PLANE WAVES IN THE GENERALIZED FIELD THEORY

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Plane wave solutions are investigated for the field equations of the macrophysical Generalized Field Theory. It is shown that when a geometry of the space-time is assumed, which is a small perturbation of the flat manifold, there exists, as expected and required, an electromagnetic wave propagating in a fixed direction. It is also shown that there can exist an unperturbed wave which does not affect the geometry; however, this wave is not electromagnetic.

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1. Introduction

It is well known that the study of gravitational radiation was initiated by Einstein himself in two early works (Refs [1, 2]) in which he showed that in a weak field approximation and under permissible coordinate conditions, the general relativistic (GR) field equations take on the form of inhomogeneous (i.e. with sources) wave equation. In empty space, the latter and the conditions were later identified as the equations of a particle of zero rest mass and spin two. Hence the graviton. In other words, graviton is a possible solution of the GR field if indeed it is sensible to consider originally purely macroscopic equations as describing a microphysical entity.

Be it as it may, all experimental investigations (Ref. [3]) aiming to show the existence of gravitational waves rely on an analysis of their material source and thus on the field equations with nonvanishing energy momentum terms. Indeed it is difficult to see how this could be otherwise in a terrestrial laboratory. It follows that radiation studies in GR, perhaps more than any other aspect of the theory, bring into it notions of "prerelativistic physics" since GR itself contains no prescription of how the source terms are to be constructed or written down.

The situation is quite different in the Generalized Field Theory (GFT, Refs. [4, 5]) which is itself but a development of the Nonsymmetric Unified Field Theory of Einstein, Straus and Kaufman (Refs. [6, 7], and the references cited in [4]). A plane wave solution has been found for the latter (Ref. [8]) but a new problem arises in GFT. It has been encountered and commented upon (loco cit.) before.

It is simply that the nonsymmetric field tensor

$$\boldsymbol{g} = (g_{\mu\nu})$$

(as always, Greek indices go from 0 to 3 and Latin, if used, from 1 to 3 with x^0 corresponding to a time-like direction) and the Riemannian metric tensor

$$\boldsymbol{a} = (a_{\mu\nu})$$

of the space-time are connected to each other ("geometrisations of physics") but only by the full set of the field equations and not by an *a priori* identification as in GR. It is therefore questionable whether one is entitled to impose restrictions such as the wave conditions on both $\boldsymbol{g}$ and $\boldsymbol{a}$. Perhaps, since they are essentially physical conditions, they should be assumed for $\boldsymbol{g}$ only but this would make the task of finding the solution enormously more complicated if not impossible. It is thus for reasons of expediency that we shall assume the components of both $\boldsymbol{g}$ and $\boldsymbol{a}$ to be functions of

$$x^0 - x^1 \tag{1}$$

only. Justification of this highhanded assumption lies in the fact that, as in GR, we wish to establish merely the existence of a plane wave solution and not to prove its uniqueness in any strict sense.

The strength of our assumption can be relaxed somewhat by requiring *a priori* that only the geometry $\boldsymbol{a}$ should be a small perturbation

$$a_{\mu\nu} = n_{\mu\nu} + \alpha_{\mu\nu}, \qquad |\alpha_{\mu\nu}| \ll 1 \tag{2}$$

of the flat, Minkowski space-time $\boldsymbol{\eta}$. We shall find that this fairly curious supposition leads unambiguously to an interesting second form of the solution though it may be too early to speculate unduly on its possible physical significance.

On the other hand, energy-momentum terms play no part in GFT until a solution has been obtained though, once it has been found, they can be readily calculated. Of course, GFT still lacks an empirical confirmation which would determine its validity as a physical theory. Indeed, our present aim is not to seek, from the wave solution, any definite predictions. It is rather to investigate whether the theory leads, at least in the first approximation, to results which are sensible from a physical point of view.

2. *Expansion of the field*

The field equations of GFT are

$$g_{\mu\nu,\lambda} - \tilde{\Gamma}^{\sigma}_{\mu\lambda} g_{\sigma\nu} - \tilde{\Gamma}^{\sigma}_{\lambda\nu} g_{\mu\sigma} = 0, \tag{1}$$

$$\mathfrak{G}^{[\mu\nu]}{}_{,\nu} = 0, \tag{2}$$

$$R_{(\mu\nu)}(\tilde{\Gamma}) = 0, \tag{3}$$

$$R_{[[\mu\nu],\lambda]} = 0 \quad \text{or} \quad R_{[\mu\nu]}(\tilde{\Gamma}) = \tfrac{2}{3}(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu}), \tag{4}$$

and

$$\tilde{\Gamma}^{\sigma}_{(\mu\nu)} = \left\{ {\sigma \atop \mu\nu} \right\}_{a}. \tag{5}$$

Here $R_{\mu\nu}$ is the Ricci tensor constructed from the "geometrical" connection $\tilde{\Gamma}^{\sigma}_{\mu\nu}$ related to another, "physical", connection $\Gamma^{\sigma}_{\mu\nu}$ by Schrödinger's equation

$$\tilde{\Gamma}^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} + \tfrac{2}{3}\,\delta^{\sigma}_{\mu}\Gamma_{\nu} \tag{6}$$

so that

$$\tilde{\Gamma}_{\mu} \equiv \tilde{\Gamma}^{\sigma}_{[\mu\sigma]} \equiv 0, \tag{7}$$

square brackets round the indices denoting skew symmetric part, and round ones the symmetric part respectively. Also

$$\mathfrak{G}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}, \qquad g = \det(g_{\mu\nu}).$$

Because of equation (5) ("metric hypothesis") which defines the geometry $\boldsymbol{a}$ of the space-time which must be (Ref. [4]) Riemannian, and adopting the notation of Mme Tonnelat (Ref. [9])

$$g_{(\mu\nu)} = h_{\mu\nu}, \qquad g_{[\mu\nu]} = k_{\mu\nu}, \tag{8}$$

with $h_{\mu\nu}$ (though not necessarily, $k_{\nu\mu}$) nonsingular, we have

$$h_{\mu\nu,\lambda} - \left\{ {\sigma \atop \mu\lambda} \right\} h_{\sigma\nu} - \left\{ {\sigma \atop \lambda\nu} \right\} h_{\mu\sigma} = \tilde{\Gamma}^{\sigma}_{[\mu\lambda]} k_{\sigma\nu} + \tilde{\Gamma}^{\sigma}_{[\lambda\nu]} k_{\mu\sigma}, \tag{9}$$

$$k_{\mu\nu,\lambda} - \left\{ {\sigma \atop \mu\lambda} \right\} k_{\sigma\nu} - \left\{ {\sigma \atop \lambda\nu} \right\} k_{\mu\sigma} = \tilde{\Gamma}^{\sigma}_{[\mu\lambda]} k_{\sigma\nu} + \tilde{\Gamma}^{\sigma}_{[\lambda\nu]} k_{\mu\sigma}, \tag{10}$$

where the Christoffel brackets are (as usual)

$$\left\{ {\sigma \atop \mu\nu} \right\} = \tfrac{1}{2}\, a^{\sigma\lambda}(a_{\lambda\nu,\mu} + a_{\mu\lambda,\nu} - a_{\mu\nu,\lambda}).$$

