FOUR-DIMENSIONAL HETEROTIC STRINGS AND THEIR FIELD THEORY LIMITS*

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The construction of heterotic string theories in $d \leqslant 10$ dimensions is reviewed within the framework of the covariant lattice approach. Methods of (super) conformal field theory, necessary to calculate string scattering amplitudes, are described and applied to a 4-dimensional model. From the calculated amplitudes we extract the (point particle) field theory which reproduces these amplitudes in the limit of infinite string tension.

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1. *Introduction*

String theory [1] is an attempt to describe in a consistent way all fundamental particles and their interactions, including gravity. Earlier attempts at a theory of everything, which were all based on point particle field theories such as for instance $N = 1$, $d = 11$ supergravity, failed because it was not possible to incorporate gravity in a consistent way. Uncancelled infinities which cannot be absorbed by renormalization spoil the success. In string theory there is a new mechanism for infinity cancellation which is due to the fact that strings are not pointlike but rather extended objects with an infinity of excitation modes. It is now possible that the divergent contributions from this infinite tower of states cancel, leading to a finite theory. This is exactly what happens for the superstring and the heterotic string (at least to one loop).

There are at least two reasons why string theory has enjoyed so much popularity in recent years: (*i*) it provides the only candidate solution for the problem of incorporating gravity in a consistent way and (*ii*) it reveals a beautiful mathematical structure whose study is interesting in its own right. It is clear that this structure has to be understood if contact with the real world is to be made. Admittedly, we are still a far cry away from understanding how this contact is made, but it is nevertheless worthwhile to see whether

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it is possible to extract a semi-realistic low energy field theory: by this we mean a theory in 4 space-time dimensions with realistic gauge group and interactions (including gravity), chiral fermions, anomaly free, etc. That this is possible will be demonstrated here.

The paper is organized as follows: in the next Section we review the construction of string theories in $d \leqslant 10$ dimensions within the framework of the covariant lattice approach. In Section 3 we show how, using methods of conformal field theory, string scattering amplitudes can be calculated. We will demonstrate this with a few examples. In Section 4 we extract the low energy field theory from these amplitudes. The last Section contains some conclusions and remarks.

2. *Strings in four dimensions*

Let us briefly review the bosonic and fermionic string theories [1]. They are described by the following fields, which are maps from the two-dimensional world-sheet (parametrized by σ and τ) to the D-dimensional target space. Bosonic string: X^μ (commuting) and the conformal ghosts b and c (anti-commuting). Fermionic string: in addition to the fields above, we have ψ^μ (anti-commuting) and the superconformal ghosts β and γ (commuting). Here $\mu = 1, \ldots D$. The absence of a conformal anomaly on the world-sheet requires $D = 26$ for the bosonic string and $D = 10$ for the fermionic string. If we now interpret D as the number of space-time dimensions, we are faced with the problem of compactifying the theory down to four dimensions. It is however possible to interpret D simply as the "number of degrees of freedom", of which only some, namely $d \leqslant D$ correspond to space-time degrees of freedom and the other, for instance to the maximal torus of some group. It is possible to do this without ever making reference to D-dimensional space-time. This has the advantage of not requiring any compactification scheme or ever having to talk about the D-dimensional field theory limit of the string theory, which is problematic anyway [2]. On the other hand this approach will not explain the number of space--time dimensions, which, after all, might be just an experimental input.

If we now try to formulate the fermionic string directly in four dimensions it turns out that while it is possible to arrive at a gauge group which is large enough to contain the SU(3) × SU(2) × U(1) standard model this never leads to a realistic particle spectrum [3]. The situation with the bosonic string is still unclear, the problem being the interpretation of some of the states in the spectrum [4–6]. This leads us to consider heterotic strings [7]. Let us recall what heterotic strings are.

Heterotic strings are closed strings. For a free closed string the world-sheet is a cylinder, parametrized by the cyclic coordinate σ and the euclidean time coordinate τ. The map $z = e^{\tau+i\sigma}$, $\bar{z} = e^{\tau-i\sigma}$ maps the cylinder onto the complex plane. In terms of the variables z and $\bar{z}$ the string action in (super)conformal gauge is (in units where the string tension $\alpha' = 1/2$):

$$S = \frac{1}{\pi}\int d^2z(\partial_z X \cdot \partial_{\bar{z}} X + \psi \cdot \partial_{\bar{z}}\psi + \bar{\psi} \cdot \partial_z \bar{\psi}$$

$$+ c^z \partial_{\bar{z}} b_{zz} + \bar{c}^{\bar{z}} \partial_z \bar{b}_{\bar{z}\bar{z}} + \bar{\gamma} \partial_z \beta_{\bar{z}} + 8 \partial_{\bar{z}} \beta_z) \tag{2.1}$$

from which we derive the following equations of motion:

$$\partial_z \partial_{\bar z} X = 0 \rightarrow X = X(z) + \bar X(\bar z),$$

$$\partial_{\bar z}\psi = 0 \rightarrow \psi = \psi(z), \qquad \partial_z \bar\psi = 0 \rightarrow \bar\psi = \bar\psi(\bar z)$$

and similarly for the other fields, i.e. the analytical and anti-analytical pieces decouple. If we go back to the original world-sheet variables, we recognize the analytical part as the right-moving and the anti-analytical part as the left-moving component of the fields. (It is clear that this separation will not work for open strings since there the boundary conditions at the ends of the string mix left- with right-movers.) Heterotic strings are now simply the combination of the right-moving part of the 10-dimensional fermionic string and the left-moving part of the 26-dimensional bosonic string.

