NUCLEON WAVE FUNCTION WITH RUNNING QUARK MASSES

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Three-quark nucleon wave function is constructed within the light-front framework. Quarks have running masses, which interpolate between the constituent and the current quark mass. In the spinor part of our model wave function all three Ioffe spin structures are needed. They are also required in the nucleon spinor currents in the QCD sum rules, if one asks for the maximal overlap with the state of the physical nucleon. In our calculations, the presence of three Ioffe spin structures is necessary to get simultaneously the negative value for the neutron charge radius and the decreasing d/u ratio in proton, which are well established experimental results. Selecting the coefficients in front of the Ioffe spin structures as: 1, 1.4 and 2.3, and the shape of a Gaussian distribution in the transverse momenta about 40% broader than that of the Isgur and Karl model, we get: $\langle r^2\rangle_{\mathrm{neutron}} = -0.11\ \mathrm{fm}^2$, $\langle r^2\rangle_{\mathrm{proton}} = 0.74\ \mathrm{fm}^2$, $\mu_{\mathrm{neutron}} = -1.7$, $\mu_{\mathrm{proton}} = 2.8$, $G_A/G_V = 1.1$, and the decreasing d/u ratio in proton's deep inelastic scattering, if x_{Bj} is increasing toward 1.

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1. Introduction

To describe the structure of hadrons one must enrich the QCD perturbative rules with the trully non-perturbative phenomena such as the quark and gluon condensates [1]. They manifest themselves in many circumstances, but most transparently in the value of the constituent quark mass [2]. For light quarks, for which we set their current quark mass in the QCD lagrangian equal to zero, the constituent quark mass is approximately equal to 1/3 of the nucleon mass. The quark mass runs, and interpolates [2] between the constituent and the current quark masses. By itself the running quark mass is a gauge dependent quantity [2], and only the hadron masses, or other hadron properties are gauge independent. If the running quark mass acts on spinors in the nucleon wave function, then the limiting value of it seems to be gauge independent as indicated in Ref. [3].

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The running quark mass depends on quark's virtuality [2], which in turn depends on the dynamical framework. In a Feynman diagram the quark virtuality is given by the square of quark's 4-momentum. However, in the light-front dynamics, in which all particles are on their mass shells, and the off-energy shell continuation is in the "—" component of total momentum, the measure of quark's virtuality is given by the inverse of free few body Green function $D_0 \equiv G_0^{-1}$. In a 3-quark system, with quark kinematics given by x_1, x_2, x_3, and transverse relative Jacobi momenta q_k and Q_k defined below in Eq. (2.1), the quantity D_0 is following [4]

$$D_0 = M^2 + q_k^2(1-x_k)(x_i x_j)^{-1} + Q_k^2(1-x_k)^{-1} x_k^{-1} - \sum_{n=1}^{3} \frac{m_n^2}{x_n}, \tag{1.1}$$

where M is the nucleon mass, and m_n are the current quark masses of individual quarks. D_0 is a quantity which frequently appears in the perturbative QCD formulated on the light front [4, 5]. One can extend the definition of D_0 to the low-energy domain, where the QCD non-perturbative features are important, by replacing the current quark mass m_n in Eq. (1.1) through the constituent quark mass $\cong \frac{1}{3} M$. More generally, in place of m_n one should put an explicit form of the running quark mass, as the function of quark's virtuality.

Strictly speaking D_0, devided by the number of partons in a given state, measures only the average parton virtuality [2]. The individual parton's virtuality is given by $x_i D_0$, where x_i is the i-th parton variable x. At low energies x_i can be approximated by the inverse of parton number, and then the average virtuality is a good measure of parton's virtuality. At high energies there appear interesting cases in which x_i takes values close to limits 1 or 0. Then, $x_i D_0$ has to be taken as the light-front measure of the individual parton virtuality. However, in some circumstances, in which the perturbative QCD rules apply, one can define a perturbative object — the distribution amplitude [4, 5]. It is a kinematical average of the current quark wave function. For this averaged QCD perturbative object one can use the average virtuality $\sim D_0$, as the measure of the average parton virtuality. If one of x_i is close to 1, then the average virtuality is large, and the running quark mass takes the value of the current quark mass, which is the quark mass in the QCD perturbative calculus. In the present paper we only consider two ends of the running quark mass values. However, if one would have an explicit form of the dependence of the running quark mass on the individual quark virtuality, then our model wave function could be used in an arbitrary case.

Beside the use of the running quark mass an important ingredient of our model is the spinor structure of the nucleon wave function. It is a slight extension of the nucleon spinor currents in the QCD sum rules [6]. We find, that for a satisfactory description of the experimental data all three Ioffe currents have to be taken into account [7], with approximately comparable weights. These three spinor structures produce in the nucleon wave function some asymmetric pieces. However, our asymmetry is much weaker than that of the Chernyak–Zhitnitsky [8] model. To illustrate that point we underline three basic differences between our approach and that of Chernyak and Zhitnitsky.

The first difference appears in the way of evaluating the low energy properties of nucleon. We consider [7] the diagram, which is the dominant one, with *no* hard gluon exchanges in it. This is the diagram in Fig. 1. In contrast to that, in the Chernyak–Zhitnitsky scheme [8] one always has to have the hard gluon exchange in the diagram if one evaluates the elastic electromagnetic form factors. The necessity of having the hard gluon in diagrams of the Chernyak-Zhitnitsky scheme follows from their choice of nucleon currents. It is implicit in their model for the distribution amplitude. The diagrams included in the Chernyak-Zhitnitsky scheme are non-leading. They are down by a factor

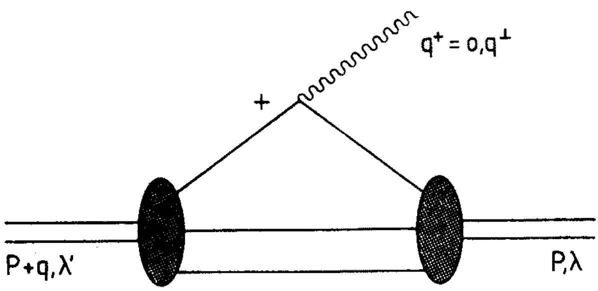

Fig. 1. The nucleon current J^+, with momentum transfer q ($q^+ = 0$), is expressed in terms of the nucleon wave functions. Note, that the incoming nucleon is on the right hand side, while the outgoing, with momentum $P+q$, on the left hand side

$\cong$ 100, in comparison with the leading diagram which has no hard gluon exchange. The factor 100 comes from $(\alpha_s/\pi)^2$, corresponding to two loops for QCD perturbative calculus of the nucleon form factor. This estimate is due to Radyushkin [9], and was also noted in Refs [10, 11]. Because the leading diagram for the electromagnetic form factor is left out in the Chernyak–Zhitnitsky scheme, and only the non-leading diagrams are kept, they are forced to consider a very asymmetric distribution amplitude to fit the experimental data. The significance of this asymmetry is however dubious, because the 100 times more important diagram is left out.

