \mathscr{D}^\mu + X). \tag{12}$$

Then divergences of $\Gamma_{(1)}$ are determined by the Minakshisundaram (Seeley-De Witt) coefficients of asymptotic expansion of the heat kernel for the operator (12), and in dimensional regularisation

$$\Gamma_{(1)}^\infty = \frac{1}{(4-n)} \frac{1}{16\pi^2} \{B_4(\Delta_{(S)}) - 2B_4(\Delta_{gh})\}, \tag{13}$$

where the B_4 coefficient for an operator of the form (12) is given by

$$B_4(\Delta_{(S)}) = \int d^4x \sqrt{-g}\, \{\tfrac{1}{180} (R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - R_{\mu\nu}R^{\mu\nu})\mathrm{Str}\, \mathbf{1}$$
$$+ \tfrac{1}{2} \mathrm{Str} \left(X + \frac{R}{6}\right)^2 + \tfrac{1}{12} \mathrm{Str}\, ([\mathscr{D}_\mu, \mathscr{D}_\nu] [\mathscr{D}^\mu, \mathscr{D}^\nu])\}. \tag{14}$$

Earlier [12] we have shown that in the Riemann-Cartan space-time U_4 the same technique is applicable to an arbitrary field theory: one should only split the original

Riemann-Cartan differential operators into purely Riemannian part and the torsion dependent part. This always can be done with the help of (2).

3. *Vector gauge field in* U_4

It is well known that the minimal torsion coupling to the vector fields in U_4 can lead to various inconsistencies. By minimal coupling we, as usual, understand the replacement of the flat space-time metric $\eta_{\mu\nu}$ and ordinary derivatives ∂_μ by the corresponding Riemann-Cartan objects — $g_{\mu\nu}$ and $\tilde{\nabla}_\mu$. According to this prescription the Maxwell tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, defined by the four-potential A_μ, must be replaced by

$$\tilde{F}_{\mu\nu} = \tilde{\nabla}_\mu A_\nu - \tilde{\nabla}_\nu A_\mu = F_{\mu\nu} + 2Q^{\alpha}_{.\mu\nu}A_\alpha. \tag{15}$$

For the massless vector field the last term in (15) evidently breaks down the gauge invariance of the theory, and hence A_μ can no longer be identified with the electromagnetic potential. The classical dynamics of this field has been considered in [20] within the framework of the Einstein-Cartan theory. Thus obtained model possesses some features of nonlinear electrodynamics, for example it predicts approximately Maxwell-like behaviour of $\tilde{F}_{\mu\nu}$ at large distances, but modifies the vector field near the source in such a way that the classical field energy of a point source becomes finite. However, the quantum version of this theory faces many serious difficulties.

For the case of massive vector field the last term in (15) seems harmless, since there is no gauge invariance. However one can show that the wave propagation of the vector field in U_4 is in general acausal [21]. Acausal anomalies disappear only when the torsion is represented by its trace or pseudotrace. On the other hand, in the first-order description of higher spin fields in U_4 the propagation of waves is causal but, instead, one encounters (for spin 1 and higher) algebraic inconsistencies which eliminate torsion [14].

An evident way out of all these difficulties is to assume simply that the vector field does not couple minimally with torsion. As for the non-minimal coupling, it should be studied separately. Here we will consider a model in which the vector field interacts with a special case of torsion in such a way, that the gauge invariance is not violated. The classical theory has been investigated in Refs. [22], and we are now interested in the quantised theory in U_4.

Let the torsion be represented by

$$Q^{\alpha}_{.\mu\nu} = \tfrac{2}{3}\, Q_{[\mu}\delta^{\alpha}_{\nu]}, \tag{16}$$

and the trace $Q_\mu = Q^{\nu}_{.\mu\nu} = -\frac{3}{2}\,\partial_\mu\varphi$ be determined by a scalar "potential" $\varphi(x)$. For example, exactly this kind of torsion appears in the conformal invariant version of the Einstein--Cartan theory [23]. Then one can easily see that the Maxwell tensor (15) (with (16) substituted in it) is invariant under the following modified gauge transformations

$$A_\mu \to A'_\mu = A_\mu + e^{\varphi}\partial_\mu\alpha, \tag{17}$$

with $\alpha(x)$ an arbitrary space-time function.

Despite we use formally (15) as in the minimal coupling scheme, we consider this gauge invariant model as a theory with non-minimal interaction. The reason is that now the scalar field φ (instead of torsion) is the true dynamical variable, and its appearance in (17) cannot be explained by the usual minimal coupling prescriptions ($\eta_{\mu\nu} \to g_{\mu\nu}$, $\partial_\mu \to \tilde{\nabla}_\mu$). There exist also generalisations on non-Abelian gauge theories [22].

Now let us consider this vector gauge model, which is quantised on the background Riemann-Cartan space-time. The action has the usual form

$$S = \int d^4x \sqrt{-g}\,(-\tfrac{1}{4}\,\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}). \tag{18}$$

Substituting

$$\tilde{F}_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu + \tfrac{2}{3}\,(Q_\mu A_\nu - Q_\nu A_\mu)$$

into (18), one gets after simple but long algebra

$$S \doteq \int d^4x \sqrt{-g}\,\{-\tfrac{1}{2}\,\nabla_\mu A_\nu \nabla^\mu A^\nu + \tfrac{1}{2}\,(\nabla_\mu A^\mu)^2 - \tfrac{2}{3}\,A_\mu Q^\mu \nabla_\nu A^\nu$$
$$+ \tfrac{1}{2}\,A_\mu A_\nu[\tfrac{2}{3}\,g^{\mu\nu}\nabla_\alpha Q^\alpha - R^{\mu\nu} - \tfrac{4}{3}\,\nabla^\mu Q^\nu - \tfrac{4}{3}\,g^{\mu\nu}Q_\alpha Q^\alpha + \tfrac{4}{9}\,Q^\mu Q^\nu]\}. \tag{19}$$

The second and the third terms in (19) suggest us to choose the gauge in the form

$$P[A] = \nabla_\mu A^\mu - \tfrac{2}{3}\,Q_\mu A^\mu = 0, \tag{20}$$

with the help of which we remove from the action all terms linear in derivatives. However, one can equally choose more general gauge

$$P_\lambda[A] = \nabla_\mu A^\mu + \lambda Q_\mu A^\mu = 0, \tag{21}$$

with arbitrary parameter λ. In Appendix A we demonstrate that the final result is independent of λ.