In the bosonic formulation of the original supersymmetric 10-dimensional heterotic string [7], the 16 left-moving dimensions, which have no right-moving counterpart which would allow to interpret them as space-time, are compactified on an even self-dual Euclidean lattice[1]. (This is required by modular invariance.) In 16 dimensions there are only two such lattices, namely the weight lattice of $Spin(32)/Z_2$ and the root lattice of $E_8 \times E_8$; hence the occurrence of these and only these gauge groups [8].

Even though 16 is the minimum number of left-moving degrees of freedom that have to be compactified, it is not the only possibility. We can compactify $16+n$ of the left bosonic fields if we also compactify n of the right bosonic fields [9]. This then leads to a theory in $d = 10-n$ space-time dimensions. The $16+2n$ coordinates have to lie on an even selfdual Lorentzian lattice $\Gamma_{16+n,n}$ with signature $(+)^{16+n}(-)^n$. There is a continuous infinity of such lattices; they are however all related by Lorentz-transformations. The resulting theories have rank $16+n$ gauge groups, the particular group depending on the particular choice of the lattice. (The gauge group is however always a product of simply laced groups, i.e. of SU(m), SO($2m$) or $E_{6,7,8}$ factors.) This was first realized by Narain [9]. He however treated the n right-moving fermionic coordinates independent from the bosonic coordinates by simply decomposing them into representations of the d-dimensional transverse Lorentz group. This leads to a non-chiral spectrum and in the case where $n = 6$, i.e. $d = 4$ to $N = 4$ supersymmetry. We can however go one step further and enlarge the lattice by bosonizing the right-handed fermions, together with the super-conformal ghosts [10–12]. The idea behind bosonization is to replace two fermionic fields by (the exponential of) one free bosonic field such as to give the same correlation functions. For instance, two fermions ψ and ψ^* are represented by one free boson ϕ as follows[2]:

$$\psi = e^{i\phi}, \qquad \psi^* = e^{-i\phi}. \tag{2.2}$$

To demonstrate that the bosonized form does indeed reproduce the anti-commutativity of ψ and ψ^*, i.e. $\psi(z)\psi^*(w) = -\psi^*(w)\psi(z)$, we calculate the singular part of the operator

[1] A lattice is even if all basis vectors have even (length)^2. It is odd if the lengths are integer but not all even. A lattice is self-dual if it is identical to its dual lattice.

[2] Here and in the following we omit normal ordering symbols.

product expansion and get[3] $\psi(z)\psi^*(w) = e^{i\phi(z)}e^{-i\phi(w)} \sim (z-w)^{-1}$. The situation with the ten fermionic fields ψ^μ is slightly more complicated. They can be represented by five free bosons ϕ^i. To do this we have to give up manifest covariance and characterize the fields by their five dimensional weight vectors $\vec{\lambda}$. For instance, the ten weights $\pm e_i = (0 \dots \pm 1 \dots 0)$ make up the vector representation of SO(10). We then have $\psi^{\vec{\lambda}} \sim e^{i\vec{\lambda}\cdot\vec{\phi}}$. This however does not yet reproduce the anticommutativity of $\psi^{\pm e_i}$ and $\psi^{\pm e_j}$ for $i \neq j$. To ensure this, we have to introduce cocycle factors $c_{\vec{\lambda}}$ whose purpose it is to introduce the necessary factors[4]; i.e.

$$\psi^{\vec{\lambda}} = e^{i\vec{\lambda}\cdot\vec{\phi}}c_{\vec{\lambda}}. \tag{2.3}$$

This can now be generalized to any weight vector of SO(10), especially to weights in either of the two spinor conjugacy classes[5]. This leads to a bosonized representation of the so--called spin fields S^α and $S^{\dot{\alpha}}$ which are necessary to represent space-time fermions in string theory.

The bosonization of the superconformal ghosts is however more intricate as they are themselves bosons. We do not want to go into any details and just state the results as needed for our purposes. We introduce one additional bosonic field, ϕ_6 and define a vector $(\vec{\phi}, \phi_6)$ and weight vectors (λ, q). Modular invariance requires the weights to lie on an odd self-dual Lorentzian lattice $\Gamma_{5,1}$ with signature $((+)^5, (-))$. q is called the ghost charge of the corresponding state. The ghost charge can be changed by one unit by the so-called picture changing operation [15] which however does not change the conjugacy class of the lattice vector.