The second difference between Chernyak–Zhitnitsky [8] and us [7] is related to the choice of the nucleon currents in the QCD sum rules. In the basic work done in the application of the QCD sum rules in the baryon sector by Ioffe [6], and also in Refs [6, 9, 10], the nucleon spinor currents are without derivatives. In Ref. [12] it is shown, that if a combination of all three Ioffe spin structures acts on the physical vacuum, then one gets the maximal overlap with the state of the physical nucleon. On the other hand, the nucleon currents chosen by Chernyak and Zhitnitsky are currents with derivatives [13] (see Section 5.4 of Ref. [13]). These currents are direct consequence of the necessity of having the hard gluon in diagrams for the elastic electromagnetic form factor. The choice of the Chernyak–Zhitnitsky currents is implicit in their model for the distribution amplitude [8]. Such currents have some non-zero overlap with the physical nucleon, but it is not the maximal overlap.

The third difference between the Chernyak-Zhitnitsky [8] and our approach [7] is in the way of evaluating the low energy properties. We take the whole wave function, depending both on the longitudinal and on the transverse variables of three quarks. On the other hand, Chernyak and Zhitnitsky [8], also at low energies, consider only a distribution amplitude, called by them a "wave function" [8, 13], which in fact is an average of the wave function. In contrast to the true wave function, their distribution amplitude depends only on the longitudinal variables of individual quarks. The information about the transverse motion of quarks is lost in the Chernyak-Zhitnitsky scheme [8] at the very begining, by using only a distribution amplitude.

The main result of our paper is, that starting from the same model of a 3-quark wave function, once describing the constituent quark wave function, the other time the QCD perturbative current quark wave function, we can account both for the static neutron and proton properties, and for the decreasing d/u ratio in proton in the deep inelastic scattering. If one would take an SU(6) symmetric wave function [4, 14], then the d/u ratio in proton would be equal to 1/2 for *all* values of x_{Bj}. However, if the SU(6) symmetric wave function is followed by a particular QCD perturbative diagram, with two gluon exchanges [14] forming the "zigzag" force [4], then at $x_{Bj} = 1$ one gets $d/u = 0.2$. This "zigzag" force breaks [4] the SU(6) symmetry, in contrast to the one-gluon exchange, which retains it, and accounts for d/u $(x = 1) \neq 1/2$.

In literature [15, 16] there were some attempts to get the nucleon wave function starting from the SU(6) symmetric input, and then boost it to the infinite momentum frame. Such attempts were successful in accounting for the static properties of nucleon, but they totally fail [7] in obtaining a decreasing d/u ratio in proton if x_{Bj} is increasing toward 1. The reason for this is the lack of the 3-rd Ioffe spin structure in the nucleon wave function. By boosting the SU(6) symmetric input one gets two Ioffe structures, not the 3-rd one. We show analytically in Section 4, that the 3-rd Ioffe structure is *necessary* to avoid $\lim_{x\to 1} d/u = 1$. So far there was the following conflict [15]: either it was possible to get the negative value for the neutron charge radius, but then the d/u ratio in proton was an increasing function of x, for x increasing toward one, or the other way around. Each case is in direct conflict with the experimental data. Our model seems to be the first one which avoids that conflict, and which simultaneously gives the negative value for the neutron charge radius, and the decreasing d/u ratio in proton.

This paper is organized in the following way. In Section 2 we construct the model of the nucleon wave function and explain the appearance of three Ioffe spin structures. Then, we test our model in the low energy domain in Section 3, where the running quark mass is approximated by 1/3 M. We calculate the charge radii of proton and neutron, their magnetic moments, and the ratio G_A/G_V. The agreement with the experimental data is on the level of 10%. In Section 4 we pass to the high energy domain and work with the QCD perturbative wave function, written for the current (massless) quarks. We construct a kinematical average of the current quark wave function, which is the distribution amplitude. Working with this QCD perturbative average we do not need an explicit form of the running quark mass dependence on the quark's virtuality, and we can set the running quark mass equal to zero. We calculate approximately the deep inelastic structure function $F_2(x)$, and show analytically that the necessary condition for the decreasing value of d/u is the presence of the 3-rd Ioffe structure. Finally, in Section 5 we summarize our conclusions and make few remarks on related papers. The first Appendix contains the definition and some properties of the light-front spinors, together with the notation used in Table I for I_1, I_2 and I_3. The second Appendix includes comments on the transverse momentum integrals.

2. *Three-quark nucleon wave function*

Our model wave function for nucleon originates from the spinor structures, which appear in the nucleon spinor current in the QCD sum rules [6, 12]. We slightly generalize these structures by allowing each of the quark position to have a different value. In spinor currents, which are used in the QCD sum rules [6, 9, 10, 12], one puts all three quark spinor fields at the *same* space-time point x. We Fourier transform the quark fields and obtain their momentum spinor structures. Then, to keep quarks together we multiply the above spinor structures by a scalar factor f, which is a decreasing function of quark's momenta. As a particular example considered in this paper we take an exponential (Gaussian) function for this factor, but the Hulthen type factors can be also used, and were successfully applied in Refs [4, 5].

The scalar factor of our model wave function is written in terms of the basic invariant of the few body dynamics [4], which is the inverse of the 3-quark free propagator $D_0 \equiv G_0^{-1}$ in the light-front dynamics. The quark's kinematics is described in terms of the light-front variables x_i, and the relativistic, relative Jacobi [4] momenta q_k in terms of the quark's individual 4-momenta p_i $(i = 1, 2, 3)$

$$x_i = p_i^+/p^+ \equiv (p_i^0+p_i^3)/(P^0+P^3), \quad P = \sum_{i=1}^{3} p_i, \quad \sum_{i=1}^{3} x_i = 1,$$
$$q_k = (x_j p_i - x_i p_j)(x_i+x_j)^{-1},$$
$$Q_k = (x_i+x_j)p_k - x_k(p_i+p_j). \tag{2.1}$$

The momentum q_k is the relativistic Jacobi relative momentum in the (i, j) quark subsystem, while Q_k is the relative Jacobi momentum of the k-th quark with respect to the (i, j) subsystem. Note, that both q_k and Q_k have their "+" components identically equal to zero, which means that both of them are space-like 4-vectors. They describe the transverse degrees of freedom, while x_i describe the longitudinal degrees. In terms of these variables we have

$$D_0 = G_0^{-1} = P^+(P^-_{\text{initial}} - P^-) = P^2_{\text{initial}} - P^2$$
$$= M^2 + q_k^2(1-x_k)(x_i x_j)^{-1} + Q_k^2(1-x_k)^{-1}x_k^{-1} - \sum_{n=1}^{3} m_n^2 x_n^{-1}, \tag{2.2}$$

where M is the nucleon mass, and m_n are the individual quark masses. In obtaining Eq. (2.2) there are used the conservation laws of the light-front dynamics [4]: $P^{+,\perp}_{\text{initial}} = P^{+,\perp}$. Both

P^2 and $P^2_{\text{initial}} = M^2$ denote the Lorentz scalar products, i.e. $P^2 \equiv P_\mu P^\mu$. Therefore, D_0 is a manifestly Lorentz invariant quantity [4], although it originates from the light-front scheme, which requires the definition of the chosen "z" direction. (A given choice of the "z" axis is implicit in the definition of variables in Eq. (2.1)). Our model wave function is fully Poincare invariant, though its representation is written in terms of the light-front variables, given in Eq. (2.1).