If now the space-time is perturbed from some ground state $\underset{0}{\boldsymbol{a}}$:

$$a_{\mu\nu} = \underset{0}{a}_{\mu\nu} + \alpha_{\mu\nu}, \qquad |\alpha_{\mu\nu}| \ll |\underset{0}{a}_{\mu\nu}| \tag{11}$$

both the inverse (metric) tensor and the brackets are expanded;

$$a^{\mu\nu} = \sum_{0}^{\infty} \underset{n}{a}^{\mu\nu}, \qquad \left\{ {\sigma \atop \mu\nu} \right\} = \sum_{0}^{\infty} \underset{n}{\left\{ {\sigma \atop \mu\nu} \right\}}, \tag{12}$$

where

$$\underset{0}{\left\{ {\sigma \atop \mu\nu} \right\}} = \left\{ {\sigma \atop \mu\nu} \right\}_{\underset{0}{a}_{\mu\nu}} \tag{13}$$

and the n^{th} terms of both series are of the order $|\alpha_{\mu\nu}|^n$. In fact, we have

$$\underset{n}{a}^{\mu\nu} = (-1)^n \underset{n-1}{a}^{\mu\sigma}\alpha^{\nu}_{\sigma}, \qquad \alpha^{\nu}_{\sigma} = \underset{0}{a}^{\nu\varrho}\alpha_{\varrho\sigma},$$

and

$$\underset{n}{\left\{ {\sigma \atop \mu\nu} \right\}} = \underset{n}{a}^{\sigma\lambda}[\mu\nu, \lambda]_{\underset{0}{a_{\mu\nu}}} + \underset{n-1}{a}^{\sigma\lambda}[\mu\nu, \lambda]_{a_{\mu\nu}},$$

where $[\mu\nu, \lambda]$ is the Christoffel bracket of the first kind formed from the tensor $\boldsymbol{b}$. If, as throughout this article, we consider only the perturbation (1.2) of the Minkowski space--time, then

$$\underset{n}{\left\{ {\sigma \atop \mu\nu} \right\}} = \underset{n-1}{a}^{\sigma\lambda}[\mu\nu, \lambda]_{\alpha} \qquad \underset{0}{\left\{ {\sigma \atop \mu\nu} \right\}} = 0. \tag{14}$$

We shall also confine ourselves in the sequel to a discussion of only the zeroth and first order terms in the (small) quantities $\alpha_{\mu\nu}$. The reason for this is that in the present state of the theory, that is in the absence of empirical evidence which would decide its physical validity, the most interesting results are those which can enhance its interpretative status. It turns out that this is achieved precisely in the two lowest order calculations rather than in clearly unobservable, higher order interactions.

Let us now consider the Ricci tensor formed from the geometrical connection $\tilde{\Gamma}^{\sigma}_{\mu\nu}$. We have

$$R_{(\mu\nu)} = -\left\{ {\sigma \atop \mu\nu} \right\}_{,\sigma} + \left\{ {\sigma \atop \mu\sigma} \right\}_{,\nu} + \left\{ {\sigma \atop \mu\varrho} \right\}\left\{ {\varrho \atop \sigma\nu} \right\} + \tilde{\Gamma}^{\sigma}_{[\mu\varrho]}\tilde{\Gamma}^{\varrho}_{[\sigma\nu]} - \left\{ {\sigma \atop \mu\nu} \right\}\left\{ {\varrho \atop \sigma\varrho} \right\},$$

and

$$R_{[\mu\nu]} = -\tilde{\Gamma}^{\sigma}_{[\mu\nu]\sigma} + \left\{ {\sigma \atop \mu\varrho} \right\}\tilde{\Gamma}^{\varrho}_{[\sigma\nu]} + \left\{ {\varrho \atop \nu\sigma} \right\}\tilde{\Gamma}^{\sigma}_{[\mu\varrho]} - \left\{ {\varrho \atop \sigma\varrho} \right\}\tilde{\Gamma}^{\sigma}_{[\mu\nu]} \equiv -\tilde{\Gamma}^{\sigma}_{[\mu\nu];\sigma}. \tag{15}$$

Hence, for the perturbation (1.2) of a flat background, the first two terms in the corresponding expansion of the components of $R_{\mu\nu}$ are

$$\underset{0}{R}_{(\mu\nu)} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\varrho]}\underset{0}{\tilde{\Gamma}}{}^{\varrho}_{[\sigma\nu]},$$

$$\underset{1}{R}_{(\mu\nu)} = -\underset{1}{\left\{ {\sigma \atop \mu\nu} \right\}}_{,\sigma} + \underset{1}{\left\{ {\sigma \atop \mu\sigma} \right\}}_{,\nu} + \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\varrho]}\underset{1}{\tilde{\Gamma}}{}^{\varrho}_{[\sigma\nu]} + \underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\varrho]}\underset{0}{\tilde{\Gamma}}{}^{\varrho}_{[\sigma\nu]},$$

$$\underset{0}{R}_{[\mu\nu]} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu],\sigma},$$

$$\underset{1}{R}_{[\mu\nu]} = -\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu],\sigma} + \underset{1}{\left\{ {\sigma \atop \mu\varrho} \right\}}\underset{0}{\tilde{\Gamma}}{}^{\varrho}_{[\sigma\nu]} + \underset{1}{\left\{ {\varrho \atop \nu\sigma} \right\}}\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\varrho]} - \underset{1}{\left\{ {\varrho \atop \sigma\varrho} \right\}}\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]}. \tag{16}$$

The components of the affine connection themselves are given by the equations

$$\underset{0}{h}_{\mu\nu,\lambda} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{0}{k}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\,\underset{0}{k}_{\mu\sigma},$$

$$\underset{0}{k}_{\mu\nu,\lambda} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{0}{h}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{0}{h}_{\mu\sigma},$$

$$\underset{1}{h}_{\mu\nu,\lambda}-\underset{1}{\begin{Bmatrix}\sigma\\ \mu\lambda\end{Bmatrix}}\underset{0}{h}_{\sigma\nu}-\underset{1}{\begin{Bmatrix}\sigma\\ \lambda\nu\end{Bmatrix}}\underset{0}{h}_{\mu\sigma} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{1}{k}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{1}{k}_{\mu\sigma}+\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{0}{k}_{\sigma\nu}+\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{0}{k}_{\mu\sigma},$$

$$\underset{1}{k}_{\mu\nu,\lambda}-\underset{1}{\begin{Bmatrix}\sigma\\ \mu\lambda\end{Bmatrix}}\underset{0}{k}_{\sigma\nu}-\underset{1}{\begin{Bmatrix}\sigma\\ \lambda\nu\end{Bmatrix}}\underset{0}{k}_{\mu\sigma} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{1}{h}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{1}{h}_{\mu\sigma}+\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{0}{h}_{\sigma\nu}+\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{0}{h}_{\mu\sigma}. \tag{17}$$

(Progressively more involved expressions occur on both sides of these equations in higher order approximations. For example, already in the second order, we find terms of the type $\underset{1}{\{\ \}}\underset{1}{(h, k)}$ or $\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]}\underset{1}{(h, k)}$ and so on. Because of such terms, it seems very difficult to devise an iterative procedure for this kind of expansion.)

We now notice that the first two of the equations (17) give, in the usual way,

$$\underset{0}{h}_{\mu\nu,\lambda}+\underset{0}{h}_{\nu\lambda,\mu}+\underset{0}{h}_{\lambda\mu,\nu} \equiv \underset{0}{h}_{[\mu\nu,\lambda]} = 0 \tag{18}$$

and

$$\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]} = \tfrac{1}{2}\underset{0}{h}{}^{\sigma\lambda}(\underset{0}{k}_{\lambda\nu,\mu}+\underset{0}{k}_{\mu\lambda,\nu}+\underset{0}{k}_{\mu\nu,\lambda}). \tag{19}$$

Similarly

$$\underset{1}{h}_{[\mu\nu,\lambda]} = 2\left(\underset{1}{\begin{Bmatrix}\sigma\\ \mu\lambda\end{Bmatrix}}\underset{0}{h}_{\sigma\nu}\ \underset{1}{\begin{Bmatrix}\sigma\\ \mu\nu\end{Bmatrix}}\underset{0}{h}_{\sigma\lambda}\ \underset{1}{\begin{Bmatrix}\sigma\\ \nu\lambda\end{Bmatrix}}\underset{0}{h}_{\sigma\mu}\right). \tag{20}$$

However, in the particular case considered below (that of the plane waves) we shall find it easier to solve the first order equations directly.