If we now combine the lattice $\Gamma_{5,1}$ with the lattice arising from the left and right moving bosonic coordinates $\Gamma_{26-d;10-d}$ we get a lattice $\Gamma_{26-d;15-d,1} = \Gamma_{26-d;15-\frac{3}{2}d} \otimes \Gamma_{\frac{d}{2},1}$; the semi-colon separates left from right movers and the signature is $((-)^{26-d}, (+)^{15-d}, (-))$. The $\Gamma_{\frac{d}{2},1}$ describes the space-time degrees of freedom of the bosonized fermions. ($\Gamma_{\frac{d}{2}}$ describes the Lorentz group SO(d).) A general lattice vector will be written as $(\vec{\lambda}_L; \vec{\lambda}_R, \vec{\lambda}'_R, q)$. Modular invariance now requires $\Gamma_{26-d;15-d,1}$ to be odd self-dual without requiring the left (Γ_{26-d}) respectively right ($\Gamma_{15-d,1}$) sublattice to be self-dual separately. To make contact with our discussion above we decompose the lattice as follows: $\Gamma_{26-d;15-d,1} = (\Gamma_{26-d;10-d})_X \otimes (\Gamma_{0;5-\frac{d}{2}} \otimes \Gamma_{0;\frac{d}{2},1})_{(\vec{\phi},q)}$. The theories considered by Narain are now those where the first factor is even self-dual and the second factor odd self-dual separately. We have already mentioned that this does not lead to phenomenologically satisfactory theories and we have to look for more general lattices.

[3] In general, the operator product of $e^{i\vec{\lambda}\cdot\vec{\phi}(z)}$ and $e^{i\vec{\mu}\cdot\vec{\phi}(w)}$, where $\vec{\phi}$ is a collection of free bosons, is given by [8] $e^{i\vec{\lambda}\cdot\vec{\phi}(z)}e^{i\vec{\mu}\cdot\vec{\phi}(w)} \sim e^{-\lambda^i\mu^j\langle\phi^i(z)\phi^j(w)\rangle}e^{i\vec{\lambda}\cdot\vec{\phi}(z)+i\vec{\mu}\cdot\vec{\phi}(w)} = (z-w)^{\vec{\lambda}\cdot\vec{\mu}}e^{i\vec{\lambda}\cdot\vec{\phi}(z)+i\vec{\mu}\cdot\vec{\phi}(w)}$.

[4] Explicit representations of the $c_{\vec{\lambda}}$ can be found in the literature [8, 13, 14]. Here it shall suffice to mention that they give rise to the covariant group theory factors in the correlation functions of chapter three below.

[5] SO($2n$) has four conjugacy classes, denoted by o, v, s and c. o contains the roots $(\pm e_i \pm e_j)$, v the weights $\pm e_i$ and s and c the weights $(\pm\frac{1}{2}, \dots, \pm\frac{1}{2})$ with an even and odd number of minus signs respectively. The difference of any two weights in any given conjugacy class is an integer linear combination of roots.

There is an infinite number of Lorentzian lattices with the given signature, but they are all related by Lorentz rotations. However not all of these lattices lead to good string theories. Severe restrictions arise from preserving world-sheet supersymmetry or, equivalently, demanding space-time Lorentz invariance, i.e. the absence of massive chiral spinors [16, 12]. A way to construct lattices satisfying all of these consistency requirements is given in [12], and a classification of them in [17]. Their number is unfortunately huge (an upper limit being 10^{38} [18]). Here we want to give just one example of a lattice, which leads to a string theory in four dimensions, which is chiral and does not possess space-time supersymmetry. Its construction is given in detail in [12]. The lattice is $(D_5 \times D_5 \times D_5 \times D_2 \times D_2 \times D_2 \times D_1)_L \times (D_3 \times D_3 \times D_3 \times D_{2,1})_R$ where D_n is the weight lattice of SO(2n). However not all conjugacy classes are present; they are restricted by the self-duality of the lattice. The gauge group is $[SO(10) \times SO(4)]^3 \times U(1)$ which has rank 22 corresponding to the dimension of the left lattice. The right part gives rise to the global symmetry $[SO(6)]^3$ of rank nine[6]. We can interpret the three SO(10)'s as grand unification groups (with two of them playing the role of hidden sectors) while the SO(4)'s and SO(6)'s can be interpreted as local and global horizontal symmetries respectively. (The U(1) factor is anomalous; the anomalies are cancelled by the Green-Schwarz mechanism [19].) In [20] we have given a complete list of all the massless states. Here we restrict ourselves to just one scalar and one fermion field. They transform under the local and global symmetry groups in the following representations:

$$\phi \sim (10, 1, 1, 1, 1 \pm 1; 6, 1, 1),$$

$$\psi_L \sim (16, 1, 1, 2, 1, 1, \tfrac{1}{2}; 4, 1, 1) + \text{h.c.}$$

These are the states that we will use in our sample calculations of the next Chapter. Other massless states, such as gravitons and gauge bosons have no excitations in the Γ_9 part of the right lattice and transform in the trivial and adjoint representations of the local gauge groups respectively.

3. *String amplitudes*

In order to check whether a given string theory is an adequate description of our world, we have to calculate scattering amplitudes which can then be compared with experiment. Or, we can look for the point particle field theory which reproduces the amplitudes in the limit where this field theory is expected to be valid, namely in the infinite string tension limit. In this Section we will explain how to calculate string scattering amplitudes using methods of conformal field theory [21–23, 15, 24, 10, 13, 14, 20], and in the next Section we will then write down the field theory which reproduces them. The techniques developed up to now allow us to calculate on-shell amplitudes only. In addition we will restrict ourselves to the evaluation of tree level amplitudes.