The scalar factor f of our model wave function is chosen to be

$$f(p_1 p_2 p_3) = \exp(D_0/6\alpha^2), \tag{2.3}$$

where the factor 6 in the exponent is simply $2 \cdot 3$, and follows from the non-relativistic limit, allowing direct comparison with the α^2 parameter used in the constituent-quark non-relativistic model of Isgur and Karl [17].

To construct the spinor part of our model wave function we start from the generic form of the nucleon spinor current $\eta(x)$, used in the QCD sum rules [6, 9, 10, 12]

$$\eta(x) = [u^{\mathrm{T}}(x)\Gamma_1 C u(x)]\Gamma_2 d(x), \tag{2.4}$$

where x is the space-time point at which all quark fields $u(x)$, $u(x)$ and $d(x)$ are taken, C is the charge conjugation matrix, and Γ_1, Γ_2 are some chosen examples of 4×4 matrices. In correspondence with Eq. (2.4) the generic form of the spinor structures of our model is

$$\bar{\eta}(p_1 p_2 p_3) u_{p\lambda} = (\bar{u}_{p_1\lambda_1}\Gamma_1 C \bar{u}^{\mathrm{T}}_{p_2\lambda_2})\bar{d}_{p_3\lambda_3}\Gamma_2 u_{p\lambda}, \tag{2.5}$$

where $u_{p\lambda}$ is the nucleon spinor, and $u_{p_1\lambda_1}$, $u_{p_2\lambda_2}$, $d_{p_3\lambda_3}$ are three quark spinors in proton. (All spinors are the light-front spinors defined in Appendix A.) The quark mass in these spinors is the running quark mass. The total 4-momentum of nucleon is denoted, for simplicity, by the same letter P as the total momentum of 3 quarks, though of course we keep in mind that $P^-_{\text{nucleon}} \equiv P_{\text{initial}} \neq \sum_{i=1}^{3} p_i^-$.

In Eq. (2.4) there are excluded derivative couplings, as argued originally by Ioffe [6]. Therefore, in Eq. (2.5) we do not allow the *relative* quark's momenta to enter in explicitly. Only the total nucleon momentum of nucleon P can appear in Eq. (2.5). The Fermi statistics requires a symmetric matrix for $\Gamma_1 C$, and that, together with the above remarks, leads us to the following 5 possibilities for the structures $\Gamma_1 C \times \Gamma_2$:

$$\gamma^\mu C \times \gamma_\mu \gamma^5, \quad \frac{-i}{2}\varepsilon_{\alpha\beta\gamma\delta} i\sigma^{\alpha\beta} \times i\sigma^{\gamma\delta}, \quad \frac{1}{M^2}\not{P}C \times \not{P}\gamma^5, \quad \frac{1}{M}\not{P}C \times \gamma^5,$$

and $\frac{1}{M} i\sigma^{\mu\nu}P_\nu C \times \gamma_\mu\gamma^5$. Upon contracting with the nucleon spinor u_p the 3-rd structure is equivalent to the 4-th one, leaving us with 4 inequivalent structures. (Nucleon is on its mass shell $P^2 = M^2$, and $\not{P}u_p = Mu_p$.)

The final reduction to three Ioffe spin structures comes from requiring the correct isospin 1/2 for nucleon. To do that we first consider an auxiliary object, denoted by prime, and defined similarly as in Eq. (2.5), but with all spinors written with the same letter "u", and with Γ's having an extra upper index k, running through 4 values: 1, 2, 3, and 5 of four inequivalent structures. We have

$$I'_k(p_1\lambda_1, p_2\lambda_2, p_3\lambda_3) \equiv (\bar{u}_{p_1\lambda_1}\Gamma^k_1 C\bar{u}^{\mathrm{T}}_{p_2\lambda_2})\bar{u}_{p_3\lambda_3}\Gamma^k_2 u_{P\lambda}, \tag{2.6}$$

and in terms of these quantities we construct an "auxiliary state" $|P\lambda\rangle'$, also carying prime, which is a superposition of states created by the quark creation operators $u^{a\dagger}$, $u^{b\dagger}$, and $d^{c\dagger}$, with a, b, c denoting colours. It is

$$|P\lambda\rangle' = N \sum_{\lambda_1\lambda_2\lambda_3} \int [\prod_{i=1}^{3} (p_i^+)^{-1} dp_i^+ d^2 p_i]\delta(\sum_{j=1}^{3} p_j^+ - P^+)\delta^{(2)}(\sum_{j=1}^{3} p_j^{\perp} - P^{\perp})$$
$$f(p_1 p_2 p_3)\,[\sum_k a_k I'_k(p_1\lambda_1, p_2\lambda_2, p_3\lambda_3)]\varepsilon^{abc} u^{a\dagger}_{p_1\lambda_1} u^{b\dagger}_{p_2\lambda_2} d^{c\dagger}_{p_3\lambda_3}|0\rangle, \tag{2.7}$$

and the constant factors a_k have to be determined later. We discuss explicit values for a_k in Sections 3 and 4.

To insure the isospin 1/2 value we must act with the isospin 1/2 projection operator $P_{1/2}$ on the auxiliary state $|P\lambda\rangle'$. The projection operator is

$$P_{1/2} = \tfrac{1}{3}(\tfrac{15}{4} - \vec{T}^2),$$

where $\vec{T}$ is the isospin operator, and if $P_{1/2}$ acts on $|P\lambda\rangle'$ it reproduces it, if I'_k are replaced by the new quantities I_k defined through the following "symmetrization" procedure

$$I_k(123) \equiv \tfrac{1}{3}\{[I_k(123) - I_k(132)] + [I_k(213) - I_k(231)]\}. \tag{2.8}$$

This procedure is in fact the antisymmetrization in 2 and 3, both in helicities and in momenta, and then the result is symmetrized in 1 and 2. Under that we find $I_1 = I'_1$ and $I_2 = I'_2$, while I_5 gives the same result as the combination $\frac{1}{2}(I_1 + I_2) + 2I_3$. Therefore, among the "symmetrized" I_k structures there are only 3 independent structures, and we take them to be: I_1, I_2, and I_3. The first two correspond precisely to the first two Ioffe currents [6], and I_3 is the 3-rd Ioffe current contracted with the nucleon's momentum P. (Our sign convention is such, that in the *static* limit, in which only the constituent quark mass terms survive, we have the same signs in all three structures I_1, I_2, and I_3. We shall not make the static limit, but it is a good checking point.)