Adding the gauge-breaking term $S_{GB} = -\int d^4x\sqrt{-g}\,\tfrac{1}{2}\,(\nabla_\mu A^\mu - \tfrac{2}{3}\,Q_\mu A^\mu)^2$, we get

$$S + S_{GB} = \int d^4x \sqrt{-g}\,\{-\tfrac{1}{2}\,A_\mu \Delta^\mu_{\cdot\nu} A^\nu\},$$

where the operator for small disturbances has the form (12)

$$\Delta^\mu_{\cdot\nu} = -(\delta^\mu_\nu \nabla^\alpha \nabla_\alpha + X^\mu_{\cdot\nu}),$$

with the matrix

$$X^\mu_{\cdot\nu} = -R^\mu_{\cdot\nu} - \tfrac{4}{3}\,\nabla^\mu Q_\nu - \tfrac{4}{9}\,\delta^\mu_\nu Q_\alpha Q^\alpha + \tfrac{2}{3}\,\delta^\mu_\nu \nabla_\alpha Q^\alpha. \tag{22}$$

We can now use the standard spectral geometry algorithm (13)–(14) to obtain the divergences of the effective action. One gets

$$[\nabla_\alpha, \nabla_\beta]^\mu_{\cdot\nu} = R^\mu_{\cdot\nu\alpha\beta}, \tag{23a}$$

$$\operatorname{tr} X = -R + \tfrac{4}{3}\,\nabla_\mu Q^\mu - \tfrac{16}{9}\,Q_\mu Q^\mu, \tag{23b}$$

$$\operatorname{tr} X^2 = R_{\mu\nu}R^{\mu\nu} + \tfrac{16}{9}\nabla_\mu Q_\nu \nabla^\mu Q^\nu$$
$$+\tfrac{8}{9} R Q_\mu Q^\mu - \tfrac{32}{27} Q_\alpha Q^\alpha \nabla_\mu Q^\mu + \tfrac{64}{81}(Q_\mu Q^\mu)^2. \tag{23c}$$

Inserting (23) into (14) (with $tr\,\mathbf{1} = 4$), we find

$$B_4(\Delta^\mu_{\cdot\nu}) = \int d^4x \sqrt{-g}\,\{-\tfrac{11}{180} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} + \tfrac{43}{90} R_{\mu\nu}R^{\mu\nu}$$
$$-\tfrac{1}{9}R^2 + \tfrac{8}{9}\nabla_\mu Q_\nu \nabla^\mu Q^\nu - \tfrac{16}{27} Q_\alpha Q^\alpha \nabla_\mu Q^\mu$$
$$+\tfrac{4}{27} R Q_\mu Q^\mu + \tfrac{2}{9} R\nabla_\mu Q^\mu + \tfrac{32}{81}(Q_\mu Q^\mu)^2\}. \tag{24}$$

The ghost operator is defined by (20) and (17),

$$\frac{\delta P[A]}{\delta\alpha} = e^\varphi[\nabla^\mu\nabla_\mu - \tfrac{4}{3} Q^\mu \nabla_\mu].$$

The factor ($\exp\varphi$) is irrelevant in the dimensional regularisation scheme, and the rest can be rearranged to the form (12) with

$$\mathscr{D}_\mu = \nabla_\mu - \tfrac{2}{3} Q_\mu, \quad X = \tfrac{2}{3}\nabla_\mu Q^\mu - \tfrac{4}{9} Q_\mu Q^\mu.$$

As a result,

$$-2B_4(\Delta_{gh}) = \int d^4x \sqrt{-g}\,\{-\tfrac{1}{90} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} + \tfrac{1}{90} R_{\mu\nu}R^{\mu\nu}$$
$$-\tfrac{1}{36}R^2 - \tfrac{4}{9}(\nabla_\mu Q^\mu)^2 + \tfrac{16}{27} Q_\alpha Q^\alpha \nabla_\mu Q^\mu$$
$$+\tfrac{4}{27} R Q_\mu Q^\mu - \tfrac{2}{9} R\nabla_\mu Q^\mu - \tfrac{16}{81}(Q_\mu Q^\mu)^2\}. \tag{25}$$

Summing (24) and (25), we finally get the infinite part of the effective action

$$\Gamma^\infty_{(1)} = \frac{1}{(4-n)}\frac{1}{16\pi^2}\int d^4x \sqrt{-g}\,\{-\tfrac{13}{180} R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$$
$$+\tfrac{22}{45} R_{\mu\nu}R^{\mu\nu} - \tfrac{5}{36} R^2 + \tfrac{8}{27} R Q_\mu Q^\mu$$
$$+\tfrac{8}{9}\nabla_\mu Q_\nu \nabla^\mu Q^\nu - \tfrac{4}{9}(\nabla_\mu Q^\mu)^2 + \tfrac{16}{81}(Q_\mu Q^\mu)^2\}. \tag{26}$$

This result can be slightly simplified, if we eliminate one of the curvature quadratic terms with the help of the Gauss-Bonnet identity $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} + R^2 = $ (*total divergence*), and make use of $Q_\mu = -\tfrac{3}{2}\partial_\mu\varphi$. Then

$$\Gamma^\infty_{(1)} = \frac{1}{(4-n)}\frac{1}{16\pi^2}\int d^4x \sqrt{-g}\,\{\tfrac{1}{5}(R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}R^2)$$
$$+(\Box\varphi)^2 - 2\partial_\mu\varphi\partial_\nu\varphi(R^{\mu\nu} - \tfrac{1}{3} R g^{\mu\nu}) + (\partial_\mu\varphi\partial^\mu\varphi)^2\}, \tag{27}$$

where $\Box = \nabla^\mu\nabla_\mu$.

Since the vector gauge field is free, only gravitational part of the action is renormalised. Divergences (26)–(27) can be removed from the total S, if we choose properly the structure of the initial action. Renormalised gravitational action can be used then (by considering its variational derivatives) to obtain semiclassical corrections to the gravity field equations, similarly to the usual General Relativity. As a final remark we should mention some problems. Firstly, the form of the effective action divergences (26) is too complicated from the point of view of the Poincaré gauge theory of gravity. More exactly, it seems unlikely that the structure of (26) can be explained with the help of Poincaré symmetry underlying the Riemann-Cartan gravitational theory. Secondly, as we see, the arising effective theory leads to the fourth-order equations both in metric and φ sectors (see (27)) and thus its unitarity is a problem.