At tree level the string world sheet is topologically a sphere. External physical states

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[6] The part of the symmetry group arising from the right part of the lattice is not gauged. This is required to get chiral fermions.

(they correspond to strings of zero length) can be inserted via vertex operators V. (The resulting picture of a sphere with physical states attached to it can be viewed as cylinder with $N-2$ tubes branching off, in the limit where the diameters of the ends of the cylinder and the tubes shrink to zero.) Then, in the language of conformal field theory, a N-particle amplitude is simply given by the following correlation function:

$$A \sim g^{N-2}\langle V_1 \dots V_N \rangle, \tag{3.1}$$

where g is the string coupling constant. To obtain non-vanishing correlation functions, three conditions have to be met: (*i*) the vertex operators, integrated over the world sheet, have to be primary fields of conformal dimension zero[7]; (*ii*) they have to be chosen such that the zero-modes of the conformal and superconformal ghosts are soaked up. (At one and higher loop order, there are other zero-modes which have to be treated carefully.) and (*iii*) space-time and lattice momentum have to be conserved. (The conservation of lattice momentum is identical to Lorentz and gauge invariance.)

The most general vertex operator is a linear combination of expressions of the following generic form:

$$\begin{aligned} V_q &= \int dz d\bar{z} \prod_j \partial^{r_j}(X)^{\nu_j} e^{q\phi_6} e^{\frac{i}{2} k_R \cdot X} e^{i\vec{\lambda}_R' \cdot \vec{\phi}_R'} e^{i\vec{\lambda}_R \cdot \vec{\phi}_R}(z) \\ &\times \prod_k \bar{\partial}^{s_k}(\bar{X})^{\varrho_k} e^{\frac{i}{2} k_L \cdot \bar{X}} e^{i\vec{\lambda}_L \cdot \vec{\phi}_L}(\bar{z}) \\ &\equiv \int dz d\bar{z} V_R(z) V_L(\bar{z}), \end{aligned} \tag{3.2}$$

where k is the space-time momentum, q the ghost charge, and we have expressed ψ^μ and S^α in bosonized form. The lattice vectors $\vec{\lambda}_L \in \Gamma_{26-d}$ and $\vec{\lambda}_R \in \Gamma_{15-d}$ are weights of the representations in which the state transforms with respect to the local and global symmetries; $\vec{\lambda}_R' \in \Gamma_{\frac{d}{2}}$. The conformal dimensions of the various components are as follows: $[dz] = [d\bar{z}] = -1$, $[\partial] = +1$, $[X] = [\phi] = 0$, $[e^{\frac{i}{2} k_{R,L} \cdot X}] = \frac{1}{8} k_{R,L}^2 = -\frac{1}{8} m_{R,L}^2$, $[e^{i\vec{\lambda}\cdot\vec{\phi}}] = \frac{1}{2}\vec{\lambda}^2$, $[\psi] = +\frac{1}{2}$, $[e^{q\phi_6}] = -\frac{1}{2}q^2 - q$ and the conformal dimension of the c and $\bar{c}$ ghosts is -1. Requiring that the conformal dimensions of V_R and V_L are zero, we then get the following mass formulae:

$$\begin{aligned} \tfrac{1}{8} m_L^2 &= N_L + \tfrac{1}{2}\vec{\lambda}_L^2 - 1, \\ \tfrac{1}{8} m_R^2 &= N_R - \tfrac{1}{2} q^2 - q + \tfrac{1}{2}\vec{\lambda}_R'^2 + \tfrac{1}{2}\vec{\lambda}_R^2 - 1, \\ m^2 &= \tfrac{1}{2} m_L^2 + \tfrac{1}{2} m_R^2, \end{aligned} \tag{3.3}$$

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[7] A primary field of conformal dimension h satisfies the following operator product relation with the energy-momentum tensor: $T(z)\phi(w) \sim \frac{h\phi(w)}{(z-w)^2} + \frac{\partial_w \phi(w)}{(z-w)} + \textit{finite}$, with no higher order poles. A similar relation holds for anti-analytic fields.

where $N_L = \Sigma s_k$ and $N_R = \Sigma r_j$ are the number of left and right bosonic oscillator excitations, respectively. (The absence of a distinguished point on a closed string leads to the condition that $m_L^2 = m_R^2$.)

To illustrate the above, let us consider the following example:

$$V_{-1} = \int d^2 z e^{-\phi_6} \psi^\mu(z) \bar{\partial} X^\nu(\bar{z}) e^{ik\cdot X(z,\bar{z})} \varepsilon_{\mu\nu}(k). \tag{3.4}$$

According to the above rules, this vertex operator has conformal weight $\frac{1}{4}k^2$, which means that the state created by it has to be massless. If we now require that V_L and V_R are primary, we get in addition the condition $k^\mu \varepsilon_{\mu\nu} = k^\nu \varepsilon_{\mu\nu} = 0$, i.e. the vertex operator creates a massless transverse 2nd rank tensor particle. When we calculate scattering amplitudes with this vertex operator we find that it can be identified with the graviton + antisymmetric tensor + dilaton.