Having insured isospin 1/2 value, we propose the following model for the 3-quark nucleon wave function

$$\psi^{P\lambda}_{\lambda_1\lambda_2\lambda_3}(p_1 p_2 p_3) = N(aI_1 + bI_2 + cI_3)\exp(D_0/6\alpha^2). \tag{2.9}$$

The normalization of this wave function includes both N and a, and it is determined by the proton charge, therefore, beside the parameter α^2, we have only two independent relative weights b/a and c/a. I_1, I_2, and I_3 are catalogued in Table I, and the notation of various spin states is explained in Appendix A.

TABLE I

The components of three Ioffe spin structures I_1, I_2, and I_3. The notation of the spin components is explained in Appendix A

	I_1	I_2	I_3
S_1	$\frac{\lambda}{2}\left[\mu_1\mu_2+\mu_3-\kappa_1(\kappa_2^*-\kappa_3^*)\right]$	$\frac{\lambda}{2}(\mu_2\mu_3+\mu_1)$	$\frac{\lambda}{12}\left[\mu_1\mu_2\mu_3-2\mu_1\mu_2+\mu_3\mu_1+\mu_2-2\mu_3+1+\kappa_1\kappa_2^*(2-\mu_3)+\kappa_3^*\kappa_1(2\mu_2-1)\right]$
S_2	$\frac{\lambda}{2}\left[\mu_1\mu_2+\mu_3-\kappa_2(\kappa_1^*-\kappa_3^*)\right]$	$\frac{\lambda}{2}(\mu_1\mu_3+\mu_2)$	$\frac{\lambda}{12}\left[\mu_1\mu_2\mu_3-2\mu_1\mu_2+\mu_2\mu_3+\mu_1-2\mu_3+1+\kappa_1^*\kappa_2(2-\mu_3)+\kappa_2\kappa_3^*(2\mu_1-1)\right]$
S_3	$-\frac{\lambda}{2}(\mu_1+\mu_2)(1+\mu_3)$	$-\lambda(\mu_1\mu_2+\mu_3)$	$-\frac{\lambda}{12}\left[2\mu_1\mu_2\mu_3-\mu_2\mu_3-\mu_3\mu_1-\mu_1-\mu_2+2+\kappa_2^*\kappa_3(1+\mu_1)+\kappa_3\kappa_1^*(1+\mu_2)\right]$
$\bar{S}_1$	$\frac{1}{2}\left[(1+\mu_3)\kappa_2-\kappa_3\right]$	$\frac{1}{2}\mu_1(2\kappa_2-\kappa_3)$	$\frac{1}{12}\left[\kappa_2(2\mu_3\mu_1-\mu_3-1)+\kappa_3(-\mu_1\mu_2-\mu_2+2)+\kappa_1^*\kappa_2\kappa_3\right]$
$\bar{S}_2$	$\frac{1}{2}\left[(1+\mu_3)\kappa_1-\kappa_3\right]$	$\frac{1}{2}\mu_2(2\kappa_1-\kappa_3)$	$\frac{1}{12}\left[\kappa_1(2\mu_2\mu_3-\mu_3-1)+\kappa_3(-\mu_1\mu_2-\mu_1+2)+\kappa_1\kappa_2^*\kappa_3\right]$
$\bar{S}_3$	$-\frac{1}{2}\left[\mu_1\kappa_2+\kappa_1\mu_2\right]$	$-\frac{1}{2}\mu_3(\kappa_1+\kappa_2)$	$-\frac{1}{12}\left[\kappa_1(\mu_2\mu_3-2\mu_2+1)+\kappa_2(\mu_1\mu_3-2\mu_1+1)+2\kappa_1\kappa_2\kappa_3^*\right]$
S_4	$\frac{1}{2}\left[\mu_1(\kappa_2^*-\kappa_3^*)+\mu_2(\kappa_1^*-\kappa_3^*)\right]$	$\frac{1}{2}\left[\kappa_1^*+\kappa_2^*-2\kappa_3^*\right]$	$\frac{1}{12}\left[\kappa_1^*(\mu_2\mu_3-2\mu_2+\mu_3)+\kappa_2^*(\mu_3\mu_1-2\mu_1+\mu_3)+\kappa_3^*(-2\mu_1\mu_2+\mu_1+\mu_2)\right]$
$\bar{S}_4$	0	$\frac{\lambda}{2}\left[(\kappa_1+\kappa_2)\kappa_3-2\kappa_1\kappa_2\right]$	$\frac{\lambda}{12}\left[(\mu_1\kappa_2+\kappa_1\mu_2)\kappa_3-2\kappa_1\kappa_2\mu_3\right]$

3. Static properties of neutron and proton

We test our model first at low energies, and evaluate the dominating diagram [9] for the electromagnetic or weak form factors, shown in Fig. 1. The virtual photon (wavy line) has the "+" component of its momentum equal to zero, and the incoming nucleon is on the right hand side of Fig. 1, while the outgoing on the left (contrary to the standard drawing). The static properties such as the electric charges, magnetic moments, and G_A/G_V are determined by the appropriate form factors, and their derivatives, taken in the limit of zero momentum of the photon. Therefore, the diagram in Fig. 1 is dominated by the low relative momenta of quarks, corresponding to low quark's virtualities. Consecutively, the running masses of such quarks are, with good approximation, the constituent quark masses $\cong \frac{1}{3} M$. We assume, as it is usually done [3], that starting from momentum $\cong \frac{1}{3}$ GeV/c, and going down to zero the running quark mass remains to be constant, equal to the constituent quark mass $\cong \frac{1}{3} M$. Therefore, the nucleon wave functions, drawn as shaded vertices in Fig. 1, are given by Eq. (2.9) with all quark masses approximated by $\frac{1}{3} M$.

Although we evaluate Fig. 1 in the limit of zero photon momentum, we have to remember that there appear various derivatives of form factors, and therefore it is crucial to have both the correct treatment of the CMS motion of the whole nucleon, as well as the CMS motions of various 2-quark subsystems. We insure that by properly boosting the nucleon wave function. Note, that in the light-front dynamics, written in terms of the relative Jacobi momenta, all center of mass motions are properly taken into account [4]. The final nucleon wave function is properly boosted, and the relative Jacobi momenta q_k and Q_k are invariant under any change of the total nucleon momentum.

