EMISSION OF SOFT-PIONS IN K^-p INTERACTIONS

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Current algebra and Partially Conserved Axial-vector Current (PCAC) hypothesis are applied to study the hyperon production processes in kaon-proton interactions involving one or two pions. The differential cross-sections for the processes are normalized to the differential cross-sections for the corresponding processes without pions. Theoretical predictions for the ratio of cross-sections at various kaon laboratory momenta are compared with the experimental data. Angular distributions are also given for different incoming kaon momenta.

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1. Introduction

It is well known that significant advances in the study of strong interaction processes became possible with the advent of current algebra. Many of the useful results using current algebra have been derived in conjunction with partially conserved axial vector current hypothesis, generally known as PCAC. The current algebra-PCAC formalism being well suited for the S-wave processes, in the recent years $\bar{p}p$ annihilations [1, 2] and kaon-nucleon [3] interactions which occur in S-state have been studied. The considerable amount of experimental data available for these processes enables one to relate a reaction amplitude to another, where one or more additional "soft" pions are present. This sort of relationship is a general feature of current algebra. In the past, the formalism has been applied mainly to strong processes other than those that involve hyperons. In the study of hyperon decays, the formalism is not found to be as successful as, say, the kaonic decays. So it is interesting to see how well the experimental results can be reproduced by the current algebra--PCAC formalism when hyperons are involved. Towards this end, in this paper we consider

$$K^-p \rightarrow \Xi K + n\pi^0,$$

where the n-pions are considered to be soft. We limit ourselves to one or two soft pions in the neutral mode, so as to simplify the theoretical manipulations.

2. The $Kp \to \Xi K + 2\pi^0$ amplitude and differential cross-section

For the processes involving "soft" pions, the amplitude is continued off the mass shell in the pion mass variable. Since the LSZ reduction formulae provide a natural basis for defining the off mass shell amplitudes we begin with

$$\int d^4x d^4y e^{-ik_1\cdot x} e^{-ik_2\cdot y} \langle \mathrm{f}| \mathrm{T}[\phi_\pi^\alpha(x), \phi_\pi^\beta(y)] |\mathrm{i}\rangle, \tag{1}$$

where i and f are initial and final states and k_1, k_2 are the momenta of the pions with isospin indices α and β and ϕ_π is the pion field operator. According to PCAC

$$\partial_\mu A_\mu^\alpha = \frac{C_\pi}{\sqrt{2}} \phi_\pi^\alpha, \tag{2}$$

where C_π is known as the PCAC constant; $G_A \simeq 1.18$, g_r is the rationalized, renormalized pion-nucleon coupling constant ($g_r^2/4\pi \approx 14.6$) and M_N and μ are the nucleon and pion masses respectively. Rewriting Eq. (1) with the help of Eq. (2) in terms of the axial-vector current and bringing the derivatives through time ordered products gives rise to various commutators.

$$\delta(x_0 - y_0) [A_0^\alpha(x), \partial_\mu A_\mu^\beta(y)] = \delta_{\alpha\beta} \sigma(x) \delta(x-y) \tag{3}$$

is dropped since they are of the same order as the PCAC correction terms. Since the pions are in the same (neutral) charge state the commutator

$$\delta(x_0 - y_0) [A_0^\alpha(x), A_\mu^\beta(y)] = i\delta(x-y) \varepsilon_{\alpha\beta\gamma} V_\mu^\gamma(x) \tag{4}$$

vanishes. Integrating the resulting expression by parts we have

$$1/2(k_1^\mu k_2^\nu + k_2^\nu k_1^\mu) M_{\mu\nu}^{\alpha\beta} = -\frac{C_\pi^2}{2} \frac{1}{(\mu^2 + k_1^2)(\mu^2 + k_2^2)} M_{2\pi}^{\alpha\beta}, \tag{5}$$

where

$$M_{\mu\nu}^{\alpha\beta} = \int d^4x d^4y e^{-ik_1\cdot x} e^{-ik_2\cdot y} \langle \mathrm{f}| \mathrm{T}[A_\mu^\alpha(x), A_\nu^\beta(y)] |\mathrm{i}\rangle \tag{6}$$

and

$$M_{2\pi}^{\alpha\beta} = \int d^4x d^4y e^{-ik_1\cdot x} e^{-ik_2\cdot y} (\mu^2 - \Box_x)(\mu^2 - \Box_y) \langle \mathrm{f}| \mathrm{T}[\phi_\pi^\alpha(x), \phi_\pi^\beta(y)] |\mathrm{i}\rangle. \tag{7}$$