On the sphere the conformal ghost c has three zero-modes and the superconformal ghost γ has two, corresponding to the number of generators of the group $\hat{\mathrm{SL}}(2)$, the supersymmetric extension of SL(2, C), the symmetry group of the sphere. There are no b or β zero-modes on the sphere. The c zero-modes are taken care of by replacing in three of the vertex operators the integral over the world sheet by a ghost insertion, i.e. $\int dz d\bar{z} \to c(z)\bar{c}(\bar{z})$, which does not change the conformal dimension of the vertex operator. It fixes the positions of three of the vertices and the amplitude must not depend on any particular choice. To absorb the γ zero-modes we have to choose the vertex operators such that their ghost charges q_i add up to -2. This is possible by the picture changing operation mentioned before, which allows one to change the ghosts charge of a given vertex operator by one unit. We do not want to go into details of how this is done and refer instead to the literature [15][8].

Since all the fields involved in the construction of the vertex operators are either free fields or can be expressed in terms of free fields via bosonization, we only need to know their two point correlation functions and can then evaluate the complete scattering amplitude using Wick's theorem. The difficulty consists in recasting the result in a manifestly Lorentz covariant form, which is lost after bosonization. The elementary two-point functions are:

$$\begin{aligned}
\langle X^\mu(z) X^\nu(w) \rangle &= -g^{\mu\nu} \ln(z-w) \\
&\to \langle e^{ik\cdot X(z)} e^{ik'\cdot X(w)} \rangle = (z-w)^{k\cdot k'}, \\
\langle \psi^\mu(z) \psi^\nu(w) \rangle &= -g^{\mu\nu}(z-w)^{-1}, \\
\langle c(z) c(w) \rangle &= \langle e^{\sigma(z)} e^{\sigma(w)} \rangle = (z-w), \\
\langle \phi^i(z) \phi^j(w) \rangle &= -\delta^{ij} \ln(z-w)
\end{aligned}$$

[8] We do need to mention however, that in the four-dimensional case the picture-changing operator splits into two parts, one that operates on space-time degrees of freedom and one that acts on the internal degrees of freedom in Γ_9 [20].

$$\to \langle e^{i\vec{\lambda}\cdot\vec{\phi}(z)} e^{i\vec{\lambda}'\cdot\vec{\phi}(w)} \rangle = (z-w)^{\vec{\lambda}\cdot\vec{\lambda}'},$$

$$\langle \phi_6(z)\phi_6(w) \rangle = -\ln(z-w)$$

$$\to \langle e^{q\phi_6(z)} e^{q'\phi_6(w)} \rangle = (z-w)^{-qq'}. \tag{3.5}$$

We are now ready to give some sample amplitude calculations [20]. The first example is the two fermion-one graviton amplitude, where by graviton we mean either a graviton, an anti-symmetric tensor particle or a dilaton. The string amplitude for the three cases differs only by the choice for the polarization tensor, which is symmetric and traceless for the graviton, anti-symmetric for the anti-symmetric tensor and pure trace for the dilaton. In addition the polarization tensor is transverse in all three cases. In the low-energy field theory however, the amplitude is reproduced by different terms corresponding to the different particles. The string amplitude is given by[9]

$$A = \sqrt{2}\, g \langle c\bar{c} V_{-1/2}^{\alpha\{a\}\{A\}}(1) c\bar{c} V_{-1/2}^{\dot{\beta}\{b\}\{B\}}(2) c\bar{c} V_{-1}^{\mu\nu}(3) \rangle u_{\alpha}^{(1)} u_{\dot{\beta}}^{(2)} \varepsilon_{\mu\nu}, \tag{3.6}$$

where

$$V_{-1/2}^{\alpha\{a\}\{A\}} = e^{-\phi_6/2} S^{\alpha} S^{\{a\}} e^{\frac{i}{2} k\cdot X}(z) S^{\{A\}} e^{\frac{i}{2} k\cdot \bar{X}}(\bar{z}) \tag{3.7}$$

is the vertex operator corresponding to a space-time fermion characterized by the spinor u_{α}. Its transformation properties under the global and local symmetry groups is given by the multi-indices $\{a\}$ and $\{A\}$. For the specific model described in Section 2 we have for instance $\{a\} = 4_{SO(6)}$ and $\{A\} = (16_{SO(10)}, 2_{SO(4)}, 1/2_{U(1)})$. So $\{a\}$ corresponds to a weight of length 3/4 and $\{A\}$ to a weight of length 2. $V_{-1/2}^{\dot{\beta}\{b\}\{B\}}$ corresponds to the conjugate spinor and $V_{-1}^{\mu\nu}$ is the graviton vertex discussed above. The correlation functions involving spin fields are:

$$\langle S^{\alpha}(z_1) S^{\dot{\beta}}(z_2) \psi^{\mu}(z_3) \rangle = \frac{1}{\sqrt{2}} (\gamma^{\mu})^{\alpha\dot{\beta}} (z_1-z_3)^{-1/2} (z_2-z_3)^{-1/2},$$

$$\langle S^{\{a\}}(z_1) S^{\{b\}}(z_2) \rangle = (z_1-z_2)^{-3/4} C^{\{a\}\{b\}},$$

$$\langle S^{\{A\}}(\bar{z}_1) S^{\{B\}}(\bar{z}_2) \rangle = (\bar{z}_1-\bar{z}_2)^{-2} C^{\{A\}\{B\}}. \tag{3.8}$$

The amplitude then becomes:

$$A = \frac{g}{2} C^{\{a\}\{b\}} C^{\{A\}\{B\}} u^{(1)} \gamma^{\mu} u^{(2)} k_1^{\nu} \varepsilon_{\mu\nu}(k_3), \tag{3.9}$$

where we have used the on-shell conditions $k^{\mu}\varepsilon_{\mu\nu}(k) = k^{\nu}\varepsilon_{\mu\nu}(k) = 0$, $\not{k}u = 0$ and $k^2 = 0$. It can be easily verified that the amplitude vanishes for the dilaton case.