We assume, that the electromagnetic quark current of constituent quarks is the same as for the current quarks, thus we use

$$J^\mu = e_u u\gamma^\mu u + e_d d\gamma^\mu d, \tag{3.1}$$

where e_u, e_d denote charges. The matrix elements of J^μ between the nucleon states of helicities λ' and λ are denoted by $M^\mu_{\lambda'\lambda}$, and they define the form factors F_1, F_2 in the standard way

$$M^\mu_{\lambda'\lambda} = \bar{u}_{P+q,\lambda'}\left[\gamma^\mu F_1 + \frac{F_2}{4M}(\not{q}\gamma^\mu - \gamma^\mu \not{q})\right] u_{P\lambda}, \tag{3.2}$$

where q is the photon momentum. To get F_1 and F_2 we take the "+" component, and find [18]

$$M^+_{\uparrow\uparrow} = 2P^+ F_1, \quad M^+_{\uparrow\downarrow} = -2P^+(q^1 - iq^2)F_2/(2M) \tag{3.3}$$

where q^1, q^2 are the transverse components of the photon's momentum. Finally, F_1 and F_2 are replaced by the electric G_E and magnetic G_M form factors

$$G_E = F_1 + (q^2/4M^2)F_2, \quad G_M = F_1 + F_2,$$

and the electric charge radius is

$$\langle r^2 \rangle = 6 \left.\frac{d}{dq^2}\right|_{q=0} G_{\mathrm{E}}. \tag{3.4}$$

The electric form factor at zero momentum is equal to 1 for proton (the normalization of our wave function), and zero for neutron. $G_{\mathrm{M}}(0)$ gives the magnetic moments of proton and neutron in the respective cases.

It is useful to introduce the following complex momenta:

$$q = q^1 + iq^2, \quad q_j = q_j^1 + iq_j^2, \quad Q_j = Q_j^1 + iQ_j^2, \quad j = 1, 2, 3,$$

and their complex conjugates q^*, q_j^*, and Q_j^*. We also abbreviate the integration volume with the sign

$$\int \sum \equiv \int dx_1 dx_2 dx_3 \delta(1 - \sum_{n=1}^{3} x_n) d^2 q_j d^2 Q_j \sum_{\lambda_1 \lambda_2 \lambda_3} .$$

In this notation we get

$$F_1(0) = \sum_{i=1}^{3} e_i \int \sum |\psi^{\dagger}_{\lambda_1 \lambda_2 \lambda_3}(q_i Q_i x_1 x_2 x_3)|^2,$$

$$F_2(0) = -2 \frac{\partial}{\partial q^*} [\langle P+q, \uparrow | J^+ | P \downarrow \rangle]_{q=0}$$

$$= -2 \sum_{i=1}^{3} e_i \left\{ \int \sum \psi^{\dagger *}_{\lambda_1 \lambda_2 \lambda_3}(q_i Q_i x_1 x_2 x_3) \left[-(1-x_i) \frac{\partial}{\partial Q_i^*} \right] \psi^{\downarrow}_{\lambda_1 \lambda_2 \lambda_3}(q_i Q_i x_1 x_2 x_3) \right\},$$

$$\langle r^2 \rangle - \tfrac{3}{2} F_2(0) = -6 \frac{\partial}{\partial q} \frac{\partial}{\partial q^*} [\langle P+q, \uparrow | J^+ | P \downarrow \rangle] \Big|_{q=0}$$

$$= -6 \sum_{i=1}^{3} e_i \left\{ \int \sum \psi^{\dagger *}_{\lambda_1 \lambda_2 \lambda_3}(q_i Q_i x_1 x_2 x_3) \left[(1-x_i)^2 \frac{\partial}{\partial Q_i} \frac{\partial}{\partial Q_i^*} \right] \psi^{\dagger}_{\lambda_1 \lambda_2 \lambda_3}(q_i Q_i x_1 x_2 x_3) \right\}. \tag{3.6}$$

In a similar way we evaluate the weak form factors G_{A} and G_{V}. We take the matrix elements

$$M^{\mu}_{\lambda' \lambda} = \bar{u}^{\mathrm{p}}_{P' \lambda'} \gamma^{\mu} (G_{\mathrm{V}} - G_{\mathrm{A}} \gamma^5) u^{\mathrm{n}}_{P \lambda}, \tag{3.7}$$

where u^{p}, and u^{n} denote the proton and neutron spinors, respectively. Chosing the "+" component, and using relations: $\bar{u}_{\lambda'} \gamma^+ u = 2P^+ \delta_{\lambda' \lambda}$, $\bar{u}_{\lambda'} \gamma^+ \gamma^5 u_{\lambda} = 2\lambda P^+ \delta_{\lambda' \lambda}$, we get

$$M^+_{\lambda \lambda} = 2P^+ (G_{\mathrm{V}} - \lambda G_{\mathrm{A}}). \tag{3.8}$$

From different values of λ we find separately G_V and G_A. To calculate the left hand side of Eq. (3.8) we assume, that the weak current of constituent quarks has the standard form $J^\mu = \bar{u}\gamma^\mu(1-\gamma^5)d$. Then, for $G_V(0)$ we get similar expression as for $F_1(0)$, while for $G_A(0)$ we find

$$G_A(0) = -\int\sum[\lambda_1\psi^{\uparrow *}_{\lambda_1\lambda_3\lambda_2}(Q_1Q_3Q_2x_1x_3x_2)+\lambda_2\psi^{\uparrow *}_{\lambda_3\lambda_2\lambda_1}(Q_3Q_2Q_1x_3x_2x_1)]$$
$$\times\,\psi^{\uparrow}_{\lambda_1\lambda_2\lambda_3}(Q_1Q_2Q_3x_1x_2x_3). \tag{3.9}$$

In Eqs (3.6) and (3.9) the Gaussian integrals can be done analytically, while x_i integrals must be evaluated numerically. Some technical details are given in Appendix B. The value of the mass of the constituent quarks is kept very close to $\frac{1}{3}M$, and the parameter α^2 not too far from the value of the Isgur–Karl [17] model. There are three adjustable parameters: α^2 and two relative ratios of Ioffe structures b/a and c/a. We find them by minimizing the square deviations of the predicted static properties and the experimental data. The result is

$$\alpha^2 = (1.425\pm0.01)\alpha^2_{\text{Isgur-Karl}},\quad m_{\text{constit}} = (0.360\pm0.005)\,M,\quad b/a = 1.4\pm0.1,\quad c/a = 2.3\pm0.3,$$

and denoting the experimental numbers with "exp" we have

$$\langle r^2\rangle_{\text{neutron}} = -0.108\pm0.001, \qquad \langle r^2\rangle_{\text{proton}} = 0.735\pm0.005,$$
$$\text{exp.}\ -0.121\pm0.001, \qquad \text{exp.}\ 0.70\pm0.03,$$
$$\mu_{\text{neutron}} = -1.699\pm0.01, \qquad \mu_{\text{proton}} = 2.783\pm0.02,$$
$$\text{exp.}\ -1.913 \qquad \text{exp.}\ 2.793,$$
$$G_A/G_V = 1.115\pm0.003,$$
$$\text{exp.}1.251\pm0.007.$$

We note approximately 10% agreement with the experimental data. Slightly too small value for G_A/G_V we interpret as the neglection of the anomalous magnetic moment of the constituent quarks. The results of Refs [16] and [19] suggest 10% correction due to the presence of the anomalous magnetic moment. Knowing the response of the results of our model to the changes in parameters we expect, that a reasonable value of the anomalous magnetic moment of the constituent quark will raise G_A/G_V to 1.25, and also will bring the neutron magnetic moment and the neutron charge radius to the experimental numbers. The ratios b/a and c/a should remain comparable, around the value 2.