3. One-dimensional plane waves

Let us adopt the coordinate system

$$(x^0, x^1, x^2, x^3) = (t, x, y, z)$$

and assume that the components of both the metric tensor $\boldsymbol{a}$ and the nonsymmetric field tensor $\boldsymbol{g}$ are functions of

$$\xi = t-x = x^0-x^1 \tag{1}$$

only:

$$a_{\mu\nu} = a_{\mu\nu}(\xi), \quad h_{\mu\nu} = h_{\mu\nu}(\xi), \quad k_{\mu\nu} = k_{\mu\nu}(\xi).$$

The components of the geometrical connection $\tilde{\Gamma}^{\sigma}_{\mu\nu}$ are necessarily functions of ξ because equations (2.1) are, in general, solvable (Ref. [9]) in terms of the tensors $\boldsymbol{h}$, $\boldsymbol{k}$ and their first derivatives. Similarly, because of the metric hypothesis (equation (2.5)), so also are the Christoffel brackets. However (see Apendix), this does not guarantee that the components of the metric tensor $\boldsymbol{a}$ will be always also functions of ξ only. To assume that they are is thus an a priori restriction on the solution

Let us impose a further simplifying assumption, namely, that as in the case of the gravitational waves, the space-time metric is of the form

$$ds^2 = dt^2 - dx^2 - (1-a)dy^2 - (1+a)dz^2 + 2b\, dydz, \tag{2}$$

where the absolute values of the functions a, b (of ξ) are small compared with unity.

We now have

$$\underset{1}{\tilde{\Gamma}^{\sigma}_{(\mu\nu)}} = \underset{1}{\begin{Bmatrix}\sigma\\ \mu\nu\end{Bmatrix}} = \tfrac{1}{2}\,\eta^{\sigma\lambda}(a_{\lambda\nu,\mu} + a_{\mu\lambda,\nu} - a_{\mu\nu,\lambda}) \tag{3}$$

and if dashes denote derivatives with respect to ξ, the only surviving brackets of order one are (dropping the order indicator)

$$\begin{Bmatrix}0\\22\end{Bmatrix} = -\tfrac{1}{2}a', \quad \begin{Bmatrix}0\\23\end{Bmatrix} = -\tfrac{1}{2}b', \quad \begin{Bmatrix}0\\33\end{Bmatrix} = a',$$

$$\begin{Bmatrix}1\\22\end{Bmatrix} = -\tfrac{1}{2}a', \quad \begin{Bmatrix}1\\23\end{Bmatrix} = -\tfrac{1}{2}b', \quad \begin{Bmatrix}1\\33\end{Bmatrix} = \tfrac{1}{2}a',$$

$$\begin{Bmatrix}2\\02\end{Bmatrix} = -\tfrac{1}{2}a', \quad \begin{Bmatrix}2\\12\end{Bmatrix} = \tfrac{1}{2}a', \quad \begin{Bmatrix}2\\03\end{Bmatrix} = -\tfrac{1}{2}b', \quad \begin{Bmatrix}2\\13\end{Bmatrix} = \tfrac{1}{2}b',$$

$$\begin{Bmatrix}3\\03\end{Bmatrix} = \tfrac{1}{2}a', \quad \begin{Bmatrix}3\\13\end{Bmatrix} = -\tfrac{1}{2}a', \quad \begin{Bmatrix}3\\02\end{Bmatrix} = -\tfrac{1}{2}b', \quad \begin{Bmatrix}3\\12\end{Bmatrix} = \tfrac{1}{2}b'. \tag{4}$$

Similarly,

$$\underset{0}{\tilde{\Gamma}^{0}_{[01]}} = \underset{0}{k'_{01}}, \qquad \underset{0}{\tilde{\Gamma}^{0}_{[02]}} = \underset{0}{k'_{02}}, \qquad \underset{0}{\tilde{\Gamma}^{0}_{[03]}} = \underset{0}{k'_{03}},$$

$$\underset{0}{\tilde{\Gamma}^{0}_{[12]}} = \tfrac{1}{2}(\underset{0}{k'_{12}} - \underset{0}{k'_{02}}), \qquad \underset{0}{\tilde{\Gamma}^{0}_{[13]}} = \tfrac{1}{2}(\underset{0}{k'_{13}} - \underset{0}{k'_{03}}), \qquad \underset{0}{\tilde{\Gamma}^{0}_{[23]}} = \tfrac{1}{2}\underset{0}{k'_{23}},$$

$$\underset{0}{\tilde{\Gamma}^{1}_{[01]}} = \underset{0}{k'_{01}}, \qquad \underset{0}{\tilde{\Gamma}^{1}_{[02]}} = \tfrac{1}{2}(\underset{0}{k'_{02}} - \underset{0}{k'_{12}}), \qquad \underset{0}{\tilde{\Gamma}^{1}_{[03]}} = \tfrac{1}{2}(\underset{0}{k'_{03}} - \underset{0}{k'_{13}}),$$

$$\underset{0}{\tilde{\Gamma}^{1}_{[12]}} = \underset{0}{k'_{12}}, \qquad \underset{0}{\tilde{\Gamma}^{1}_{[13]}} = \underset{0}{k'_{13}}, \qquad \underset{0}{\tilde{\Gamma}^{1}_{[23]}} = \tfrac{1}{2}\underset{0}{k'_{23}},$$

$$\underset{0}{\tilde{\Gamma}^{2}_{[01]}} = \tfrac{1}{2}(\underset{0}{k'_{12}} + \underset{0}{k'_{02}}), \qquad \underset{0}{\tilde{\Gamma}^{2}_{[03]}} = -\tfrac{1}{2}\underset{0}{k'_{23}}, \qquad \underset{0}{\tilde{\Gamma}^{2}_{[13]}} = \tfrac{1}{2}\underset{0}{k'_{23}},$$

$$\underset{0}{\tilde{\Gamma}^{3}_{[01]}} = \tfrac{1}{2}(\underset{0}{k'_{13}} + \underset{0}{k'_{02}}), \qquad \underset{0}{\tilde{\Gamma}^{3}_{[02]}} = \tfrac{1}{2}\underset{0}{k'_{23}}, \qquad \underset{0}{\tilde{\Gamma}^{3}_{[12]}} = -\tfrac{1}{2}\underset{0}{k'_{23}}. \tag{5}$$

It is now easy to show that the equations

$$\underset{0}{h}_{[\gamma\mu\nu,\lambda]} = 0$$

imply that

$$\underset{0}{h}_{\mu\nu} = \text{constant}. \tag{6}$$

In fact, letting $\mu = \nu = 0$, $\lambda = 0, 1, 2, 3$ gives immediately

$$\underset{0}{h}_{0\lambda} = \text{constant}.$$

Similarly, $\mu = 0$, $\nu = 1$, $\lambda = 1, 2, 3$ gives

$$\underset{0}{h}_{11}, \underset{0}{h}_{12}, \underset{0}{h}_{13} = \text{constant},$$

$\mu = 0$, $\nu = 2$, $\lambda = 2, 3$ gives

$$\underset{0}{h}_{22}, \underset{0}{h}_{23} = \text{constant},$$

and, finally, $\mu = 0$, $\nu = 3$, $\lambda = 3$ gives

$$\underset{0}{h}_{33} = \text{constant}.$$

Without loss of generality, we can therefore take

$$\underset{0}{h}_{\mu\nu} = \eta_{\mu\nu}. \tag{7}$$

To determine $\underset{0}{k}_{\mu\nu}$ we proceed as follows. Let

$$\underset{0}{k}_{01} = p, \quad \underset{0}{k}_{02} = q, \quad \underset{0}{k}_{03} = r, \quad \underset{0}{k}_{12} = u, \quad \underset{0}{k}_{13} = v, \quad \underset{0}{k}_{23} = w.$$