$M_{2\pi}^{\alpha\beta}$ and $M_{\mu\nu}^{\alpha\beta}$ are respectively the matrix elements for the emission of two pions and two axial-vector currents with momenta k_1 and k_2 in the process i $\to$ f. In the soft pion limit [4] the left hand side of Eq. (5) vanishes unless it has pole terms that go as k^{-2}. Such pole terms arise when axial-vector currents are attached to non-terminating external baryon lines [5]. The diagrams that give rise to pole terms of order k^{-2} are given in Fig. 1. The central interaction in the diagram is

$$m = A + B\not{Q}, \tag{8}$$

where $Q = q_1 + q_2$, the sum of the kaon momenta and A and B are scalar functions of Q^2, K^2 and $Q \cdot K$ with $K = p_1 + p_2$, the sum of the momenta of initial proton and final

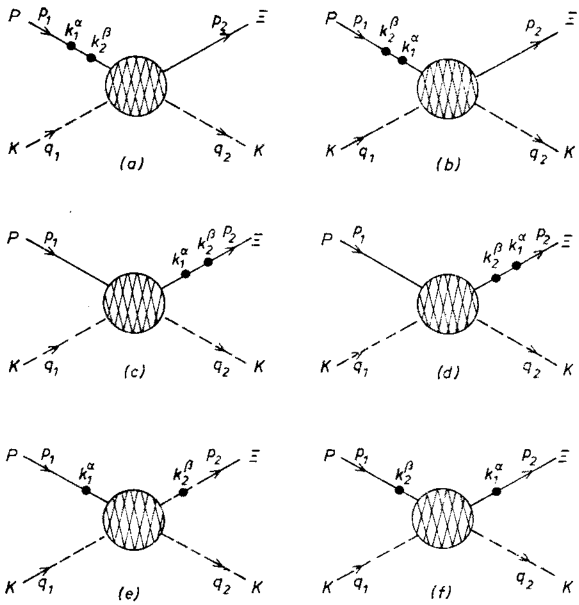

Fig. 1. Diagrams of order k^{-2} contributing to $M_{\mu\nu}^{\alpha\beta}$

hyperon. To facilitate computation we drop the terms proportional to A as in the earlier studies [3, 6], where the results seem to indicate that the terms proportional to A^2 are indeed small compared to B^2 terms as assumed initially. For a process like $\alpha \to \beta + m\pi$ where α and β are different hadronic states, the matrix element is of zeroth order in the pion momenta [7]. So retaining only zeroth order terms in pion momenta, we have

$$M^{33}_{2\pi} = -\frac{g_r^2}{M_N^2} B\bar{u}(p_2)\left\{\frac{1}{(a+b)a}[-M_N^2\not{Q}\not{k}_2\not{k}_1 + iM_N b\not{Q}\not{k}_1 - iM_N a\not{Q}\not{k}_2 - ab\not{Q}]\right.$$

$$+\frac{1}{(a+b)b}[-M_N^2\not{Q}\not{k}_1\not{k}_2 + iM_N a\not{Q}\not{k}_2 - iM_N b\not{Q}\not{k}_1 - ab\not{Q}]$$

$$+\frac{1}{(c+d)c}[-M_H^2\not{k}_1\not{k}_2\not{Q} + iM_H d\not{k}_1\not{Q} - iM_H c\not{k}_2\not{Q} - cd\not{Q}]$$

$$+\frac{1}{(c+d)d}[-M_H^2\not{k}_2\not{k}_1\not{Q} + iM_H c\not{k}_2\not{Q} - iM_H d\not{k}_1\not{Q} - cd\not{Q}]$$

$$+\frac{1}{ad}[-M_N M_H\not{k}_2\not{Q}\not{k}_1 - iM_H a\not{k}_2\not{Q} - iM_N d\not{Q}\not{k}_1 + ad\not{Q}]$$

$$\left.+\frac{1}{bc}[-M_N M_H\not{k}_1\not{Q}\not{k}_2 - iM_H b\not{k}_1\not{Q} - iM_N c\not{Q}\not{k}_2 + bc\not{Q}]\right\} u(p_1), \qquad (9)$$