4. The d/u ratio in proton

Not having an explicit dependence of the running quark mass on the quark's virtuality we test our model at the other limit than the constituent quark mass, i.e. at the current quark mass value. We can do that, within some approximation, at high energies. In the deep inelastic scattering of proton, restricting our attention to large values of $x_{Bj} \geqslant 0.6$, we have the case when one of the valence quarks has large value of x, while the remaining two quarks have small x. In that kinematical situation the average quark's virtuality is large, since it is given by D_0, which contains the large factor $(1-x)^{-1}$. Therefore, if we permit ourselves to work with average quantities, like the distribution amplitudes [4, 5], then our quarks are the current, massless, QCD-perturbative quarks. If we would know the explicit form of the running quark mass formula, then we could evaluate directly the deep inelastic structure function $F_2(x)$, in which small values of the individual quark's virtualities, given by $x_i D_0$, are needed.

We now show how an approximate formula [4] for $F_2(x)$ can be found, if we have only the distribution amplitude, which by definition is a kinematical average of the QCD perturbative current quark wave function. It is advantageous to work with the partial wave distribution amplitudes, which were introduced in Ref. [4]. They are defined as follows

$$\phi_{l_i}(x_1x_2x_3\tilde{Q}) \equiv [d_F(\tilde{Q}^2)]^{-3/2} \int dq_k^2 dQ_k^2 \theta(\tilde{Q}^2+D_0)\psi_{l_i}(x_1x_2x_3q_kQ_k), \tag{4.1}$$

where ψ_{l_i} is the partial wave amplitude of the current quark wave function, l_i denotes the eigenvalue of the z-component of angular momentum, and $d_F(\tilde{Q}^2)$ is the renormalization factor, related to the quark field anomalous dimension γ by the equation

$$\gamma = -\tilde{Q}^2 \frac{d}{d\tilde{Q}^2} \ln d_F^{-1}.$$

$\tilde{Q}$ is the high energy scale, of the order of few GeV/c.

Together with the definition of ϕ_{l_i} it is useful to write down the explicit form of the appropriate projection operator

$$P_{\tilde{Q}} \equiv (16\pi^2)^2\delta(q_i^2)\delta(Q_i^2)\theta(\tilde{Q}^2+D_0). \tag{4.2}$$

It acts on ψ_{l_i} in the following way

$$P_{\tilde{Q}}\psi_{l_i} = (16\pi^2)^2\delta(q_i^2)\delta(Q_i^2) \int (16\pi^2)^{-2}dq_i'^2dQ_i'^2\theta(\tilde{Q}^2+D_0(q_i'Q_i'))\psi_{l_i}(x_1x_2x_3q_i'Q_i'). \tag{4.3}$$

In terms of $P_{\tilde{Q}}\psi_{l_i}$ we have an expansion of the partial wave current quark wave function ψ_{l_i}. We write this expansion, omitting the index l_i everywhere below for simplicity,

$$\psi = P_{\tilde{Q}}\psi+(1-P_{\tilde{Q}})G_0 \sum_{n=0}^{\infty} [V(1-P_{\tilde{Q}})G_0]^n \; P_{\tilde{Q}}\psi \cong P_{\tilde{Q}}\psi+(1-P_{\tilde{Q}})G_0VP_{\tilde{Q}}\psi.. \tag{4.4}$$

Beside the free 3-quark propagator $G_0 = D_0^{-1}$, there appears in Eq. (4 4) the interaction V, which is the kernel in the Weinberg equation for the three current quark wave function. From this expansion and from Eqs (4.1) and (4.3) we can obtain an approximate expression for the renormalized current quark wave function $d_F^{-3/2}\psi$, in terms of the distribution amplitude ϕ. We get

$$\begin{aligned} d_F^{-3/2}\psi \cong d_F^{-3/2}P_{\tilde{Q}}\psi+(1-P_{\tilde{Q}})G_0V d_F^{-3/2}P_{\tilde{Q}}\psi &= \phi\delta(q_k^2)\delta(Q_k^2)+(1-P_{\tilde{Q}})\bar{V}\phi \\ &\cong \phi\delta(q_k^2)\delta(Q_k^2). \end{aligned} \tag{4.5}$$

In Eq. (4.5) a new kernel $\bar{V}$ appears. In contrast to V the kernel $\bar{V}$ can be fully evaluated within the perturbative QCD. On the other hand the kernel V contains some QCD non--perturbative parts, which are yet unknown. This is one of the advantages of using the distribution amplitude ϕ, and connected with it the kernel $\bar{V}$, in contrast to the wave function ψ and V.

The distribution amplitudes ϕ obey evolution equations, which are integro-differential equations for ϕ, with $\bar{V}$ being the kernel. The evolution equations being partly differential equations require an input in order to find their solutions. We studied numerically in Ref. [4] various solutions of evolution equations, and found that quantitatively the output differs very little from the input. Thus, in practice to find the solution of evolution equations means finding a "sensible" input. The Hulthen and Gaussian inputs are among such sensible classes. Using the three current quark wave function, with the quark mass equal to zero in Eq. (2.9) and the definition of ϕ given by Eq. (4.1), we can produce a sensible input for the evolution equation, which numerically is almost identical with the solution of evolution equations.

Finally, we can show that the last line of Eq. (4.5) gives a good approximation of $d_F^{-3/2}\psi$, in the sense of approximations which are made in deriving the evolution equations [4, 5]. We distinguish two cases corresponding to the kernel $\bar{V}$ originating either from the one-, or from two-gluon exchanges. In the first case the term with $\bar{V}$ in Eq. (4.5) has an extra power of α_s in comparison with the retained term $\phi\delta\delta$. Thus, in the QCD perturbative regime, where α_s is small, the neglected term in Eq. (4.5) is down by a power of α_s in comparison with the retained term. In the second case $\bar{V}$ corresponds to two-gluon exchange diagrams. Among these diagrams there is the leading one, called the "zigzag" force [4], and the remaining 2-gluon exchange diagrams. The leading contribution in $F_2(x)$ comes from the "zigzag" force [4, 14], understood as an input for $\phi(x)$, and results in the $(1-x)^3$ behaviour in $F_2(x)$. The omitted non-leading term results in the $(1-x)^5$ behaviour in $F_2(x)$. Therefore, by powers of α_s or by the leading behaviour in $(1-x)$ the retained term in Eq. (4.5) is more important than the omitted one.