Then, substituting the above expressions for $\underset{0}{\Gamma}{}^{\sigma}_{[\mu\nu]}$ in terms of the derivatives of $\underset{0}{k}_{\mu\nu}$ into the equations (viz. (2.17))

$$\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{0}{k}_{\sigma\nu} + \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{0}{k}_{\mu\sigma} = 0$$

we obtain the 24 equations:

$$\begin{aligned}
2pp' + q(u' + q') + r(v' + r') &= 0,\\
p(q' - u') + rw' &= 0,\\
p(r' - v') - qw' &= 0,\\
2pp' - u(u' + q') - v(v' + r') &= 0,\\
2p(q' - u') - vw' + rw' &= 0,
\end{aligned}$$

$$
\begin{aligned}
2p(r'-v')+uw'-qw' &= 0,\\
2qp'+2up'+2pu'-w(v'+r')-rw' &= 0,\\
2qq'+u(q'-u')-ww' &= 0,\\
2qr'+u(r'-v')-pw' &= 0,\\
2rp'+2vp'+2pv'+w(u'+q')+qw' &= 0,\\
2rq'+v(q'-u')+pw' &= 0,\\
2rr'+v(r'-v')-ww' &= 0,\\
p(u'-q')+vw' &= 0,\\
p(v'-r')-uw' &= 0,\\
-2qp'-2up'-2pq'+w(v'+r')+vw' &= 0,\\
2uu'+ww'+q(u'-q') &= 0,\\
2uv'+pw'+q(v'-r') &= 0,\\
2rp'+2vp'+2pr'+w(u'+q')+uw' &= 0,\\
2vu'-pw'+r(u'-q') &= 0,\\
2vv'+ww'+r(v'-r') &= 0,\\
(q+u)w' &= 0,\\
2rq'+2qr'+v(q'-u')+u(r'-v') &= 0,\\
2vu'+quv'+r(u'-q')+q(v'-r') &= 0,\\
(r+v)w' &= 0.
\end{aligned}
\tag{8}
$$

It is not difficult to show that (neglecting irrelevant, additive, constants of integration) the only solution of these equations is given by

$$
p = w = 0, \quad q = -u, \quad r = -v. \tag{9}
$$

Then, the nonzero components of $\underset{0}{k}_{\mu\nu}$ and $\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]}$ are

$$
\begin{aligned}
&\underset{0}{k}_{02} = -\underset{0}{k}_{12} = -u, && \underset{0}{k}_{03} = -\underset{0}{k}_{13} = -v,\\
&\underset{0}{\tilde{\Gamma}}{}^{0}_{[02]} = \underset{0}{\tilde{\Gamma}}{}^{\prime}_{[02]} = -u', && \underset{0}{\tilde{\Gamma}}{}^{0}_{[03]} = \underset{0}{\tilde{\Gamma}}{}^{\prime}_{[03]} = -v',\\
&\underset{0}{\tilde{\Gamma}}{}^{0}_{[12]} = \underset{0}{\tilde{\Gamma}}{}^{\prime}_{[12]} = u', && \underset{0}{\tilde{\Gamma}}{}^{0}_{[13]} = \underset{0}{\tilde{\Gamma}}{}^{\prime}_{[13]} = v'.
\end{aligned}
\tag{10}
$$

We may note that the equations

$$
\underset{0}{\tilde{\Gamma}}_{\mu} = 0
$$

are identically satisfied as required.

It is also evident that the field equations

$$\underset{0}{R}_{(\mu\nu)} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\varrho]}\underset{0}{\tilde{\Gamma}}{}^{\varrho}_{[\sigma\nu]} = 0 \tag{11}$$

are satisfied. For example,

$$\underset{0}{R}_{(23)} = \underset{0}{\tilde{\Gamma}}{}^{0}_{[20]}\underset{0}{\tilde{\Gamma}}{}^{0}_{[03]}+\underset{0}{\tilde{\Gamma}}{}^{0}_{[21]}\underset{0}{\tilde{\Gamma}}{}^{1}_{[03]}+\underset{0}{\tilde{\Gamma}}{}^{1}_{[20]}\underset{0}{\tilde{\Gamma}}{}^{0}_{[13]}+\underset{0}{\tilde{\Gamma}}{}^{1}_{[21]}\underset{0}{\tilde{\Gamma}}{}^{1}_{[13]},$$

$$= -u'v'+u'v'+u'v'-u'v' = 0,$$

$$\underset{0}{R}_{(33)} = \underset{0}{\tilde{\Gamma}}{}^{0}_{[30]}\underset{0}{\tilde{\Gamma}}{}^{0}_{[03]}+\underset{0}{\tilde{\Gamma}}{}^{0}_{[31]}\underset{0}{\tilde{\Gamma}}{}^{1}_{[03]}+\underset{0}{\tilde{\Gamma}}{}^{1}_{[30]}\underset{0}{\tilde{\Gamma}}{}^{0}_{[13]}+\underset{0}{\tilde{\Gamma}}{}^{1}_{[31]}\underset{0}{\tilde{\Gamma}}{}^{1}_{[13]},$$

$$= -v'^2+2v'^2-v'^2 = 0.$$

The remaining eight equations (11) can be checked out in the same way. We still have to satisfy equations

$$\underset{0}{R}_{[\mu\nu],\lambda}+\underset{0}{R}_{[\nu\lambda],\mu}+\underset{0}{R}_{[\lambda\mu],\nu} = 0$$

with

$$R_{[\mu\nu]} = -\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu],\sigma}.$$

However, when

$$\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]}(x^0-x^1),$$

we have

$$\underset{0}{R}_{[\mu\nu]} = -\underset{0}{\tilde{\Gamma}}{}^{0}_{[\mu\nu],0}-\underset{0}{\tilde{\Gamma}}{}^{1}_{[\mu\nu],1} = -\underset{0}{\tilde{\Gamma}}{}^{0}_{[\mu\nu]}{}'+\underset{0}{\tilde{\Gamma}}{}^{1}_{[\mu\nu]}{}'$$

and a glance at the solution (10) above shows that

$$R_{[\mu\nu]} \equiv 0,$$

(because for all μ, ν, $\tilde{\Gamma}^{0}_{[\mu\nu]} = \tilde{\Gamma}^{1}_{[\mu\nu]}$).

Now, in GFT, $R_{[\mu\nu]}$ is identified as (proportional to) the electromagnetic field tensor. In other words, although we have shown above that there exists a plane wave solution of the GFT field equations (actually, a class of solutions) for which the geometry of the space-time is nearly Minkowskian, but the skew field is not infinitesimal (a zeroth order approximation), no free electromagnetic field exists.

The functions a, b, u and v of x^0-x^1 are quite arbitrary.

4. One-dimensional waves in the first approximation

In the first approximation, the components of $\underset{1}{\Gamma}{}^{\sigma}_{[\mu\nu]}$ are now given by

$$\underset{1}{h}_{\mu\nu,\lambda}-\underset{1}{\begin{Bmatrix}\sigma\\ \mu\lambda\end{Bmatrix}}\eta_{\sigma\nu}-\underset{1}{\begin{Bmatrix}\sigma\\ \lambda\nu\end{Bmatrix}}\eta_{\mu\sigma} = \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{1}{k}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{1}{k}_{\mu\sigma}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{0}{k}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{0}{k}_{\mu\sigma} \tag{1}$$

and

$$\underset{1}{k}_{\mu\nu,\lambda}-\underset{1}{\begin{Bmatrix}\sigma\\ \mu\lambda\end{Bmatrix}}\underset{0}{k}_{\sigma\nu}-\underset{1}{\begin{Bmatrix}\sigma\\ \lambda\nu\end{Bmatrix}}\underset{0}{k}_{\mu\sigma} = \underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\eta_{\sigma\nu}+\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\eta_{\mu\sigma}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\lambda]}\underset{1}{h}_{\sigma\nu}+\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{1}{h}_{\mu\sigma}. \tag{2}$$

From equation (2), we obtain, in the usual way,

$$\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]} = \tfrac{1}{2}\eta^{\sigma\lambda}(\underset{1}{k}_{\lambda\nu,\mu}+\underset{1}{k}_{\mu\lambda,\nu}+\underset{1}{k}_{\mu\nu,\lambda})-\eta^{\sigma\lambda}\left(\underset{1}{\begin{Bmatrix}\varrho\\ \lambda\mu\end{Bmatrix}}\underset{0}{k}_{\varrho\nu}+\begin{Bmatrix}\varrho\\ \nu\lambda\end{Bmatrix}\underset{0}{k}_{\mu\varrho}+\underset{0}{\tilde{\Gamma}}{}^{\varrho}_{[\mu\nu]}\underset{1}{h}_{\varrho\lambda}\right). \tag{3}$$