4. *Interacting fields in* U_4

Perturbative renormalisability is an important property of a quantum field theory. An interesting question is whether this property, proved in a flat space-time, is preserved also in a curved manifold. This problem has been discussed recently for the case of Riemannian geometry (see e.g. [8, 17, 18, 24]), and it was shown that the flat-space renormalisable theories of interacting fields remain renormalisable in curved space-times. However, it has been long ago recognised that the minimal coupling recipe is not sufficient to preserve renormalisability. This is clearly seen for those models which include interacting scalar fields φ it is well known an additional non-minimal coupling term of the form $\xi R\varphi^2$ is necessarily required.

The same is expected for scalars in the Riemannian-Cartan space-time with torsion. For example, the simple analysis for the quantum theory of interacting scalar φ and spinor fields ψ in U_4 with the Lagrangian

$$L = \tfrac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \tfrac{1}{2}\xi R\varphi^2 + \frac{\lambda}{4!}\varphi^4 + g\varphi\bar{\psi}\psi$$

$$+ i\bar{\psi}\gamma^\mu(\nabla_\mu + \alpha\gamma_5\check{Q}_\mu)\psi, \qquad \lambda, g, \xi, \alpha = \text{const},$$

shows that the effective Lagrangian, determined by loop contributions, contains divergences of the type $\check{Q}_\mu\check{Q}^\mu\varphi^2$. Hence, the latter non-minimal torsion coupling term should be included in the bare action, in order to achieve renormalisability in the Riemann-Cartan space-time.

As compared to the purely Riemannian case, the number of different non-minimal torsion interaction terms (which could naturally arise in various models) is rather great. This is connected with the well-known fact [2] that irreducible decomposition of the curvature and torsion tensors is more complicated, and hence the number of independent invariants (constructed from $Q^{\alpha}_{\cdot\mu\nu}$ and $\tilde{R}^{\alpha}_{\cdot\beta\mu\nu}$) increases greatly [25]. For example, decomposition of torsion into irreducible parts reads

$$Q^{\alpha}_{\cdot\mu\nu} = \tfrac{2}{3} Q_{[\mu}\delta^{\alpha}_{\nu]} + \varepsilon^{\alpha}_{\cdot\mu\nu\beta}\check{Q}^{\beta} + \bar{Q}^{\alpha}_{\cdot\mu\nu},$$

where $Q_\mu = Q^{\nu}_{\cdot\mu\nu}$ is the torsion trace, $\check{Q}_\mu = \frac{1}{6}\varepsilon_{\mu\alpha\beta\gamma}Q^{\alpha\beta\gamma}$ is the so-called pseudotrace, and $\bar{Q}^{\alpha}_{\cdot\mu\nu}$ is the traceless and pseudotraceless reducible tensor $\bar{Q}^{\nu}_{\cdot\mu\nu} = 0$, $\varepsilon^{\mu\alpha\beta\gamma}\bar{Q}_{\alpha\beta\gamma} = 0$, which can be decomposed into self-dual (antiself-dual) parts.

Therefore it would be useful to study the general problem: which types of non-minimal torsion coupling terms do not disturb renormalisability of a given flat-space theory. Recently, it has been supposed that [26] any multiplicatively renormalisable theory remains multiplicatively renormalisable in U_4, when non-minimal coupling includes for scalars and spinors, respectively,

$$\tfrac{1}{2}\Lambda(R, Q^2)\varphi^2,$$

$$\Lambda = \xi_1 R + \xi_2 Q_\mu Q^\mu + \xi_3 \check{Q}_\mu \check{Q}^\mu + \xi_4 \bar{Q}^\alpha{}_{\mu\nu} \bar{Q}_{\alpha}{}^{\mu\nu}, \tag{28}$$

$$\alpha Q_\mu \bar{\psi}\gamma^\mu\psi + \beta\check{Q}_\mu\bar{\psi}\gamma^\mu\gamma_5\psi, \tag{29}$$

where $\xi_1, \ldots, \xi_4, \alpha, \beta$ are the coupling constants.

In this section we investigate the one-loop renormalisability of an Abelian gauge model with spinors and scalars, and non-minimal interactions (28)–(29). This is the spinor electrodynamics coupled through a Yukawa-type interaction with a real nonlinear scalar field. Its Lagrangian in U_4 reads

$$\begin{aligned} L = & -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\nabla_\mu\psi + eA_\mu\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi \\ & + \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tfrac{1}{2}M^2\varphi^2 - \frac{f}{4!}\varphi^4 + h\varphi\bar{\psi}\psi \\ & + \alpha Q_\mu\bar{\psi}\gamma^\mu\psi + \beta\check{Q}_\mu\bar{\psi}\gamma^\mu\gamma_5\psi + \tfrac{1}{2}\Lambda(R, Q^2)\varphi^2. \end{aligned} \tag{30}$$

Here m and M are respectively masses of spinor ψ and scalar fields φ, while e, h and f are the coupling constants. Electromagnetic field A_μ does not interact with torsion, and hence as usually $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

In the framework of the background-field method we decompose all fields into classical and quantum parts:

$$A_\mu = \hat{A}_\mu + q_\mu, \qquad \varphi = \hat{\varphi} + \lambda, \qquad \psi = \hat{\psi} + \eta, \qquad \bar{\psi} = \hat{\bar{\psi}} + \bar{\eta}. \tag{31}$$