where $a = p_1 \cdot k_1$, $b = p_1 \cdot k_2$, $c = p_2 \cdot k_1$ and $d = p_2 \cdot k_2$. M_H is the mass of the hyperon Ξ. From the above equation one can find $|M_{2\pi}^{33}|^2$ averaged over the initial and summed over the final spin states. However, in the present case, the direct trace calculations happen to be a near impossible task. Therefore we take the limit $k_1 = k_2 = (\vec{0}, i\mu)$ and arrive at the required expression after some tedious manipulations. The expression thus arrived at for $\langle |M_{2\pi}^{33}|^2 \rangle$ is given in AUPH Report [8]. The differential cross-section for the reaction $\mathrm{Kp} \to \Xi\,\mathrm{K} + 2\pi^0$ is given by the expression

$$(d\sigma)^{2\pi^0} = \frac{M_N M_H}{2[(p_1 \cdot q_1)^2 - m_K^2 M_N^2]^{1/2}} \frac{\langle |M_{2\pi}^{33}|^2 \rangle}{(2\pi)^8} dm_{\Xi K}^2 dm_{\pi\pi}^2$$

$$\times d(\cos\theta_p) d(\cos\theta_K) d(\cos\theta_\pi) d\phi_1 d\phi_2 d\phi_3$$

$$\times \frac{1}{8M_{Kp}^2} [(M_{Kp}^2 + m_{\pi\pi}^2 - m_{\Xi K}^2)^2 - 4m_{\pi\pi}^2 M_{Kp}^2]^{1/2}$$

$$\times \frac{1}{4m_{\Xi K}^2} [(m_{\Xi K}^2 + m_\Xi^2 - m_K^2)^2 - 4m_{\Xi K}^2 m_\Xi^2]^{1/2} \frac{1}{8m_{\pi\pi}} (m_{\pi\pi}^2 - 4\mu^2)^{1/2}, \tag{10}$$

where we have considered $\mathrm{Kp}(p_1 + q_1 = R)$ system decaying into $\Xi\mathrm{K}(p_2 + q_2 = R_1)$ and $2\pi(k_1 + k_2 = R_2)$ and integrated over four of the twelve variables trivially. The remaining variables are chosen to be $m_{\Xi K}^2 = -R_1^2$, $m_{\pi\pi}^2 = -R_2^2$, θ_p (the angle between R_2 and p_1), θ_K (the angle between p_2 and R_1), θ_π (the angle between R_2 and k_2), ϕ_1 (the azimuthal angle between the R and R_2 in the R_1 rest frame), ϕ_2 (the azimuthal angle between R and R_1 in the R_2 rest frame) and ϕ_3 (the azimuthal angle between R_2 and R_1 in the R rest frame). The relevant equations connecting the terms in $\langle |M_{2\pi}^{33}|^2 \rangle$ with the variables in (10) are given in Ref. [8].

3. The Kp → Ξ Kπ⁰ amplitude and differential cross-section

In the last Section we have used current commutation relations together with the PCAC notion. However it is possible to use PCAC independent of the current algebra. Therefore to test the PCAC hypothesis we consider a single soft pion emission in $\mathrm{Kp} \to \Xi\,\mathrm{K}$. There is no time ordered product of pion field operators and therefore we have

$$ik_\mu M_\mu^3 = \frac{M_N G_A \mu^2}{g_r(\mu^2 + k^2)} M_\pi^3. \tag{11}$$

This is analogous to Eq. (5) in the last Section. Pole terms that go as k^{-1} on the left hand side have to be evaluated as before. Retaining only zeroth order terms and again dropping terms proportional to A one has

$$\langle |M^{\pi^0}|^2 \rangle = \frac{g_r^2 B^2}{2M_N^3 M_H} \left[\frac{1}{(p_1 \cdot k)^2} (-M_N^3 M_H Q \cdot Q k \cdot k - 4M_N^2 p_1 \cdot k Q \cdot p_2 Q \cdot k \right.$$

$$+2M_N^2 Q\cdot p_1 k\cdot kQ\cdot p_2+2M_N^2 Q\cdot Qk\cdot p_1 k\cdot p_2-M_N^2 Q\cdot Qk\cdot kp_1\cdot p_2)$$

$$+\frac{1}{(p_2\cdot k)^2}(-M_H^3 M_N Q\cdot Qk\cdot k-4M_H^2 Q\cdot kQ\cdot p_1 k\cdot p_2$$