However, instead of using this solution which is quite general, it is preferable, even if tedious, to write out the 64 equations inserting the zeroth order solution derived in the last section directly. We employ lexicographic order of indices, using at each stage results obtained previously. This reduces somewhat the number of independent equations that remain. Dropping the order indicator, we have

$$h'_{00} = 0,$$
$$0 = u\tilde{\Gamma}^{2}_{[01]}+v\tilde{\Gamma}^{3}_{[01]},$$
$$0 = u'k_{01}+u\tilde{\Gamma}^{2}_{[02]}+v\tilde{\Gamma}^{3}_{[02]},$$
$$0 = v'k_{01}+u\tilde{\Gamma}^{2}_{[03]}+v\tilde{\Gamma}^{3}_{[03]},$$
$$h'_{01} = 0,$$
$$0 = -u'k_{01}+u\tilde{\Gamma}^{2}_{[12]}+v\tilde{\Gamma}^{3}_{[12]},$$
$$0 = -v'k_{01}+u\tilde{\Gamma}^{2}_{[13]}+v\tilde{\Gamma}^{3}_{[13]},$$
$$h'_{02} = 0,$$
$$0 = u(-\tilde{\Gamma}^{0}_{[01]}+\tilde{\Gamma}^{1}_{[01]}),$$
$$0 = -u'(k_{02}+k_{12})-u\tilde{\Gamma}^{0}_{[02]}+u\tilde{\Gamma}^{1}_{[02]},$$
$$0 = -v'(k_{02}+k_{12})-u\tilde{\Gamma}^{0}_{[03]}+u\tilde{\Gamma}^{1}_{[03]}+u\tilde{\Gamma}^{2}_{[23]}+v\tilde{\Gamma}^{3}_{[23]},$$
$$h'_{03} = 0,$$
$$0 = v(-\tilde{\Gamma}^{0}_{[01]}+\tilde{\Gamma}^{1}_{[01]}),$$
$$0 = -u'(k_{03}+k_{13})-v\tilde{\Gamma}^{0}_{[02]}+v\tilde{\Gamma}^{1}_{[02]}-u\tilde{\Gamma}^{2}_{[23]}-v\tilde{\Gamma}^{3}_{[23]},$$
$$0 = -v'(k_{03}+k_{13})-v\tilde{\Gamma}^{0}_{[03]}+v\tilde{\Gamma}^{1}_{[03]},$$
$$h'_{11} = 0,$$
$$h'_{12} = u(\tilde{\Gamma}^{0}_{[01]}-\tilde{\Gamma}^{1}_{[01]}) = 0,$$

$$
\begin{aligned}
0 &= u'(k_{02}+k_{12})-u\tilde{\Gamma}^0_{[12]}+u\tilde{\Gamma}^1_{[12]},\\
0 &= v'(k_{02}+k_{12})-u\tilde{\Gamma}^0_{[13]}+u\tilde{\Gamma}^1_{[13]}-u\tilde{\Gamma}^2_{[23]}-v\tilde{\Gamma}^3_{[23]},\\
h'_{13} &= 0,\\
0 &= u'(k_{03}+k_{13})-v\tilde{\Gamma}^0_{[12]}+v\tilde{\Gamma}^1_{[12]}+u\tilde{\Gamma}^2_{[23]}+v\tilde{\Gamma}^3_{[23]},\\
&= v'(k_{03}+k_{13})-v\tilde{\Gamma}^0_{[13]}+v\tilde{\Gamma}^1_{[13]},\\
h'_{22}-a' &= 0,\\
0 &= u(-\tilde{\Gamma}^0_{[23]}+\tilde{\Gamma}^1_{[23]}),\\
h'_{23}-b' &= u'(k_{03}+k_{13})+v'(k_{02}+k_{12})+v\tilde{\Gamma}^0_{[02]}-v\tilde{\Gamma}^1_{[02]}+u\tilde{\Gamma}^0_{[03]}-u\tilde{\Gamma}^1_{[03]}\\
-h'_{23}+b' &= -u'(k_{03}+k_{13})-v'(k_{02}+k_{12})+v\tilde{\Gamma}^0_{[12]}-v\tilde{\Gamma}^1_{[12]}+u\tilde{\Gamma}^0_{[13]}-u\tilde{\Gamma}^1_{[13]}\\
0 &= v(-\tilde{\Gamma}^0_{[23]}+\tilde{\Gamma}^1_{[23]}),\\
h'_{33}+a' &= 0,
\end{aligned}
\tag{4}
$$

and

$$
\begin{aligned}
k'_{01} &= 0,\\
0 &= -\tilde{\Gamma}^1_{[02]}-\tilde{\Gamma}^0_{[12]}-2u'h_{01}-u'h_{11}-u'h_{00},\\
0 &= -\tilde{\Gamma}^1_{[03]}-\tilde{\Gamma}^0_{[13]}-2v'h_{01}-v'h_{11}-v'h_{00},\\
k'_{02}-\tfrac{1}{2}a'u-\tfrac{1}{2}b'v &= \tilde{\Gamma}^0_{[02]}-u'h_{00}-u'h_{01} = \tilde{\Gamma}^2_{[01]}-\tilde{\Gamma}^0_{[12]}-u'h_{00}-u'h_{01},\\
0 &= -\tilde{\Gamma}^2_{[02]}-u'h_{02}-u'h_{12} = -\tilde{\Gamma}^2_{[03]}-\tilde{\Gamma}^0_{[23]}-v'h_{02}-v'h_{12},\\
k'_{03}-\tfrac{1}{2}b'u+\tfrac{1}{2}a'v &= \tilde{\Gamma}^0_{[03]}-v'h_{00}-v'h_{01} = \tilde{\Gamma}^3_{[01]}-\tilde{\Gamma}^0_{[13]}-v'h_{00}-v'h_{01},\\
0 &= -\tilde{\Gamma}^3_{[03]}-v'h_{13} = -\tilde{\Gamma}^3_{[02]}+\tilde{\Gamma}^0_{[23]}-u'h_{03}-u'h_{13},\\
k'_{12}+\tfrac{1}{2}a'u+\tfrac{1}{2}b'v &= \tilde{\Gamma}^2_{[01]}-\tilde{\Gamma}^1_{[02]}-u'h_{01}-u'h_{11} = \tilde{\Gamma}^1_{[12]}-u'h_{01}-u'h_{11},\\
0 &= -\tilde{\Gamma}^2_{[12]}+u'h_{02}+u'h_{12} = -\tilde{\Gamma}^2_{[13]}+\tilde{\Gamma}^1_{[23]}+v'h_{02}+v'h_{12},\\
k'_{13}+\tfrac{1}{2}b'u-\tfrac{1}{2}a'v &= \tilde{\Gamma}^3_{[01]}-\tilde{\Gamma}^1_{[03]}-v'h_{01}-v'h_{11} = \tilde{\Gamma}^1_{[13]}-v'h_{01}-v'h_{11},\\
0 &= -\tilde{\Gamma}^3_{[12]}-\tilde{\Gamma}^1_{[23]}+u'h_{03}+u'h_{13} = -\tilde{\Gamma}^3_{[13]}+v'h_{03}+v'h_{13},\\
k'_{23} &= \tilde{\Gamma}^3_{[02]}-\tilde{\Gamma}^2_{[03]}+u'h_{03}+u'h_{13}-v'h_{02}-v'h_{12}\\
&= \tilde{\Gamma}^2_{[13]}-\tilde{\Gamma}^3_{[12]}+u'h_{03}+u'h_{13}-v'h_{02}-v'h_{12},\\
0 &= \tilde{\Gamma}^3_{[23]}.
\end{aligned}
\tag{5}
$$