The over cups denote classical fields. However, in what follows we we will drop them, since only background parts enter the formulas given below. The one-loop effective action is determined by the quadratic term in the expansion of (30) in powers of quantum parts. We compute $\Gamma^{\infty}_{(1)}$ using the algorithm [17], which we have outlined in Sect. 2. Details of calculations are given in Appendix B, and here we present only the form of supermatrices $\mathcal{D}_\mu$ and X which determine superoperator (12) for the Lagrangian (30). They are as follows, with the upper-left, middle and lower-right blocks referring, respectively, to electromagnetic field, scalars and spinors,

$$\mathcal{D}_\mu = \left(\begin{array}{c:c:c} g^{\alpha\beta}\overset{1}{\nabla}_\mu & 0 & ie\bar{\psi}\gamma^\alpha\gamma_\mu \\ \hdashline 0 & \overset{0}{\nabla}_\mu & -ih\bar{\psi}\gamma_\mu \\ \hdashline 0 & 0 & D_\mu \end{array}\right), \tag{32}$$

$$X = \left(\begin{array}{c|c|c} -R^{\alpha\beta} & 0 & -e(i\overset{1/2}{\nabla}_\mu\bar{\psi}\gamma^\alpha\gamma^\mu + \bar{\psi}\gamma^\alpha N) \\ \hline 0 & M^2 - \Lambda + \dfrac{f}{2}\,\varphi^2 & h(i\overset{1/2}{\nabla}_\mu\bar{\psi}\gamma^\mu + \bar{\psi}N) \\ \hline -e\gamma^\beta\psi & -h\psi & Z \end{array}\right), \tag{33}$$

where

$$N = -eA_\mu\gamma^\mu - \alpha Q_\mu\gamma^\mu + \beta\check{Q}_\mu\gamma^\mu\gamma_5 + 2h\varphi - 2m, \tag{34}$$

$$Z = m^2 - \frac{R}{4} - i\sigma^{\mu\nu}\left(\frac{e}{2}\,F_{\mu\nu} + \alpha\nabla_\mu Q_\nu\right) + 3h^2\varphi^2$$
$$- i\beta\gamma_5\nabla_\mu\check{Q}^\mu + 4\beta h\varphi\check{Q}_\mu\gamma^\mu\gamma_5 - 2\beta^2\check{Q}_\mu\check{Q}^\mu, \tag{35}$$

$$D_\mu = \overset{1/2}{\nabla}_\mu - ieA_\mu - i\alpha Q_\mu - i\beta\sigma_{\mu\nu}\gamma_5\check{Q}^\nu - ih\varphi\gamma_\mu, \tag{36}$$

and $\overset{1/2}{\nabla}_\mu$, $\overset{0}{\nabla}_\mu$ and $\overset{1}{\nabla}_\mu$ denote covariant derivatives, respectively, for spinors, scalars and vectors.

The ghost operator is much simpler: $\Delta_{\mathrm{gh}} = -\overset{0}{\nabla}{}^\mu\overset{0}{\nabla}_\mu$.

All necessary traces of X, X^2 and $[\mathscr{D}_\mu, \mathscr{D}_\nu]^2$ are calculated in Appendix B. Inserting them into (14), we finally obtain

$$\Gamma^\infty_{(1)} = \frac{1}{(4-n)}\,\frac{1}{16\pi^2}\int d^4x\,\sqrt{-g}\,\Big\{-\tfrac{17}{360}\,R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} + \tfrac{91}{180}\,R_{\mu\nu}R^{\mu\nu}$$
$$-\tfrac{5}{36}\,R^2 + \tfrac{1}{2}\,\Lambda^2 - \tfrac{1}{6}\,R\Lambda - \Lambda M^2 + R\left(\frac{M^2}{6} + \frac{m^2}{3}\right) + \frac{M^4}{2} - 2m^4$$
$$+ 8m^2\beta^2\check{Q}_\mu\check{Q}^\mu - \tfrac{2}{3}\,\beta^2(\nabla_\mu\check{Q}_\nu - \nabla_\nu\check{Q}_\mu)(\nabla^\mu\check{Q}^\nu - \nabla^\nu\check{Q}^\mu)$$
$$-\tfrac{2}{3}\,\alpha^2(\nabla_\mu Q_\nu - \nabla_\nu Q_\mu)(\nabla^\mu Q^\nu - \nabla^\nu Q^\mu) - \tfrac{8}{3}\,\alpha e F^{\mu\nu}\nabla_\mu Q_\nu$$
$$-\tfrac{2}{3}\,e^2F_{\mu\nu}F^{\mu\nu} + 2h^2\partial_\mu\varphi\partial^\mu\varphi + \left(\frac{fM^2}{2} - 12m^2h^2\right)\varphi^2$$
$$+ R\varphi^2\left(\frac{f}{12} + \frac{h^2}{3}\right) + 8\beta^2h^2\check{Q}_\mu\check{Q}^\mu\varphi^2 - \tfrac{1}{2}\,\Lambda f\varphi^2$$
$$+ \left(\frac{f^2}{8} - 2h^4\right)\varphi^4 + (2e^2 + h^2)\,(i\bar{\psi}\gamma^\mu\nabla_\mu\psi + eA_\mu\bar{\psi}\gamma^\mu\psi$$
$$+ \alpha Q_\mu\bar{\psi}\gamma^\mu\psi) + (2e^2 - h^2)\beta\check{Q}_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$$
$$+ (8e^2 - 2h^2)\,(h\varphi\bar{\psi}\psi - m\bar{\psi}\psi)\Big\}. \tag{37}$$

Let us briefly analyse this result. The purely gravitational part is very complicated and, like for the model of Sect. 3, it seems impossible to give any reasonable explanation (in the framework of the Poincaré gauge theory) to this sum of curvature and torsion quadratic terms. As concerns material interacting fields contribution, one notices a disturbing term $F^{\mu\nu}\nabla_\mu Q_\nu$. In its absence the theory would be indeed multiplicatively renormalisable, similarly to the case of flat space. This term disappears only on the mass shell, when the classical Maxwell equations for the electromagnetic field $\nabla_\mu F^{\mu\nu} = -e\bar{\psi}\gamma^\nu\psi$ are used. Elimination of divergences, which vanish on the mass shell, can be considered [27] as a shift of a quantised field

$$A_\mu \to A_\mu + \varepsilon Q_\mu$$

by divergent contribution of the background geometry. In the second and higher loops this leads to new complications. Hence we conclude that the model (30) cannot be considered as multiplicatively renormalisable in presence of non-minimal term $\alpha Q_\mu \bar{\psi}\gamma^\mu\psi$. Such a torsion coupling should be excluded in the Abelian gauge models.