Note, that the distribution amplitude ϕ, which is the coefficient in front of the retained term in Eq. (4.5), contains all information present in ψ in the longitudinal direction, though it looses the information in the transverse directions. For large values of the Bjorken parameter, i.e. $x_{Bj} \geqq 0.6$, the main contribution to the deep inelastic structure function $F_2(x)$ comes from the 3-quark valence sector, which is described by $d_F^{-3/2}\psi$. In turn, the last object can be approximated by $\phi\delta\delta$, therefore we have the following sequence of approximations, valid for large x ($x \geqq 0.6$)

$$\begin{aligned} F_2(x) &\cong Ax \sum_a e_a^2 \sum_{b=a} \int [dx] d^2q_k d^2Q_k |d_F^{-3/2}\psi|^2 \delta(x_b - x) \\ &\cong Bx \sum_a e_a^2 \sum_{b=a} \int [dx]\, |\phi|^2 \delta(x_b - x). \end{aligned} \tag{4.6}$$

The first normalization constant A shows what is the total contribution of the three current quark sector in $F_2(x)$. It differs from the normalization constant determined in the previous Section, since there we had the "heavy" constituent quarks. The second normalization constant B in Eq. (4.6) is connected with the fact, that we have two Dirac δ functions in Eq. (4.5). Strictly speaking they should be understood as limits of the proper functions. Then, if we square $d_F^{-3/2}\psi$ we do not get an apparent singularity $\delta(0)$, but only the change of normalization from A to B. Both of these constants are irrelevant if we are only interested in the ratio $d(x)/u(x)$.

In the limiting case $x \to 1$ we can evaluate analytically the ratio d/u from our model wave function for three current quarks. Using the second approximation in Eq. (4.6), and the fact that the leading contribution to ϕ comes from the zero partial wave, since the other partial waves are down by the inverse powers of the scale $\tilde{Q}$, we get relatively simple expression for $F_2(x)$. (Many terms, which originally appear in Table I do not contribute if the quark mass is the current quark mass = 0 from the perturbative QCD, and if we only keep the leading partial wave distribution amplitude with $l_i = 0$.) The limiting value of the d/u ratio turns out to be only a function of the ratio of the weights of the 3-rd Ioffe spin structure I_3 and the 1-st one I_1. We get

$$\lim_{x\to 1} \frac{d(x)}{u(x)} = \frac{4\left(3-\frac{c}{a}\right)^2}{\left(\frac{c}{a}\right)^2 + \left(6-\frac{c}{a}\right)^2}, \tag{4.7}$$

and we see, that if $c/a = 0$ then the d/u ratio necessarily tends to 1, if $x \to 1$. Similarly, if $\frac{a}{c} = 0$, then $\lim_{x\to 1} d/u = 2$. Since the experimental data point towards $\lim_{x\to 1} d/u$ close to zero, neither the absence of the 3-rd Ioffe structure, nor the absence of the 1-st one are allowed by the data. If we take $c/a = 2$, then $\lim_{x\to 1} d/u = 0.2$, i.e. the same result as obtained many years ago [14] from the QCD perturbative rules. This result also corresponds to the presence of the particular "zigzag" force [4] in the kernel of evolution equations.

For x slightly smaller than 1 we can still expect the dominance of the 3-current-quark sector, and use our model to evaluate the behaviour of the d/u ratio as a function of x. We show in Fig. 2 several curves for the d/u ratio at different values of the ratio of the 3-rd and the 1-st Ioffe spin structure: $c/a \equiv c = 2$, or 2.4, or 2.6. From the variation of these curves we see, that the ratio $c/a = 2.3$, taken in Section 3 for evaluating the charge radii, magnetic moments, and G_A/G_V, corresponds to the decreasing behaviour of $d(x)/u(x)$ as x increases towards 1, and that our curve is well within the experimental errors.

An important feature of our model of 3-quark nucleon wave function is that we can *simultaneously* account for the *decreasing* behaviour of $d(x)/u(x)$, if x is increasing toward 1, and that we get the correct static properties of neutron and proton, including the *negative* value of the neutron charge radius (with the absolute value within 10% of the experimental data). There are two essential ingredients of our model which are responsible for these results. First is the presence of all three Ioffe spin structures I_1, I_2, and I_3, and second is the fact that the quark mass runs, and interpolates between the constituent and the current quark masses. To our knowledge, the nucleon model wave function given by Eq. (2.9) is the only one which gives both the decreasing value of the d/u ratio, and the negative value for the neutron charge radius. In all other models there is always a conflict with the experimental data, since either one has the correct static properties, but then the d/u ratio increases, or the other way around.

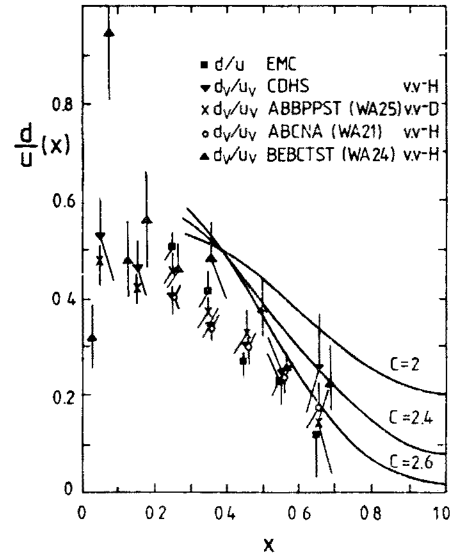

Fig. 2. The d/u ratio in proton in the deep inelastic scattering. $c \equiv c/a$ measures the weight of the I_3 spin structure with respect to the weight of the I_1 structure. The experimental data are from the review K. Rith, in Proceedings of HEP 83, Intern. Europhys. Conf. on High Energy Physics, Brighton 83, eds. J. Guy and C. Costain, Rutherford Appleton Lab., Chilton, Didcot, UK

The particular form of the scalar factor f of our model, in the form of the Gaussian function, is not crucial for obtaining our results. We took it only to simplify the numerical work. Other choices of f such as the Hulthen function should also work, and in general f should be a decreasing function of momenta. It is useful to have f as a function of the principal 3-quark invariant function D_0, which appears very frequently in the QCD perturbative calculations on the light front [4], and which has the direct analogy with the non--relativistic form of the kinetic energy of three constituent quarks.

5. *Conclusions and remarks*

Our model calculations lead to the following conclusions:

(*i*) if the running quark mass, interpolating between $\frac{1}{3}M$ and 0, is incorporated, then one can get both the low, and the high energy results,

(*ii*) the negative value of the neutron charge radius, and the decreasing d/u ratio in proton can be simultaneously obtained within one framework,

(*iii*) all three Ioffe spin structures: I_1, I_2 and I_3 are required for explaining the experimental data,

(*iv*) essentially 3 free parameters: the ratio of weights b/a, c/a, and the width of the Gaussian distribution α^2 are sufficient to get 5 static properties: $\langle r^2 \rangle_{p,n}$, $\mu_{p,n}$, and G_A/G_V, as well as the decreasing d/u ratio in proton (the constituent quark mass $\approx \frac{1}{3}M$, $b/a = 1.4$, $c/a = 2.3$, and α^2 is approximately 40% broader than that of Isgur and Karl [17]),

(*v*) a 10% deviation between the model prediction and the experimental data can be corrected if the anomalous magnetic moment of the constituent quarks is taken into account.