Thus h_{00}, h_{01}, h_{02}, h_{03}, h_{11}, h_{12} and h_{13} are necessarily constant and there seems to be no reason why these components should not be taken as zero. This leaves us with the symmetric field exactly the same as in GR:

$$h_{22} = -h_{33} = a, \qquad h_{23} = b. \tag{6}$$

Of course, the same result follows if

$$u = v = 0. \tag{7}$$

Let us consider this case first. Then the physical field $\boldsymbol{g}$ itself becomes weak and

$$\underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]} = 0. \tag{8}$$

The components

$$\underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\mu\nu]} = \tilde{\Gamma}^{\sigma}_{[\mu\nu]}$$

are now given by

$$\begin{aligned}
&\tilde{\Gamma}^{0}_{[01]}l = k'_{01},\\
&\tilde{\Gamma}^{0}_{[12]} = -\tilde{\Gamma}^{1}_{[02]}, \quad \tilde{\Gamma}^{0}_{[13]} = -\tilde{\Gamma}^{1}_{[03]}, \quad \tilde{\Gamma}^{0}_{[23]} = -\tilde{\Gamma}^{2}_{[03]} = \tilde{\Gamma}^{3}_{[02]},\\
&\tilde{\Gamma}^{1}_{[23]} = \tilde{\Gamma}^{2}_{[13]} = -\tilde{\Gamma}^{3}_{[12]}, \quad \tilde{\Gamma}^{1}_{[23]} = \tilde{\Gamma}^{2}_{[13]} = -\tilde{\Gamma}^{3}_{[12]},\\
&\tilde{\Gamma}^{3}_{[02]} - \tilde{\Gamma}^{2}_{[02]} = \tilde{\Gamma}^{2}_{[13]} - \tilde{\Gamma}^{3}_{[12]} = k'_{23},\\
&\tilde{\Gamma}^{0}_{[02]} = \tilde{\Gamma}^{2}_{[01]} - \tilde{\Gamma}^{0}_{[12]} = k'_{02}, \quad \tilde{\Gamma}^{0}_{[03]} = \tilde{\Gamma}^{3}_{[01]} - \tilde{\Gamma}^{0}_{[13]} = k'_{03},\\
&\tilde{\Gamma}^{1}_{[12]} = \tilde{\Gamma}^{2}_{[01]} - \tilde{\Gamma}^{1}_{[02]} = k'_{12},\\
&\tilde{\Gamma}^{1}_{[13]} = \tilde{\Gamma}^{3}_{[01]} - \tilde{\Gamma}^{1}_{[03]} = k'_{13}
\end{aligned} \tag{9}$$

and

$$\tilde{\Gamma}^{2}_{[02]} = \tilde{\Gamma}^{3}_{[03]} = \tilde{\Gamma}^{2}_{[12]} = \tilde{\Gamma}^{3}_{[13]} = 0 = \tilde{\Gamma}^{3}_{[23]}.$$

Because also

$$\underset{1}{\tilde{\Gamma}}_{\mu} = 0, \tag{10}$$

the last five of the equations (9) give

$$\tilde{\Gamma}^{1}_{[01]} = \tilde{\Gamma}^{0}_{[10]} = 0, \quad \tilde{\Gamma}^{0}_{[20]} = -\tilde{\Gamma}^{1}_{[21]} = \tilde{\Gamma}^{1}_{[12]}.$$

Hence, we get immediately

$$k'_{01} = (k_{02} + k_{12})' = \tilde{\Gamma}^{2}_{[01]} = 0$$

and the nonvanishing components of the skew connection as

$$\tilde{\Gamma}^0_{[02]} = -\tilde{\Gamma}^0_{[12]} = \tilde{\Gamma}^1_{[02]} = -\tilde{\Gamma}^1_{[12]} = -k'_{12} \quad (= +k'_{02}),$$

$$\tilde{\Gamma}^0_{[23]} = \tilde{\Gamma}^1_{[23]} = -\tilde{\Gamma}^2_{[03]} = -\tilde{\Gamma}^3_{[12]} = \tilde{\Gamma}^3_{[02]} = \tilde{\Gamma}^2_{[13]} = \tfrac{1}{2}\, k'_{23},$$

$$\tilde{\Gamma}^0_{[03]} = k'_{03}, \quad \tilde{\Gamma}^1_{[13]} = k'_{13},$$

$$\tilde{\Gamma}^3_{[01]} = -(k'_{03}+k'_{13}), \quad \tilde{\Gamma}^1_{[03]} = -\tilde{\Gamma}^0_{[13]} = \tfrac{1}{2}(k'_{03}-k'_{13}),$$

$$\tilde{\Gamma}^2_{[23]} = -(k'_{03}+k'_{13}). \tag{11}$$

For this solution (equations (8) and the second of (2.16))

$$\underset{1}{R}_{(\mu\nu)} = -\begin{Bmatrix}\sigma\\ \mu\nu\end{Bmatrix}_{,\sigma} = -\begin{Bmatrix}0\\ \mu\nu\end{Bmatrix}' + \begin{Bmatrix}1\\ \mu\nu\end{Bmatrix}' = 0 \tag{12}$$

and

$$\underset{1}{R}_{[\mu\nu]} = -\tilde{\Gamma}^\sigma_{[\mu\nu],\sigma} = (-\tilde{\Gamma}^0_{[\mu\nu]}+\tilde{\Gamma}^1_{[\mu\nu]})'. \tag{13}$$

Hence the only nonzero components of the skew symmetric part of the Ricci tensor (in the first approximation) are

$$\underset{1}{R}_{[03]} = -\tfrac{1}{2}(k_{03}+k_{13})'' = -\underset{1}{R}_{[13]} \tag{14}$$

and it is easy to check that these satisfy

$$\underset{1}{R}_{[[\mu\nu],\lambda]} = 0.$$

With the GFT identification of the electromagnetic field tensor $f_{\mu\nu}$

$$\underset{1}{R}_{[\mu\nu]} = kf_{\mu\nu}, \quad k = \text{constant}, \tag{15}$$

and the standard relations

$$f_{0k} = -E_k, \quad f_{ij} = B_k, \quad i, j, k \text{ cyclic } 1, 2, 3$$

our result gives indeed a wave propagating in the positive x-direction (direction $\hat{i}$):

$$\boldsymbol{E} \wedge \boldsymbol{B} \propto ((k_{03}+k_{13})'')^2\hat{i}. \tag{16}$$

5. *On energy*

It is known (Ref. [4]) that a suitable definition of the energy-momentum tensor in GFT is given by

$$T_{\mu\nu} = +\frac{1}{\kappa}\left(R_{\mu\nu}\left(\begin{Bmatrix}\alpha\\ \beta\gamma\end{Bmatrix}\right) - \tfrac{1}{2}\, a_{\mu\nu}R\right), \tag{1}$$

where

$$R = a^{\mu\nu} R_{\mu\nu}\left(\begin{Bmatrix}\alpha \\ \beta\gamma\end{Bmatrix}\right).$$

(Such a tensor automatically satisfies the conservation equation

$$T^{\mu\nu}{}_{;\nu} = 0.$$

For the Christoffel brackets given by the equations (3.4)

$$\underset{1}{T}_{\mu\nu} = \frac{1}{\kappa}(\underset{1}{R}_{\mu\nu} - \eta_{\mu\nu}\underset{1}{R}) = \frac{1}{\kappa}\underset{1}{R}_{\mu\nu} \tag{2}$$

since

$$\underset{1}{R} = \eta^{\mu\nu}\underset{1}{R}_{\mu\nu} = 0. \tag{3}$$

Moreover, the only nonzero components of the symmetric Ricci tensor are

$$R_{00} = R_{11} = \tfrac{1}{2}(a'^2 + b'^2) = -R_{01}. \tag{4}$$

We shall now show that for the solution (4.14) also only these three components of the Maxwell energy-stress-momentum tensor (in the first approximation)

$$E_{\mu\nu} = f_{\mu\alpha} f^{\alpha}_{\nu} + \tfrac{1}{4}\eta_{\mu\nu} f_{\alpha\beta} f^{\alpha\beta} \tag{5}$$

(indices being, of course, raised with the Minkowski tensor η) of the classical electromagnetic field do not vanish identically.