5. *Asymptotic properties of coupling constants*

Recently [28, 29] renormalisation group equation technique has been generalised to the case of quantum field theory in curved space-time. Since the momentum or coordinate scaling cannot be properly introduced in general non-flat space, one studies behaviour of the Green functions under the scaling of the metric tensor $g_{\mu\nu} \to S^{-2}g_{\mu\nu}$ with the constant parameter S. Large S corresponds to the case of large curvature (high energy or short distances) limit. In [28] the opposite low-energy (large distance) case has been discussed for the scalar field theory with quartic self-interaction. In particular, it was shown that asymptotic value of non-minimal scalar curvature coupling constant equals 1/6. Here we present analogous analysis of renormalisable Abelian gauge field model in U_4, discussed in the previous section. In accordance with the above discussion we set $\alpha = 0$ and suppose that the structure of purely gravitational bare Lagrangian is defined by (37). One-loop divergences (37) can be eliminated by renormalization of fields and coupling constants:

$$e_B = \mu^{2-\frac{n}{2}} Z_e e, \quad (A_\mu)_B = \mu^{\frac{n}{2}-2} Z_A^{1/2} A_\mu, \quad (Z_e Z_A^{1/2} = 1) \tag{38.1}$$

$$\psi_B = \mu^{\frac{n}{2}-2} Z_\psi \psi, \quad \bar{\psi}_B = \mu^{\frac{n}{2}-2} Z_\psi \bar{\psi}, \quad m_B = Z_m m, \tag{38.2}$$

$$\varphi_B = \mu^{\frac{n}{2}-2} Z_S^{1/2} \varphi, \quad M_B^2 = Z_M M^2, \quad f_B = \mu^{4-n} Z_f f, \tag{38.3}$$

$$h_B = \mu^{2-\frac{n}{2}} Z_h h, \quad (\xi_i)_B = Z_{\xi_i} \xi_i, \quad i = 1, 2, 3, 4, \quad \beta_B = Z_\beta \beta. \tag{38.4}$$

Here the subscript B denotes bare quantities (the renormalized ones are without additional marks), μ is an arbitrary mass scale parameter which makes the action dimensionless in dimension $n \neq 4$. Formulas (38) give renormalization of the material gauge theory (30). As for renormalization of the gravity action, it is trivial and we will not write it out explicitly.

From (37) one easily obtains renormalization constants $\delta Z = Z-1$. Denoting $\varepsilon = 16\pi^2(n-4)$ we get

$$\delta Z_e = -\frac{1}{\varepsilon}\tfrac{4}{3}e^2, \tag{39.1}$$

$$\delta Z_S = \frac{1}{\varepsilon}4h^2, \tag{39.2}$$

$$f\delta Z_f = \frac{1}{\varepsilon}(-3f^2-8h^2f+48h^2), \tag{39.3}$$

$$M^2\delta Z_M = \frac{1}{\varepsilon}[24m^2h^2-M^2(f+4h^2)], \tag{39.4}$$

$$\xi_1\delta Z_{\xi_1} = -\frac{1}{\varepsilon}(\xi_1-\tfrac{1}{6})(f+4h^2), \tag{39.5}$$

$$\delta Z_{\xi_2} = \delta Z_{\xi_4} = -\frac{1}{\varepsilon}(f+4h^2), \tag{39.6}$$

$$\xi_3\delta Z_{\xi_3} = \frac{1}{\varepsilon}[16\beta^2h^2-\xi_3(f+4h^2)], \tag{39.7}$$

$$\delta Z_\varphi = \frac{1}{\varepsilon}(2e^2+h^2), \qquad \delta Z_m = \frac{1}{\varepsilon}(6e^2-3h^2), \tag{39.8}$$

$$\delta Z_h = \frac{1}{\varepsilon}(6e^2-5h^2), \tag{39.9}$$

$$\delta Z_\beta = -\frac{1}{\varepsilon}2h^2. \tag{39.10}$$

With the help of (39) we can now obtain the curved space renormalization group equations [28, 29] for effective (running) coupling constants as a function of the scaling parameter $t = \ln S$. For the gauge charge $\bar{e}^2(t)$ one gets

$$16\pi^2\frac{d\bar{e}^2}{dt} = \tfrac{8}{3}\bar{e}^4. \tag{40}$$

Integrating this with the initial condition $\bar{e}^2(0) = e^2$ we see that the theory is infrared-stable, and the effective charge

$$\bar{e}^2(t) = \frac{e^2}{1-\dfrac{be^2t}{16\pi^2}} \tag{41}$$

goes to zero when $t \to -\infty$ ($S \to 0$), since $b = 8/3$. Hence (analogously to the case of the pure scalar field theory [28]) we can analyse the long-distance (small curvature) asymptotic behaviour of the effective charges, using the perturbation theory results. Since in view of (41) the limit $S \to 0$ coincides with $\bar{e}^2 \to 0$, it is convenient to consider all coupling constants as functions of $\bar{e}^2$. More precisely, let us make a change of variables

$$\bar{h}^2(t) = k_1(t)\bar{e}^2(t), \quad \bar{f}(t) = k_2(t)\bar{e}^2(t), \tag{42}$$

and introduce a new scaling parameter τ, defined from (41) by

$$\tau = \frac{1}{b}\ln\left(1 - \frac{be^2 t}{16\pi^2}\right). \tag{43}$$

For $S = 1$ ($t = 0$) we have $\tau = 0$, while for $S \to 0$ ($t \to -\infty$) we get $\tau \to \infty$.

Then the renormalization group equations are as follows:

$$\dot{k}_1 = -10k_1^2 + \tfrac{44}{3}k_1, \tag{44.1}$$

$$\dot{k}_2 = -3k_2^2 - \tfrac{16}{3}k_2 + 48k_1^2, \tag{44.2}$$

$$\dot{\sigma} = -\sigma(k_2 + 4k_1), \tag{44.3}$$

$$\dot{\beta}^2 = -4k_1\beta^2, \tag{44.4}$$

$$\dot{\xi}_3 = -\xi_3(k_2 + 4k_1) + 16k_1\beta^2. \tag{44.5}$$

Here the dot denotes derivative with respect to τ, and $\sigma(\tau)$ stands for $\xi_1 - \frac{1}{6}$ or ξ_2, or ξ_4.