$$+2M_H^2 k\cdot p_1 Q\cdot Qk\cdot p_2+2M_H^2 k\cdot kQ\cdot p_1 Q\cdot p_2-M_H^2 k\cdot kQ\cdot Qp_1\cdot p_2)$$

$$+\frac{1}{p_1\cdot kp_2\cdot k}(-4M_H^2 M_N^2 Q\cdot kQ\cdot k+2M_H^2 M_N^2 k\cdot kQ\cdot Q$$

$$-4M_N M_H Q\cdot kQ\cdot kp_1\cdot p_2+4M_N M_H Q\cdot p_1 Q\cdot kk\cdot p_2$$

$$+4M_N M_H Q\cdot kp_1\cdot kQ\cdot p_2-4M_N M_H Q\cdot p_1 k\cdot kQ\cdot p_2$$

$$\left.-4M_N M_H Q\cdot Qk\cdot p_1 k\cdot p_2+2M_N M_H Q\cdot Qk\cdot kp_1\cdot p_2)\right], \tag{12}$$

and

$$d\sigma^{\pi^0}=\frac{M_N M_H \pi}{[(p_1\cdot q_1)^2-m_K^2 M_N^2]^{1/2}}\frac{\langle|M^{\pi^0}|^2\rangle}{8(2\pi)^5}dm_{\Xi K}^2 d(\cos\theta_{p_2})d(\cos\theta_\pi)d\phi$$

$$\times\frac{1}{2m_{\Xi K}^2}[(m_{\Xi K}^2+m_K^2-m_\Xi^2)^2-4m_K^2 m_{\Xi K}^2]^{1/2}$$

$$\times\frac{1}{2M_{Kp}^2}[(M_{Kp}^2+m_{\Xi K}^2-m_K^2)^2-4M_{Kp}^2 m_{\Xi K}^2]^{1/2}. \tag{13}$$

Where $m_{\Xi K}^2 = -R_1^2$ is the invariant mass of the final system. θ_π is the angle between the pion and the proton in the $R(=p_1+q_1)$ rest frame. θ_{p_2} is the angle between the final particle Ξ and the $R_1(=p_2+q_2)$. ϕ is the relative azimuthal angle between the $p_1 q_1$ plane and the R_1 decay plane. The terms in $\langle|M^{\pi^0}|^2\rangle$ have to be expressed in terms of variables in (13) as in the two pion case, before any integration is carried out, to get the cross-section σ^{π^0}.

4. *Differential scattering cross-section for Kp → Ξ K*

The differential cross-sections of (10) and (13) have to be normalized to the corresponding process Kp → Ξ K where soft-pions are absent. The relevant expression is

$$d\sigma=\frac{M_N M_H}{[(p_1\cdot q_1)^2-m_K^2 M_N^2]^{1/2}}\frac{\langle|M|^2\rangle}{16\pi M_{Kp}^2}d(\cos\theta_{p_2})$$

$$\times[(M_{Kp}^2+m_\Xi^2-m_K^2)^2-4M_{Kp}^2 m_\Xi^2]^{1/2}, \tag{14}$$

where

$$\langle|M|^2\rangle=\frac{-B^2}{2M_N M_H}[M_N M_H Q\cdot Q-2Q\cdot p_1 Q\cdot p_2+Q\cdot Qp_1\cdot p_2]. \tag{15}$$

5. *Discussion and comparison of results*

The expressions given in (10) and (13) are valid only for low energies near threshold, but following the general practice are generalized to hold at other centre of mass energies, so as to make a comparison with experimental data. The data on experimental cross-sections for various kaon induced reactions is well tabulated [9–14]. Therefore we numerically integrate the expressions given in Eqs (10) and (13) over the relevant kinematic variables to find the ratio of cross-sections for various incoming kaon momenta. For the two pion production case the kaon threshold momentum in the lab system is 1.6417 GeV/c.