Indeed, if

$$f_{03} = -f_{13}$$

are the only nonzero components of the field tensor $f_{\mu\nu}$, then

$$f_{\alpha\beta} f^{\alpha\beta} \equiv 0 \tag{6}$$

and

$$E_{00} = E_{11} = -E_{01} = f_{03}^2 = f_{13}^2. \tag{7}$$

Hence we can have, as in the Einstein-Maxwell theory,

$$E_{\mu\nu} \propto R_{(\mu\nu)}\left(\begin{Bmatrix}\sigma \\ \mu\nu\end{Bmatrix}\right) \tag{8}$$

and, with a clearly permissible adjustment of scale,

$$(k_{03} + k_{13})'' = \pm\sqrt{a'^2 + b'^2}. \tag{9}$$

If we choose the solution (see, e.g., Misner, Thorne & Wheeler, Ref. [10]) so that

$$b = 0, \tag{10}$$

we obtain a further restriction on the field

$$k'_{03}+k'_{13} = \pm a = 2\int_1 R_{[13]}d\xi \tag{11}$$

(ignoring again the constant of integration). In this way, geometry of the space-time becomes explicitly dependent on the electromagnetic field. Of course, when $b' \neq 0$ we can still define arbitrary functions A', θ by

$$a' = A' \cos\theta, \qquad b' = A' \sin\theta$$

and obtain

$$\pm A = k'_{03}+k'_{13}, \qquad \theta = \tan^{-1}\left(\frac{db}{da}\right)$$

but the geometric dependence on the field will be only implicit.

6. *The case when* $u, v \neq 0$

When u and v do not vanish, there is a non-weak (i.e. of zeroth order) field which apparently does not disturb the fact that (by hypothesis) the geometry of space-time differs only infinitesimally (i.e. to the first order) from Minkowskian. The above field is characterised by the skew field

$$g_{[\mu\nu]}$$

since we have retained only the weak components

$$h_{22} = -h_{33} = a, h_{23}$$

of the symmetric part. We shall show that there is no electromagnetic field in this case.

From equations (4.4), (4.5) we immediately have

$$\begin{aligned}
&\tilde{\Gamma}^1_{[01]} = \tilde{\Gamma}^0_{[01]} = \tilde{\Gamma}^3_{[13]} = \tilde{\Gamma}^2_{[12]} = \tilde{\Gamma}^3_{[03]} = \tilde{\Gamma}^2_{[02]} = \tilde{\Gamma}^3_{[23]} = 0,\\
&\tilde{\Gamma}^0_{[23]} = \tilde{\Gamma}^1_{[23]} = \tilde{\Gamma}^3_{[02]} = \tilde{\Gamma}^2_{[13]} = -\tilde{\Gamma}^2_{[03]} = -\tilde{\Gamma}^3_{[12]},\\
&\tilde{\Gamma}^3_{[02]}-\tilde{\Gamma}^2_{[03]} = \tilde{\Gamma}^2_{[13]}-\tilde{\Gamma}^3_{[12]},\\
&\tilde{\Gamma}^0_{[12]} = -\tilde{\Gamma}^1_{[02]}, \quad \tilde{\Gamma}^0_{[02]} = \tilde{\Gamma}^2_{[01]}-\tilde{\Gamma}^0_{[12]}, \quad \tilde{\Gamma}^1_{[12]} = \tilde{\Gamma}^2_{[01]}-\tilde{\Gamma}^1_{[02]},
\end{aligned}$$

whence

$$\tilde{\Gamma}^2_{[01]} = \tilde{\Gamma}^3_{[01]} = 0, \tag{1}$$

and so

$$\tilde{\Gamma}^0_{[13]} = -\tilde{\Gamma}^1_{[03]} = -\tilde{\Gamma}^0_{[03]} = \tilde{\Gamma}^1_{[13]}$$

which implies that

$$\tilde{\Gamma}^2_{[23]} = 0,$$

and (since $\tilde{\Gamma}^3_{[23]} = 0$)

$$\tilde{\Gamma}^0_{[02]} = -\tilde{\Gamma}^1_{[12]} = -\tilde{\Gamma}^0_{[12]} = \tilde{\Gamma}^1_{[02]}.$$

Equations (4.4) also give

$$u'(k_{02}+k_{12}) = u'(k_{03}+k_{13}) = v'(k_{02}+k_{12}) = v'(k_{03}+k_{13}) = 0. \tag{2}$$

If we reject the possibility

$$u' = v' = 0$$

as effectively leading to the case considered in Section 4,

$$k_{02} = -k_{12}, \quad k_{03} = -k_{13}. \tag{3}$$

It now follows that

$$h_{23} = b \tag{4}$$

as before.

We also have

$$k'_{01} = 0 = u'k_{01}+v\tilde{\Gamma}^3_{[02]} = v'k_{01}+u\tilde{\Gamma}^2_{[03]}. \tag{5}$$

Thus, either

$$k_{01} = 0 \tag{6}$$

or

$$k_{01} = \text{constant} \neq 0. \tag{7}$$

In the first case

$$\tilde{\Gamma}^3_{[02]} = 0 = k'_{23} \tag{8}$$

and the only nonzero components of $\tilde{\Gamma}^\sigma_{[\mu\nu]}$ are

$$\begin{aligned}\tilde{\Gamma}^0_{[03]} = -\tilde{\Gamma}^0_{[13]} = \tilde{\Gamma}^1_{[03]} = -\tilde{\Gamma}^1_{[13]} &= -k'_{13}-\tfrac{1}{2}\,b'u+\tfrac{1}{2}\,a'v,\\ \tilde{\Gamma}^0_{[02]} = -\tilde{\Gamma}^1_{[12]} = -\tilde{\Gamma}^0_{[12]} = \tilde{\Gamma}^1_{[02]} &= -k'_{12}-\tfrac{1}{2}\,a'u-\tfrac{1}{2}\,b'v.\end{aligned} \tag{9}$$

In the second case

$$uu'+vv' = 0 \quad \text{or} \quad u^2+v^2 = \text{constant}$$

and

$$\tilde{\Gamma}^3_{[02]} = \text{constant}. \tag{10}$$

Only the first case needs to be considered. Then, however, we easily check that both

$$\underset{1}{R}_{(\mu\nu)} = \underset{1}{R}_{[\mu\nu]} = 0. \tag{11}$$

7. *Conclusions*

We have considered in this work the first steps required for a full discussion of what kind of wave propagation is predicted by the Generalised Field Theory of macrophysics. There is, of course, no doubt that wave-like motion exists in Nature at every level of physical reality (there are two only that need to be specified; microphysical at which the Uncertainty Principle cannot be ignored, and macrophysical at which it is emphatically not taken into account).

We have seen that the GFT field equations ((2.1) through to (2.5)) allow at least plane-wave solutions of a very general kind although we have restricted ourselves to the case where the geometry of space-time is only slightly perturbed from its background, flat state. We can expect such situation to occur in regions free of material sources of the field itself. This does not tell us what the waves themselves might be. No problem arises in classical physics because there we know from the start what we are talking about. If we are discussing the structure of the electromagnetic field, our waves will be electromagnetic waves even if the theory may lead to identification of light as an electromagnetic phenomenon. If the theory is that of the gravitational field, then any wave-like solution automatically involves the postulate that it represents gravitational waves whether these are observed or not. Their empirical discovery would provide a significant confirmation of the theoretical description but, unlike more tangible predictions, nondiscovery cannot be regarded as destructive. We can never be certain *a priori* that any proposed apparatus is suitable or that the observation preventing noise has been adequately eliminated.