The system (44) can be easily integrated. Indeed, let us notice that in the physical region ($k_1 > 0$, $k_2 > 0$) equations (44.1), (44.2) possess the stable fixed point $k_1^* = 22/15$, $k_2^* = \frac{8}{9}(\frac{\sqrt{1114}}{5} - 1) \approx 5$. Hence, for arbitrary initial conditions integral curves $\{k_1(\tau), k_2(\tau)\}$ asymptotically ($\tau \to \infty$) approach (k_1^*, k_2^*). Thus the Yukawa and quartic self-interaction effective coupling constants (42) are asymptotically zero at large distances, and this fact supports the use of perturbation theory results for the study of renormalization group equations.

Now let us consider the large-distance behaviour of non-minimal curvature and torsion coupling constants σ, β, ξ_3. From (44.3)–(44.5) we see immediately that all of them tend to zero when $\tau \to \infty$ ($S \to 0$). In fact, as soon as $k_1(\tau)$ and $k_2(\tau)$ are known, equations (44.3)–(44.4) can be directly integrated to give

$$\sigma(\tau) = \sigma(0)\exp\left\{-\int_0^\tau [k_2(\tau') + 4k_1(\tau')]d\tau'\right\}$$

$$\underset{\tau\to\infty}{\sim} e^{-(4k_1^* + k_2^*)\tau} = \left(1 - \frac{be^2 t}{16\pi^2}\right)^{-\frac{4k_1^* + k_2^*}{b}} \to 0, \tag{45}$$

$$\bar{\beta}^2(\tau) = \bar{\beta}^2(0) \exp\left(-\int_0^\tau 4k_1(\tau')d\tau'\right)$$

$$\underset{\tau\to\infty}{\sim} e^{-\frac{88\tau}{15}} = \left(1 - \frac{be^2 t}{16\pi^2}\right)^{-\frac{11}{5}} \to 0. \tag{46}$$

A little bit more complicated is integration of inhomogeneous equation (44.5). Here the solution can be find in a form $\bar{\xi}_3(\tau) = \eta(\tau)\bar{\beta}^2(\tau)$, where the function η satisfies

$$\dot{\eta} = -k_2\eta + 16k_1.$$

Thus, when $\tau \to \infty$ we have $\eta \to \eta^* = \frac{16k_1^*}{k_2^*}$, and hence for large τ the coupling $\bar{\xi}_3$ behaves like $\bar{\beta}^2$ and also tends to zero.

Finally, one can find (analogously to [28]) that in the limit $S \to 0$ ($\tau \to \infty$) effective masses $\bar{m}(\tau)$, $\bar{M}^2(\tau)$ become infinite. The dominant term, which determines their behaviour, comes from the naive scaling properties of masses, and it is present already in flat space-time (see [28] for the relevant discussion). Massless case is an exception, then $\bar{m}(\tau) = \bar{M}^2(\tau) = 0$.

In conclusion, we have analysed the asymptotic behaviour of the effective coupling constants in the Abelian gauge field model (30) in the Riemann-Cartan space-time. In the limit of large distances the model becomes infrared-free in all coupling constants; $\bar{e}^2$, $\bar{h}^2$, $\bar{f}$ tend to zero when $S \to 0$. As concerns the gravity-matter non-minimal coupling constants, it is seen that the interaction with torsion becomes asymptotically weak for all types of couplings ($\xi_2, \xi_3, \xi_4, \beta$), while scalar curvature coupling constant ξ_1 asymptotically approaches the conformal invariant (for $M^2 = 0$) limit value 1/6.

An interesting problem is to make the renormalization group analysis of asymptotically free non-Abelian gauge field models, in which asymptotic properties of ξ_i and β could be studied in the limit of small distances (high curvature).

6. *Conclusion*

In this paper we have started the discussion of renormalizability properties of quantised gauge field theories in the Riemann-Cartan space-time U_4. Two Abelian models with non--minimal torsion coupling were considered. Our results correct some shortcomings of Ref. [26]. In the forthcoming paper we will consider generalisation on the case of renormalizable non-Abelian gauge models.

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APPENDIX A

Let us consider the calculation of the effective action for the vector model (Sect. 3) in general gauge (21). The gauge-breaking term $S_{GB} = -\int d^4x\sqrt{-g}\,\frac{1}{2}(\nabla_\mu A^\mu + \lambda Q_\mu A^\mu)^2$ does not eliminate in (19) terms which are linear in derivatives, and hence the operator for small disturbances $\Delta_{(\lambda)\cdot\nu}^{\mu}$ reduces to the form (12) with

$$(\mathscr{D}_\alpha)^\mu_\nu = \delta^\mu_\nu \nabla_\alpha + \left(\tfrac{1}{3} + \frac{\lambda}{2}\right)(\delta^\mu_\alpha Q_\nu - Q^\mu g_{\alpha\nu}), \tag{A.1}$$

$$X^\mu_{\cdot\nu} = -R^\mu_{\cdot\nu} + \tfrac{2}{3}\,\delta^\mu_\nu \nabla_\alpha Q^\alpha + (\lambda - \tfrac{2}{3})\nabla^\mu Q_\nu$$

$$+\left(-\tfrac{1}{3} + \frac{\lambda}{3} + \frac{\lambda^2}{4}\right)\delta^\mu_\nu Q_\alpha Q^\alpha + \left(\tfrac{2}{3} + \frac{2\lambda}{3} - \frac{\lambda^2}{2}\right) Q^\mu Q_\nu. \tag{A.2}$$

After long and tedious algebra we get the basic traces

$$\tfrac{1}{2}\,\mathrm{tr}\,X^2 = \tfrac{1}{2}\,R_{\mu\nu}R^{\mu\nu} + \tfrac{8}{9}\,\nabla_\mu Q_\nu \nabla^\mu Q^\nu - \left(\tfrac{1}{3} + \frac{\lambda}{2}\right) R\nabla_\mu Q^\mu$$

$$+\left(\tfrac{1}{3} - \frac{\lambda}{3} - \frac{\lambda^2}{4}\right) R Q_\mu Q^\mu + \tfrac{1}{2}(\lambda^2 - \tfrac{4}{9})(\nabla_\mu Q^\mu)^2$$

$$+\lambda\left(\frac{\lambda^2}{2} + \tfrac{2}{3}\right) Q_\alpha Q^\alpha \nabla_\mu Q^\mu + \tfrac{1}{2}\left(\tfrac{2}{3} + \frac{\lambda^2}{2}\right)^2 (Q_\mu Q^\mu)^2, \tag{A.3}$$