TABLE I

Calculated ratios of cross-sections for various centre of mass energies

$K^{\pm}$ lab momentum (GeV/c)	Centre of mass energies (GeV)	$\frac{\sigma(Kp \to \Xi K + 2\pi^0)}{\sigma(Kp \to \Xi K)}$	$\frac{\sigma(Kp \to \Xi K\pi^0)}{\sigma(Kp \to \Xi K)}$
1.49	2.019		0.000588
1.546	2.043		0.001196
1.606	2.07		0.002089
1.7	2.11		0.004332
1.8	2.152		0.007272
1.935	2.215		0.01231
2.331	2.367	0.0187	
2.412	2.398	0.02176	
2.516	2.437	0.03797	
2.64	2.484	0.07015	
2.87	2.568	0.19708	
3.00	2.614	0.5226	

However the results are given only from 2.331 GeV/c, since the numerical integration process became feasible only at that energy. The results thus obtained are given in Table I and comparison with the experimental data is made in Figs 2 and 3. For $Kp \to \Xi K + \pi\pi$, the theoretical values agree to a reasonable extent with experiment. For $Kp \to \Xi K\pi$, the theoretical values are smaller than the experimental ones. The discrepancy could be explained to some extent by noting that, though PCAC can be tested independent of the idea of current commutation relations, it is the combination of current algebra and PCAC that gives impressive predictions [15]. In the study of single soft pion production by Shrauner [16], the theoretical cross-sections seemed to be underestimated by a factor of 7. He argued that the large discrepancy might be due to the neglected S-wave pion-pion interactions. In the case of double pion production, the difficulty could be avoided since the equal time commutators of two axial-vector currents can be replaced by the iso-spin operator.

In comparing the theoretical results which are strictly valid for very low energies and for 'soft' pion production, we have not made any attempt at selecting experimental data relevant to soft pions. Also in the numerical integration that is carried out no attempt

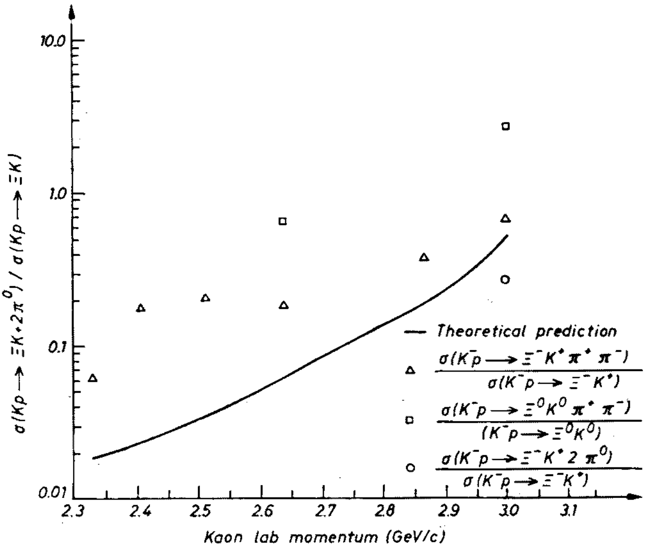

Fig. 2. Comparison of theoretical predictions of $\sigma(Kp \rightarrow \Xi K + 2\pi^0)/(Kp \rightarrow \Xi K)$, with experiment at various kaon laboratory momenta

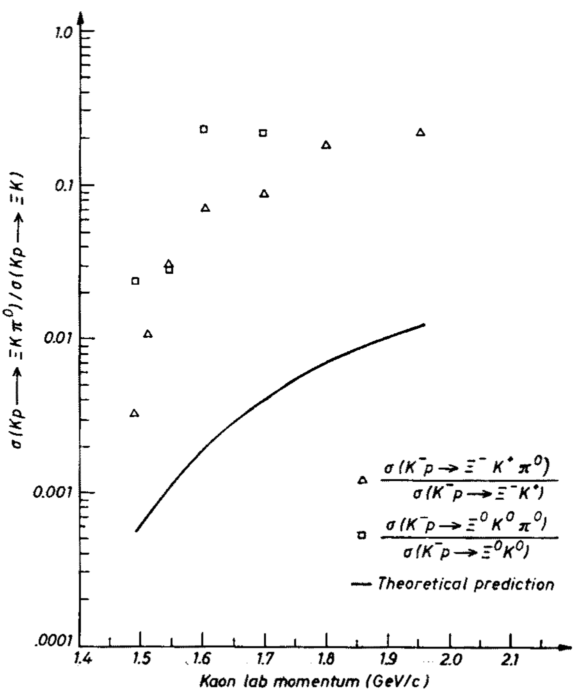

Fig. 3. Comparison of theoretical predictions of $\sigma(Kp \rightarrow \Xi K\pi^0)/\sigma(Kp \rightarrow \Xi K)$, with experiment at various kaon laboratory momenta