[9] The amplitudes are normalized as follows: each vertex operator with $q = -1(-1/2)$ is accompanied by a factor of $2(2^{-1/4})$ and each world-sheet integral by a factor of $\frac{1}{2\pi}$. Then the vertex operators create states that are normalized according to particles in field theory.

The next amplitude we want to present is the scattering of two scalars and one gauge boson. This requires the gauge-boson vertex operator in the $q=0$ picture:

$$V_0^i = \left[\partial X^\mu + \frac{i}{2}(k\cdot\psi)\psi^\mu\right] e^{\frac{i}{2}k\cdot X}(z) J^i e^{\frac{i}{2}k\cdot\bar{X}}(\bar{z})\varepsilon_\mu(k), \tag{3.10}$$

where ε_μ is a polarization vector and $J^i = \frac{1}{2}\psi^M\psi^N(T^i)^{MN}$ a gauge current which satisfies $[J^i, J^j] = if^{ijk}J^k$. (The T^i's are representation matrices.) The amplitude is then

$$\begin{aligned} A^i &= 4g\langle c\bar{c}V_{-1}^{\{m\}\{M\}}(1)c\bar{c}V_{-1}^{\{n\}\{N\}}(2)c\bar{c}V_0^{\mu i}(3)\rangle\varepsilon_\mu \\ &= g\varepsilon\cdot(k_1-k_2)\delta^{\{m\}\{n\}}(T^i)^{\{M\}\{N\}}. \end{aligned} \tag{3.11}$$

The last three-point amplitude we want to quote is the two fermion–one scalar (Yukawa) amplitude. Since all the ingredients for its evaluation have already been given, we only quote the result:

$$A = \sqrt{2}\, g(u^{(1)}Cu^{(2)})\left(\frac{1}{\sqrt{2}}(\Gamma^{\{m\}})^{\{a\}\{b\}}\right)\left(\frac{1}{\sqrt{2}}(\Gamma^{\{M\}})^{\{A\}\{B\}}\right). \tag{3.12}$$

The last example we want to present is the scattering of two fermions, one scalar and one graviton. It is given by the following correlation function:

$$A = \frac{g^2}{\sqrt{2\pi}}\int dz d\bar{z}\langle c\bar{c}V_{-1/2}^{\alpha\{a\}\{A\}}(\infty)c\bar{c}V_{-1/2}^{\beta\{b\}\{B\}}(1)V_0^{\mu\nu}(z,\bar{z})c\bar{c}V_{-1}^{\{m\}\{M\}}(0)\rangle u_\alpha^{(1)}u_\beta^{(2)}\varepsilon_{\mu\nu}. \tag{3.13}$$

Here we have used the SL(2, C) invariance (or equivalently the fact that there are three (complex) c zero-modes on the sphere) to fix the positions of three of the vertices at the values $z_1=\infty$, $z_2=1$ and $z_4=0$.

$$V_{-1} = e^{-\Phi_6}\psi^{\{m\}}e^{\frac{i}{2}k\cdot X}(z)\psi^{\{M\}}e^{\frac{i}{2}k\cdot\bar{X}}(\bar{z}) \tag{3.14}$$

is the vertex operator for a space-time scalar transforming in the vector representation of the internal symmetry group. For the model discussed in Section 2, we have $\{m\} = 6_{SO(6)}$ and $\{M\} = (10_{SO(10)}, \pm 1_{U(1)})$.

$$V_0 = \left[\partial X^\mu + \frac{i}{2}(k\cdot\psi)\right] e^{\frac{i}{2}k\cdot X}(z)\bar{\partial}\bar{X}^\nu e^{\frac{i}{2}k\cdot\bar{X}}\varepsilon_{\mu\nu}(k) \tag{3.15}$$

is the graviton vertex operator in the $q=0$ ghost picture. The various correlation functions are easy to evaluate with the results given above, the only new one being

$$\langle S^\alpha(z_1)S^\beta(1)\psi^\varrho\psi^\mu(z)\rangle = -\tfrac{1}{2}\frac{(\gamma^{\varrho\mu})^{\alpha\beta}}{2(1-z)}z_1^{-1/2}. \tag{3.16}$$