$$\tfrac{1}{6}\,R\,\mathrm{tr}\,X = -\frac{R^2}{6} + \left(\tfrac{1}{3} + \frac{\lambda}{6}\right) R\nabla_\mu Q^\mu + \left(-\tfrac{1}{9} + \frac{\lambda}{3} + \frac{\lambda^2}{12}\right) R Q_\mu Q^\mu, \tag{A.4}$$

$$\tfrac{1}{12}\,\mathrm{tr}\,([\mathscr{D}_\mu, \mathscr{D}_\nu][\mathscr{D}^\mu, \mathscr{D}^\nu]) = -\tfrac{1}{12}\,R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - \left(\tfrac{1}{3} + \frac{\lambda}{2}\right)^2 (\nabla_\mu Q^\mu)^2$$

$$+\tfrac{1}{3}\left(\tfrac{1}{3} + \frac{\lambda}{2}\right) R\nabla_\mu Q^\mu + \tfrac{1}{3}\left(\tfrac{1}{3} + \frac{\lambda}{2}\right)^2 R Q_\mu Q^\mu$$

$$-2\left(\tfrac{1}{3} + \frac{\lambda}{2}\right)^3 Q_\alpha Q^\alpha \nabla_\mu Q^\mu - \left(\tfrac{1}{3} + \frac{\lambda}{2}\right)^4 (Q_\mu Q^\mu)^2. \tag{A.5}$$

Summing all these, when using (14), one gets

$$B_4(\Delta_{(\lambda)\cdot\nu}^{\mu}) = \int d^4x \sqrt{-g}\left\{-\tfrac{11}{180}\,R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} + \tfrac{43}{90}\,R_{\mu\nu}R^{\mu\nu}\right.$$

$$-\tfrac{1}{9}\,R^2 + \tfrac{8}{9}\,\nabla_\mu Q_\nu \nabla^\mu Q^\nu + (\nabla_\mu Q^\mu)^2\left[-\tfrac{4}{9} + \left(\tfrac{1}{3} - \frac{\lambda}{2}\right)^2\right]$$

$$+\left(\tfrac{1}{9} - \frac{\lambda}{6}\right) R\nabla_\mu Q^\mu + 2\left(\frac{\lambda}{2} - \tfrac{1}{3}\right)^3 Q_\alpha Q^\alpha \nabla_\mu Q^\mu$$

$$\left. + R Q_\mu Q^\mu\left[\tfrac{8}{27} - \tfrac{1}{3}\left(\tfrac{1}{3} - \frac{\lambda}{2}\right)^2\right] + (Q_\mu Q^\mu)^2\left[\tfrac{16}{81} + \left(\tfrac{1}{3} - \frac{\lambda}{2}\right)^4\right]\right\}. \tag{A.6}$$

However, one easily sees that the ghost operator is now

$$-\Delta_{\text{gh}}^{(\lambda)} = \frac{\delta P_\lambda[A]}{\delta\alpha} = e^{\varphi}[\nabla^\mu\nabla_\mu+(\lambda-\tfrac{2}{3})Q^\mu\nabla_\mu], \tag{A.7}$$

and it also reduces to the form (12) with

$$\mathscr{D}_\mu = \nabla_\mu+\left(\frac{\lambda}{2}-\tfrac{1}{3}\right)Q_\mu, \tag{A.8}$$

$$X = \left(\tfrac{1}{3}-\frac{\lambda}{2}\right)\nabla_\mu Q^\mu-\left(\tfrac{1}{3}-\frac{\lambda}{2}\right)^2 Q_\mu Q^\mu. \tag{A.9}$$

With these we obtain

$$-2B_4(\Delta_{\text{gh}}^{(\lambda)}) = \int d^4x\sqrt{-g}\left\{-\tfrac{1}{90}R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}+\tfrac{1}{90}R_{\mu\nu}R^{\mu\nu}\right.$$

$$-\tfrac{1}{36}R^2+\left(\frac{\lambda}{6}-\tfrac{1}{9}\right)R\nabla_\mu Q^\mu+\tfrac{1}{3}\left(\tfrac{1}{3}-\frac{\lambda}{2}\right)^2 RQ_\mu Q^\mu-\left(\tfrac{1}{3}-\frac{\lambda}{2}\right)^2(\nabla_\mu Q^\mu)^2$$

$$\left.+2\left(\tfrac{1}{3}-\frac{\lambda}{2}\right)^3 Q_\alpha Q^\alpha\nabla_\mu Q^\mu-\left(\tfrac{1}{3}-\frac{\lambda}{2}\right)^4(Q_\mu Q^\mu)^2\right\}. \tag{A.10}$$

Sum of (A.10) and (A.6) evidently is independent of λ and gives the final result (26) for the divergent part of the effective action of the Abelian vector gauge model.

APPENDIX B

In the model (30) it is most convenient to choose the standard Lorentz gauge $\nabla_\mu q^\mu = 0$. Then, expanding the Lagrangian (30) with the help of (31), one obtains the operator for small disturbances Δ in the form (7), where the boson (vector-scalar) part is

$$\Delta_{(\text{B})} = \begin{pmatrix} \overset{1}{\nabla}_\mu\overset{1}{\nabla}{}^\mu g^{\alpha\beta}-R^{\alpha\beta} & 0 \\ 0 & -\overset{0}{\nabla}_\mu\overset{0}{\nabla}{}^\mu-M^2+\Lambda-\dfrac{f}{2}\varphi^2 \end{pmatrix}, \tag{B.1}$$

the fermion operator is

$$\Delta_{(\text{F})} = i\gamma^\mu\overset{1/2}{\nabla}_\mu-m+eA_\mu\gamma^\mu+\alpha Q_\mu\gamma^\mu+\beta\check{Q}_\mu\gamma^\mu\gamma_5+h\varphi, \tag{B.2}$$

and boson-fermion (fermion-boson) off diagonal terms are

$$K = \begin{pmatrix} e\bar{\psi}\gamma^\alpha \\ h\bar{\psi} \end{pmatrix}, \quad L = [e\gamma^\beta\psi \,\vdots\, h\psi]. \tag{B.3}$$

With the help of

$$\overset{1/2}{\Delta^*_{(F)}} = i\gamma^\mu\nabla_\mu + m + eA_\mu\gamma^\mu + \alpha Q_\mu\gamma^\mu + \beta\check{Q}_\mu\gamma^\mu\gamma_5 + h\varphi, \tag{B.4}$$

one gets for the square of the Dirac operator

$$\Delta_{(F)}\Delta^*_{(F)} = -(D_\mu D^\mu + Z), \tag{B.5}$$

where D_μ and Z are given respectively by (36) and (35), and as usually $\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$.