The role of the anomalous magnetic moment of constituent quarks was emphasized in Refs [16], and [19]. In one of them Belyaev and Kogan [19] explicitly included the interaction of the weak current with the quark condensate. They were able to explain the value of (G_A/G_V-1) as being caused by the presence of the extra QCD non-perturbative interaction. This is connected with the structure of the constituent quark.

In Ref. [16] a model of the nucleon wave function was constructed, in which an SU(6) symmetric wave function was boosted to the infinite momentum frame. Such procedure can miss out some terms, which go to zero in the non-relativistic limit. Indeed that model does not have terms contained in the 3-rd Ioffe spin structure I_3, but has only a superposition of the I_1, and I_2 terms. Therefore, in such scheme one is bound to get the incorrect, increasing d/u ratio in proton, although it accounts well for the static properties of nucleon.

In calculations of: the nucleon mass [6, 10, 12], the static properties of nucleon [6, 20], and the electromagnetic elastic form factors of nucleon [9], with the QCD sum rule technique, there is usually considered the nucleon spinor current, which corresponds only to the 1-st Ioffe spin structure I_1. The reason for that is the desire to satisfy two conflicting criteria [21], which are important within the QCD sum rule technique. One of them is to have the maximal overlap between the nucleon spinor current, acting on the physical vacuum, and the state of the physical nucleon. The other criterion is to have a minimal contribution from the higher order terms in the Wilson operator product expansion. The 1-st Ioffe spin structure I_1 makes a compromise [21] between these two conflicting criteria. We do not have to worry about the second criterion, if we only construct the model of the nucleon wave function. Staying with the 1-st criterion, of the maximal overlap with the state of the physical nucleon, it was shown in Ref. [12] that indeed the nucleon spinor current with all three Ioffe spin structures has the maximal overlap. Our results confirm these findings.

Calculations of the present paper should be extended in several directions. One is the test of some explicit formulas for the running quark mass, as the function of quark's virtuality. Then, one can evaluate the whole behaviour of the elastic electromagnetic form factors on the momentum, beside geting the static properties of nucleon. Also one can then directly evaluate the deep inelastic structure function $F_2(x)$, without the need of approximating $d_F^{-3/2}\psi$ by $\phi\delta\delta$. The second direction is the study of various forms of the scalar factor f. The Hulthen form should be tested, and some non-symmetric forms of f should be tried out, which would also allow more spinor structures than contained in I_1, I_2, and I_3. Finally, one should include the anomalous magnetic moment of the constituent quark, and verify numerically how large is its effect on the electromagnetic properties of nucleon.

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APPENDIX A

Light front spinors

To boost a particle of mass m from its rest frame to the frame where it has momentum p we denote the Lorentz transformation by L_{pm}. In terms of the light front components of the x-space 4-vector x it takes the form

$$x_p^- = \frac{m}{p^+} x_m^- + \frac{p^{\perp 2}}{p^+ m} x_m^+ + \frac{2p^\perp}{p^+} x_m^\perp, \quad x_p^+ = \frac{p^+}{m} x_m^+, \quad x_p = \frac{p^\perp}{m} x_m^+ + x_m^\perp,$$

where x_m, and x_p denote the components of x in two frames, respectively. The light front dynamics is invariant [4, 5] under the above boosts, for arbitrary values of m, p^+, and $p^\perp$.

The spinor representation of these boosts, denoted by $S(L_{pm})$, defines the light front spinors $u(p, \lambda)$, and also the meaning of the spin index λ. We have

$$u(p, \lambda) = S(L_{pm}) u(0, \lambda), \tag{A.1}$$

where $u(0, \lambda)$ is the Dirac spinor at rest [22]. In terms of the projection matrices [5] $\Lambda_\pm \equiv \frac{1}{2}(1 \pm \alpha^3)$, where $\alpha^3 \equiv \gamma^0\gamma^3$, and $\alpha^\perp \equiv \gamma^0\gamma^\perp$, we get [23]

$$S(L_{pm}) = \sqrt{\frac{2}{p^+}} \left[m\Lambda_- + (p^+ + \alpha^\perp \cdot p^\perp)\Lambda_+\right]. \tag{A.2}$$

One of the important properties of the light front spinor $u(p, \lambda)$ defined by Eq. (A.1) is, that it does not undergo the Wigner rotation for an arbitrary boost L_{qn}. This means that, for the *same* value of λ on both sides, the following relation holds

$$S(L_{qn}) u(p, \lambda) = u(L_{qn} p, \lambda). \tag{A.3}$$

Three Ioffe spin structures I_1, I_2, and I_3, which are listed in Table I, are denoted in the following convention. There are $2^3 = 8$ spin components of each wave function, corresponding to the given value λ of the spin of nucleon. The indices i, j, k run through 1, 2, 3, and different spin states of three quarks are denoted as follows:

$$s_i \equiv s_i^\lambda(\lambda_1\lambda_2\lambda_3) \equiv \delta_{\lambda_i\bar\lambda}\delta_{\lambda_j\lambda}\delta_{\lambda_k\lambda}, \quad \bar s_i \equiv s_i^{\bar\lambda}(\lambda_1\lambda_2\lambda_3),$$

$$s_4 \equiv s_4^\lambda(\lambda_1\lambda_2\lambda_3) \equiv \delta_{\lambda_1\lambda}\delta_{\lambda_2\lambda}\delta_{\lambda_3\lambda}, \quad \bar s_4 \equiv s_4^{\bar\lambda}(\lambda_1\lambda_2\lambda_3),$$

where $\bar\lambda \equiv -\lambda$.

The symbols κ_j and μ_j in Table I mean

$$\kappa_j \equiv (Q_j^1 + i\lambda Q_j^2) x_j^{-1}, \quad \mu_j \equiv \mu x_j^{-1}, \quad \mu \equiv m M^{-1}.$$

Note, that here, i.e. in Table I, the imaginary part of the complex quantities κ_j depend on the nucleon spin λ, in contrast to the complex momenta Q_j, which are used in Eq. (3.6).

APPENDIX B

Transverse momentum integrals

The scalar factor f of our model wave function depends on the transverse momenta through D_0, which is an even function of q_j and Q_j. Therefore, only such transverse momentum integrals are non-zero, which have coefficients of the form $|q_j|^{2m}|Q_j|^{2n}$, originating either from I_1, I_2, and I_3, and/or from the appropriate derivatives in Eq. (3.6). To evaluate such integrals it is advantageous to rescale q_j and Q_j, and replace them by the "universal" q and Q, so that we get

$$(D_0-M^2)(3\alpha^2)^{-1} = -|q|^2-|Q|^2-M^2\mu^2(x_1^{-1}+x_2^{-1}+x_3^{-1})(3\alpha^2)^{-1},$$

and

$$\int d^2q_j d^2Q_j = x_1x_2x_3\pi^2(3\alpha^2)^2\int d|q|^2 d|Q|^2.$$

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