The z-integration can be done using the following general formula [7]:

$$\int\frac{d^2z}{\pi}|z|^\alpha|1-z|^\beta z^n(1-z)^m = (-)^{m+n}\frac{\Gamma(1+n+\frac{1}{2}\alpha)\Gamma(1+m+\frac{1}{2}\beta)\Gamma(-1-n-m-\frac{1}{2}(\alpha+\beta))}{\Gamma(-\frac{1}{2}\alpha)\Gamma(-\frac{1}{2}\beta)\Gamma(2+\frac{1}{2}(\alpha+\beta))} \tag{3.17}$$

which is valid for integer m and n. Introducing the Mandelstam variables $s = -(k_1+k_2)^2$, $t = -(k_1+k_4)^2$ and $u = -(k_1+k_3)^2$ we finally get

$$A = \sqrt{2}\, g^2 \frac{\Gamma\left(-\frac{s}{8}\right)\Gamma\left(-\frac{t}{8}\right)\Gamma\left(-\frac{u}{8}\right)}{\Gamma\left(\frac{s}{8}\right)\Gamma\left(\frac{t}{8}\right)\Gamma\left(\frac{u}{8}\right)} \frac{(\Gamma^{(m)})^{\{a\}\{b\}}}{\sqrt{2}} \frac{(\Gamma^{\{M\}})^{\{A\}\{B\}}}{\sqrt{2}}$$

$$\times \left[\frac{1}{s}(k_4 \varepsilon k_4) u^{(1)} C u^{(2)} - \frac{1}{2t} k_2^\nu k_4^\varrho u^{(1)} \gamma^\varrho \gamma^\mu u^{(2)} \varepsilon_{\mu\nu} - \frac{1}{2u} k_1^\nu k_4^\varrho u^{(1)} \gamma^\mu \gamma^\varrho u^{(2)} \varepsilon_{\mu\nu}\right]. \quad (3.18)$$

All other amplitudes can now be calculated in a similar way. We have calculated all three- and four-point amplitudes for the model of Section 2 and refer for the results to [20]. Most of the results given there are in fact model independent and apply to any model constructed from a covariant lattice. This concludes our discussion of string amplitudes.

4. *Low energy field theory*

We now want to find the point particle field theory Lagrangian that reproduces the results of the previous Section in the limit of infinite string tension. (The Lagrangian is of course required to be invariant under all the symmetries of the theory.) This will also allow us to relate the field theory coupling constants to the string coupling constant g. Our strategy is as follows [25]: we first try to reproduce the three particle amplitudes with a field theory Lagrangian $\mathscr{L}_{3\text{pt}}$. The four particle amplitudes can then be partially reproduced by $\mathscr{L}_{3\text{pt}}$ either by direct couplings or via exchange diagrams. The remainder then gives rise to new interactions $\mathscr{L}_{4\text{pt}}$. Let us illustrate this with a few examples.

The two scalar–one gauge boson amplitude Eq. (3.11) is reproduced by

$$\mathscr{L}_{3\text{pt}} = \tfrac{1}{2}\sqrt{g}\, g^{\mu\nu}(D_\mu \Phi)^{m\{M\}\dagger}(D_\nu \Phi)^{m\{M\}}, \quad (4.1)$$

with

$$(D_\mu \Phi)^{m\{M\}} = \partial_\mu \Phi^{m\{M\}} - g_4 A_\mu^{\{M\}\{N\}} \Phi^{m\{N\}},$$

where g_4 is the four-dimensional gauge coupling constant. Comparison with the string result gives $g_4 = g$, i.e. the gauge coupling constant for each simple factor of the gauge group is equal to the string coupling constant. Likewise, the two fermion–one graviton amplitude Eq. (3.9) is reproduced by the kinetic energy term:

$$\mathscr{L}_{3\text{pt}} = \sqrt{g}\; \bar{\Psi}^{\alpha\{a\}\{A\}}(\not{D}\Psi)_{\alpha\{a\}\{A\}}, \quad (4.2)$$

where $\not{D}\Psi$ is the gauge and general coordinate covariant derivative acting on the spinor Ψ. To verify that this indeed reproduces the amplitude we expand $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa_4 h_{\mu\nu}$ to find the two fermion–one graviton vertex [26]. (κ_4 is related to Newton's constant $\kappa_4 = \sqrt{8\pi G}$.)

Comparison with the string result gives $\kappa_4 = \frac{g}{2}\sqrt{2\alpha'}$, where we have re-introduced the string tension. Above expression does however not reproduce the two fermion-one antisymmetric tensor part of the amplitude. For this we have to introduce a new term

$$\mathscr{L}_{3pt} = \lambda_4^{(1)} C^{\{a\}\{b\}} C^{\{A\}\{B\}} e^{-cD} \Psi_{\{a\}\{A\}} \gamma^{\mu\nu\varrho} \Psi_{\{b\}\{B\}} H_{\mu\nu\varrho}, \tag{4.3}$$

where $H_{\mu\nu\varrho}$ is the field strength for the antisymmetric tensor $B_{\mu\nu}$, given by $H_{\mu\nu\varrho} = \partial_{[\varrho}B_{\mu\nu]}$ + Chern-Simons terms. Comparison gives $\lambda_4^{(1)} = \frac{\kappa_4}{2} = \frac{g_4}{4}\sqrt{2\alpha'}$. A comment on the dilaton coupling is in order here. Four dimensional conformal invariance requires a factor $e^{c(1+w)D}$ in front of each term in the Lagrangian with conformal weight w. The conformal weight is determined as follows: $g_{\mu\nu}(\gamma_\mu)$ and $g_{\mu\nu}(\gamma^\mu)$ have conformal weight $+1(+1/2)$ and $-1(-1/2)$ respectively and Ψ has conformal weight $-1/4$; all other fields have conformal weight zero. The constant c will be determined shortly.