Now the supermatiix (9) is completely constructed and with a little effort one can prove that it can be reduced to the foi m (12) with $\mathscr{D}_\mu$ and X as given in (32) and (33). Evidently,

$$\mathrm{Str}\,[\mathscr{D}_\mu, \mathscr{D}_\nu]^2 = \mathrm{tr}\,[\overset{1}{\nabla}_\mu, \overset{1}{\nabla}_\nu]^2 - \mathrm{tr}\,[D_\mu, D_\nu]^2,$$

and

$$[D_\mu, D_\nu] = \tfrac{1}{4}\sigma^{\alpha\beta}R_{\alpha\beta\mu\nu} - ieF_{\mu\nu} - i\alpha(\nabla_\mu Q_\nu - \nabla_\nu Q_\mu)$$
$$+2i\beta\gamma_5\sigma_{[\mu}{}^{\alpha}\nabla_{\nu]}\check{Q}_\alpha + 2ih\gamma_{[\mu}\nabla_{\nu]}\varphi + 2\beta^2(\check{Q}_\alpha\check{Q}^\alpha\sigma_{\mu\nu} - \sigma_{[\mu}{}^{\alpha}\check{Q}_{\nu]}\check{Q}_\alpha)$$
$$-2h^2\varphi^2\sigma_{\mu\nu} - 4i\beta h\varphi\varepsilon_{\mu\nu\alpha\beta}\check{Q}^\alpha\gamma^\beta. \tag{B.6}$$

Then after a long computation one gets

$$\tfrac{1}{12}\,\mathrm{Str}\,([\mathscr{D}_\mu, \mathscr{D}_\nu]\,[\mathscr{D}^\mu, \mathscr{D}^\nu]) = -\tfrac{1}{24}\,R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$$
$$-\tfrac{4}{3}\,\beta^2R_{\mu\nu}\check{Q}^\mu\check{Q}^\nu + \tfrac{2}{3}\,\beta^2R\check{Q}_\mu\check{Q}^\mu - \tfrac{4}{3}\,\beta^2\nabla_\mu\check{Q}_\nu\nabla^\mu\check{Q}^\nu$$
$$-\tfrac{2}{3}\,\beta^2(\nabla_\mu\check{Q}^\mu)^2 + 8\beta^4(\check{Q}_\mu\check{Q}^\mu)^2 + \frac{e^2}{3}\,F_{\mu\nu}F^{\mu\nu} + 2h^2\partial_\mu\varphi\partial^\mu\varphi$$
$$+\tfrac{4}{3}\,e\alpha F^{\mu\nu}\nabla_\mu Q_\nu + \tfrac{1}{3}\,\alpha^2(\nabla_\mu Q_\nu - \nabla_\nu Q_\mu)\,(\nabla^\mu Q^\nu - \nabla^\nu Q^\mu)$$
$$-\tfrac{2}{3}\,h^2\varphi^2R - 48\beta^2h^2\varphi^2\check{Q}_\mu\check{Q}^\mu + 16h^4\varphi^4. \tag{B.7}$$

For the traces of the matrix X we obtain

$$\tfrac{1}{2}\,\mathrm{Str}\,X^2 = \tfrac{1}{2}\,R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{8} + \tfrac{1}{2}\,\Lambda^2 - \Lambda M^2 + \frac{M^4}{2} - 2m^4 + m^2R$$
$$-2\beta^2R\check{Q}_\mu\check{Q}^\mu - 8\beta^4(\check{Q}_\mu\check{Q}^\mu)^2 + 8m^2\beta^2\check{Q}_\mu\check{Q}^\mu + 2\beta^2(\nabla_\mu\check{Q}^\mu)^2$$
$$-\alpha^2(\nabla_\mu Q_\nu - \nabla_\nu Q_\mu)\,(\nabla^\mu Q^\nu - \nabla^\nu Q^\mu) - 4e\alpha F^{\mu\nu}\nabla_\mu Q_\nu - e^2F_{\mu\nu}F^{\mu\nu}$$
$$+56\beta^2h^2\varphi^2\check{Q}_\mu\check{Q}^\mu + 3h^2R\varphi^2 - \tfrac{1}{2}f\Lambda\varphi^2 + \varphi^2\left(\frac{M^2f}{2} - 12m^2h^2\right)$$

$$+\varphi^4\left(\frac{f^2}{8}-18h^4\right)+(2e^2+h^2)\,(i\bar{\psi}\gamma^\mu\nabla_\mu\psi+eA_\mu\bar{\psi}\gamma^\mu\psi+\alpha Q_\mu\bar{\psi}\gamma^\mu\psi)$$

$$+(2e^2-h^2)\beta\check{Q}_\mu\bar{\psi}\gamma^\mu\gamma_5\psi+(8e^2-2h^2)\,(h\varphi\bar{\psi}\psi-m\bar{\psi}\psi), \tag{B.8}$$

$$\tfrac{1}{6}\,R\,\mathrm{Str}\,X\;=\;-\tfrac{1}{6}\,\Delta R+\tfrac{4}{3}\,\beta^2R\check{Q}_\mu\check{Q}^\mu$$

$$+\tfrac{1}{6}\,R(M^2-4m^2)+\left(\frac{f}{12}-2h^2\right)R\varphi^2. \tag{B.9}$$

Inserting (B.7)–(B.9) into (13)–(14), we get the final result (37), where one must keep in mind that Str $\mathbf{1}=1$, and the ghost operator $\Delta_{\mathrm{gh}}=-\overset{0}{\nabla}_\mu\overset{0}{\nabla}{}^\mu$ gives contribution

$$-2B_4(\Delta_{\mathrm{gh}})=\int d^4x\,\sqrt{-g}\,\{-\tfrac{1}{90}\,R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}+\tfrac{1}{90}\,R_{\mu\nu}R^{\mu\nu}-\tfrac{1}{36}\,R^2\}. \tag{B.10}$$

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