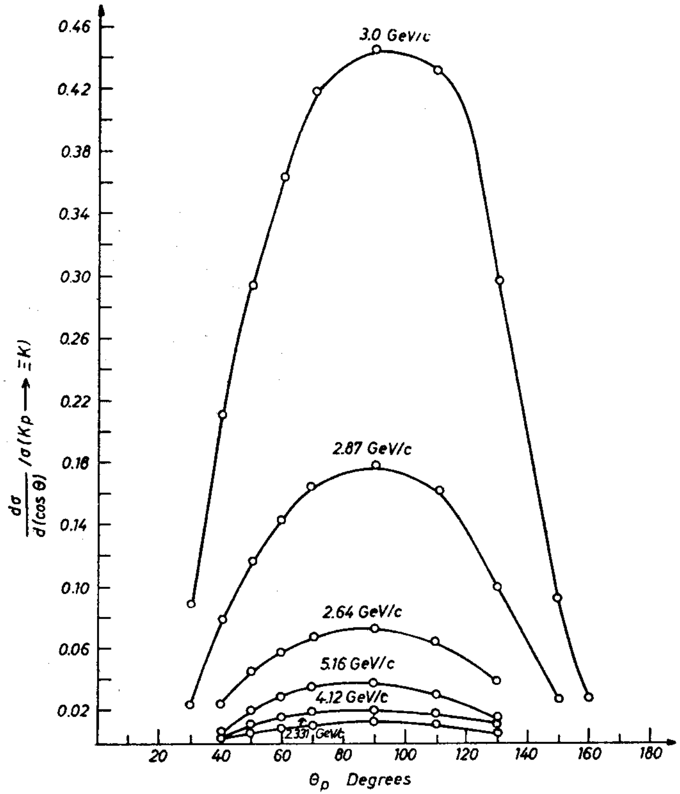

Fig. 4

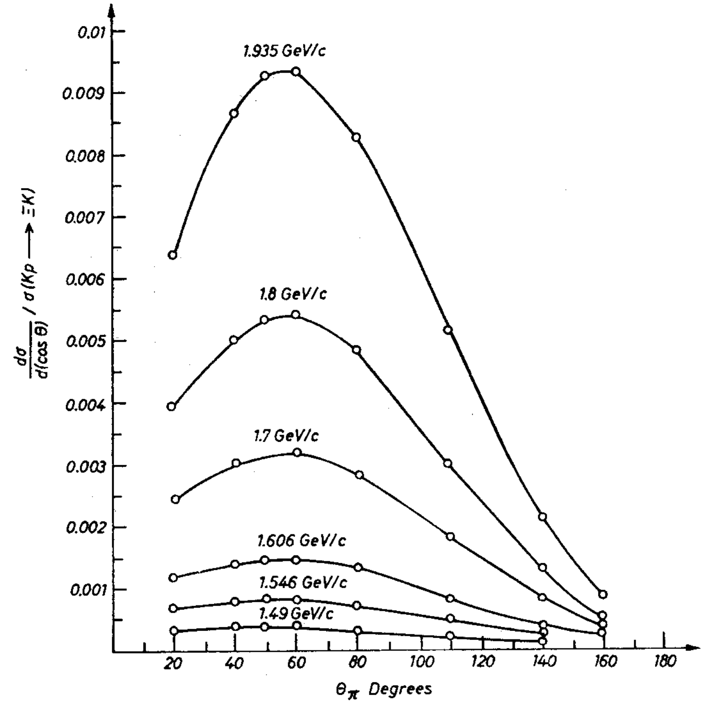

Fig. 5

Fig. 4. Variation of differential cross-section, for the reaction $Kp \rightarrow \Xi K + 2\pi^0$, with θ_p, the angle between the incoming proton and the (soft) dipion system

Fig. 5. Variation of differential cross-section, for the reaction $Kp \rightarrow \Xi K \pi^0$, with θ_π, the angle between the incoming proton and the (soft) pion

has been made to restrict the soft pion momenta, there by requiring a considerable extrapolation of PCAC. In this context it is worth noting that the expressions given in the most differential form are better suited for comparison with experiment. Towards this end integrating over some of the variables we obtain angular distributions which are given in Figs. 4 and 5.

It is indeed gratifying to find that the soft pion formalism applied in a straightforward way, without considering the intermediate resonance production and so on, does fit the experimental data to at least within an order of magnitude.

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