The Yukawa amplitude is reproduced by the following term in the Lagrangian:

$$\mathscr{L}_{3pt} = \tfrac{1}{2}\sqrt{g}\, h_4 e^{\frac{c}{2}D} C^{\alpha\beta} \frac{(\Gamma^m)^{\{a\}\{b\}}}{\sqrt{2}} \frac{(\Gamma^{\{M\}})^{\{A\}\}B\}}}{\sqrt{2}}$$

$$\times \Psi_{\alpha\{a\}\{A\}} \Psi_{\beta\{b\}\{B\}} \Phi^{m\{M\}}, \tag{4.4}$$

and we get $h_4 = \sqrt{2}g_4$, i.e the Yukawa coupling constant is expressed in terms of the gauge coupling constant.

It can be shown that all possible three-point amplitudes are reproduced by the Lagrangian in Eqs (4.1–4.4), with the exception of the ones involving only gauge and gravitational particles. They require

$$\mathscr{L}_{3pt} = \sqrt{g}\left\{\frac{1}{2\kappa_4^2} R - \tfrac{1}{6} e^{-2cD} H_{\mu\nu\varrho} H^{\mu\nu\varrho} - \tfrac{1}{2}(\nabla_\mu D)(\nabla^\mu D)\right.$$

$$\left. -\tfrac{1}{8} e^{-c\kappa_4 D}\,\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})\right\} + \textit{other terms} \tag{4.5}$$

where '*other terms*' stands for various higher derivative terms.

Let us now look at the four-particle amplitude, Eq. (3.18). Here we have to distinguish three different cases. In the case of the graviton, there is no direct four point interaction coming from Eq. (4.4). (The term linear in the graviton field arising from the expansion of $\sqrt{g} = 1 + \kappa_4 h_\mu^\mu$ vanishes due to the tracelessness of an on-shell graviton.) However the terms in $\mathscr{L}_{3pt}$ give rise to exchange contributions. There are three diagrams: an s-channel diagram with the exchanged particle being a scalar, and a t- and u-channel diagram via fermion exchange. If we add the contributions of all three diagrams, we reproduce exactly the string result, i.e. no new contact terms are needed. The second case involves instead of a graviton an antisymmetric tensor particle. Again, there is no direct interaction arising from $\mathscr{L}_{3pt}$, but as in the previous case $\mathscr{L}_{3pt}$ suffices to reproduce Eq. (3.18) for this case, namely through fermion exchange in the t- and u-channel. Finally, the two fermion–one scalar–one dilaton amplitude is completely reproduced via a contact term arising from Eq. (4.3) upon expanding the exponent to first order in the dilaton field. Comparing this with the string result also allows us to fix the constant c to $c = \frac{2}{\sqrt{d-2}}\kappa_d$, which is valid in d dimensions. So far we have not been forced to add new quartic terms to the Lagrangian. In fact most of the four-point amplitudes can be reproduced by $\mathscr{L}_{3pt}$, in general as a combination of direct couplings and exchange contributions. There are however contributions to $\mathscr{L}_{4pt}$. They arise (to lowest non-trivial order in α') from examining the four scalar amplitude, the four fermion amplitude and the two fermion–two scalar amplitude and they can be found in [20].

To summarize, we have extracted from the string amplitudes a field theory Lagrangian, which reproduces them and we have related all coupling constants to the string coupling constant. The effective Lagrangian contained the Einstein-Hilbert Lagrangian, as well as the Yang-Mills Lagrangian and bosons and fermions coupled to gravity and Yang--Mills theory. We have also found interactions among the matter fields, such as quartic scalar and fermion interactions and Yukawa couplings. The Lagrangian has all the basic ingredients a successful field theory Lagrangian is believed to have except for may-be a mass term for the scalars, but they can either be produced by string-loop effects, or if we find some other-stringy-mechanism to break the gauge symmetry may not even be necessary.

5. Conclusions

There are many open problems in string theory that have to be solved in order to turn it into a fully acceptable theory of the fundamental particles and forces. But it is encouraging to see, that we can make at least formal contact with the real world by being able to show that we can construct from string theory an 'acceptable' field theory. We have tried to demonstrate how this is done within the framework of the covariant lattice approach, in which all the calculations involved are rather straightforward (even though tedious at times).

Two immediate problems come however to mind. The value of the cosmological constant and the rank of the gauge group. There is hope to find a satisfactory solution to the first problem, even in non-supersymmetric theories. (In the case of supersymmetric theories it is solved automatically by boson-fermion cancellation on each mass level.) This is based on the recent discovery [27] of a symmetry of the string integrand which arises in the evaluation of the cosmological constant. It is identically zero in the supersymmetric case. The integral over moduli space can however vanish if the integrand shows what is known as Atkin-Lehner symmetry. The second problem, concerning the rank of the gauge group might be solved along the lines of Refs [28, 29] by dividing out discrete symmetry groups. Work on this is in progress.

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