Now there is an important aspect in which GFT differs from General Relativity. It is that it does not specify *a priori* the physical nature of the field whose structure it claims to describe. Indeed, it was Einstein's mistake (however natural if not actually forced by the vague character of what he sought) to equate over-hastily $h_{\mu\nu}$ with gravitation and $k_{\nu\mu}$ with electromagnetic intensity. Instead, and more so than any other proposal for macrophysical unification of Nature, GFT speaks of the total (macrophysical) field specified by the tensor $\boldsymbol{g}$, even if when it is an (0, 2) tensor in a four-dimensional world, little room seems to be left for anything more than gravitation and electromagnetism. This follows from the structure of both GR and Maxwell's theory.

Nevertheless, there is room if Γ_μ is postulated as (proportional to) the electromagnetic vector potential rather than $k_{\mu\nu}$ or something like it as the intensity field. This (equation (2.4)) is equivalent to regarding the tensor $R_{[\mu\nu]}(\tilde{\Gamma}^\sigma_{(\mu\nu)})$ as giving the latter. The reason is that then we have effectively twenty functions (sixteen components of $\boldsymbol{g}$ and four of Γ_μ) with which to describe macrophysical reality. It is very gratifying to note that the solution presented in Section 3 indicates that GFT appears to be more comprehensive than a mere combination of gravitational and electromagnetic theories. What we have shown there was that GFT permits macrophysical, and for that matter not even necessarily "weak", waves which do not disturb the geometry of space-time. Equations (3.12) and the second of (6.11) show that these waves are not electromagnetic in the sense that, to the order of approximation considered, they correspond to a zero electromagnetic field. Taking into account the premises of GFT and, in particular, the way in which energy-momentum components are calculated, it is difficult to indicate how such waves could be experimentally detected. For this reason, as mentioned previously, we have refrained from speculating, at the present stage, whether waves such as these are definitely gravitational or whether they represent something more than either gravity or electromagnetism. Their apparent possibility, however, is instructive and will be considered further when other than only plane-polarised waves are studied. At present, we can note that their existence is fully in keeping with the spirit of GFT.

This brings us to a point which is frequently glossed over in theoretical physics. When symmetry of the field and of the connections is restored GFT collapses, as it should in accordance with its claim that it is its natural extension, into classical General Relativity. Indeed, because of the peculiar (Hermitian) symmetry of the field equations and especially of equation (2.1), this collapse is immediately brought about if we assume only the field g to be symmetric since then

$$\tilde{\Gamma}^{\sigma}_{\mu\nu} = \left\{ {\sigma \atop \mu\nu} \right\}_{h\equiv a}. \tag{1}$$

If only the connection $\tilde{\Gamma}$ is symmetric, equation (2.1) reduces to

$$h_{\mu\nu;\lambda} = 0 = k_{\mu\nu;\lambda}, \tag{2}$$

the first of which determines the brackets while the second implies (Ref. [1]) the existence of a four-vector ϕ_μ such that

$$k_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu}. \tag{3}$$

It is then easily shown that the field equations imply

$$h^{\alpha\beta}\phi_{\mu;\alpha\beta} = \phi^{\lambda}{}_{;\lambda\mu} \tag{4}$$

so that a wave solution is the consequence of a single gauge condition

$$\phi^{\lambda}_{;\lambda} = 0. \tag{5}$$

The bifurcation of gravitation and electromagnetism is then complete and, as far as their connection is concerned, we are left with the standard, Einstein-Maxwell theory. It can be regarded as a further confirmation of GFT, in the absence of direct empirical evidence, that the primitive physical situation of plane waves allows relation (5.8) to hold and to lead merely to a partial determination of the nonzero components of $k_{\mu\nu}$. It must be pointed out that, in general, there seems to be no reason why the tensor $E_{\mu\nu}$ constructed from

$$f_{\mu\nu} \propto R_{[\mu\nu]}\left(\left\{ {\sigma \atop \mu\nu} \right\}\right) \tag{6}$$

should always allow such determination when assumed to be proportional to

$$R_{(\mu\nu)}\left(\left\{ {\sigma \atop \mu\nu} \right\}\right).$$

Maybe, only those solutions should be regarded as physical for which this is the case but to demand it could be an unwarranted restriction on GFT. On the other hand, and this is the point we want to stress, any physical theory contains two kinds of assumptions. There are those which determine the structure of the theory. As far as GFT is concerned, the example in question is provided by the postulate of Hermitian symmetry of the field equations related by Einstein with their invariance under conjugation of the electric charge. Perhaps, more than anything else, it implies the macrophysical nature of the resulting theory because we know that such invariance can be broken at microphysical level.

Once the structure of the theory is established, another kind of assumption is involved in hypothesising a physical interpretation for the quantities appearing within it. Again, there is rarely any problem in distinguishing the two classes of assumptions in classical physics. The distinction, however, is important when exploring reality whose structure may not be clear from the outset. The reason is simply that the mathematical framework of a theory may be perfectly valid under one interpretation and lead to contradictions or, rather, paradoxes under another. To some extent, the postulate of charge conjugation already belongs to the second class of assumptions. A more direct example, however, is provided by our identification of

$$R_{[\mu\nu]}\left(\begin{Bmatrix}\sigma\\ \mu\nu\end{Bmatrix}\right)$$

as the electromagnetic field intensity tensor.

Now, originally (Ref. [4]), a different tensor was proposed as the intensity field, namely

$$\omega_{\mu\nu} = a^{\alpha\beta}k_{\mu\nu;\alpha\beta}. \tag{7}$$

The reasons why this proposal was made and why eventually that given by equation (6) was preferred need not concern us here. It is interesting, however, to note that for the plane wave solution of Section 4

$$\underset{0}{\omega}_{\mu\nu} = \underset{1}{\omega}_{\mu\nu} = 0.$$

In other words, our solution must be regarded as a deciding argument why the choice of $R_{[\mu\nu]}$, rather than of $\omega_{\mu\nu}$, is correct as well as, in view of equation (4.16), as another indication in favour of the Generalised Field Theory.

APPENDIX

When the field $\boldsymbol{g}$ itself is weak so that the components of the tensors $\boldsymbol{h}$ and $\boldsymbol{k}$ are of the same order of magnitude as those of the metric $\boldsymbol{a}$ and

$$\boldsymbol{g} = \boldsymbol{\eta}+\boldsymbol{h}+\boldsymbol{k}+\ \dots, \tag{1}$$

then

$$\tilde{\Gamma}^{\sigma}_{\mu\nu} = \underset{1}{\tilde{\Gamma}}{}^{\sigma}_{\mu\nu}+\text{higher orders}. \tag{2}$$

The third of the equations (2.17) and equations (2.5) then give

$$\tfrac{1}{2}\,\eta^{\sigma\lambda}(h_{\lambda\nu,\mu}+h_{\mu\lambda,\nu}-h_{\mu\nu,\lambda}) = \begin{Bmatrix}\sigma\\ \mu\nu\end{Bmatrix} \tag{3}$$

or

$$h_{\lambda\mu,\nu} = a_{\lambda\mu,\nu} \tag{4}$$

and it follows that

$$a_{\mu\nu} = a_{\mu\nu}(\xi).$$

However, we have seen (equations (3.10)) that a solution of the form

$$g = \underset{0}{\eta} + \underset{1}{k} + \underset{1}{h} + k + \dots \tag{5}$$

is also possible. The metric hypothesis then gives

$$\underset{1}{a}_{\lambda\mu,\nu} = h_{\lambda\mu,\nu} - \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{1}{k}_{\sigma\mu} - \underset{0}{\tilde{\Gamma}}{}^{\sigma}_{[\nu\mu]}\underset{1}{k}_{\lambda\sigma} - \underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\lambda\nu]}\underset{0}{k}_{\sigma\mu} - \underset{1}{\tilde{\Gamma}}{}^{\sigma}_{[\nu\mu]}\underset{0}{k}_{\lambda\sigma} \tag{6}$$

and, although on the right-hand-side we have functions of ξ only, the components $a_{\lambda\mu}$ can appear to be (linear) functions of x^2 and x^3 in general.

Editorial note. This article was proofread by the editors only, not by the author.

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