PHENOMENOLOGICAL SUPERPARTICLES*

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The concept of a phenomenological superparticle is discussed as a base of an "engineering approach" rather than a recourse to first principles. It is a hypothetical object described by the position four-vector and a Dirac bispinor as well as three dynamical variables responsible for color and two responsible for flavor. Then, the spectrum of such a superparticle arises from an interplay of three types of excitations related to spin, color and flavor. All leptons and quarks are presumed to correspond to the spin-1/2 sector of this spectrum. Two flavor dynamical variables obeying Fermi statistics generate in a natural way a lefthanded SO(5) group that unifies the electroweak SU(2) with two lepton and quark generations. Two other generations appear as a spin excitation of the two former. A form of the effective mass operator is tentatively proposed in order to describe the mass spectrum and mixing parameters of leptons and quarks in consistency with experimental data.

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1. Introduction

Under the name of a *phenomenological superparticle* [1] we will understand a hypothetical physical object described by the position four-vector x^μ, $\mu = 0, 1, 2, 3$, and a Dirac bispinor ψ_α, $\alpha = 1, 2, 3, 4$, (and their canonical momenta p_μ and $\bar{\psi}_\alpha = (\psi^+\beta)_\alpha$) as well as dynamical variables responsible for color and flavor. On the level of quantum mechanics or the first-quantization level all these dynamical variables are operators acting on the state vector Ψ of the superparticle. In particular,

$$[x^\mu, p_\nu] = -i\delta^\mu_\nu, \quad [x^\mu, x^\nu] = 0 = [p_\mu, p_\nu] \tag{1}$$

and

$$\{\psi_\alpha, \bar{\psi}_\beta\} = \delta_{\alpha\beta}, \quad \{\psi_\alpha, \psi_\beta\} = 0 = \{\bar{\psi}_\alpha, \bar{\psi}_\beta\}. \tag{2}$$

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In the Heisenberg picture they depend on the proper time τ and act on the τ-independent Ψ, while in the Schrödinger picture they become τ-independent and are acting on the $\Psi(\tau)$ that, in principle, satisfies the state equation

$$i \frac{d}{d\tau} \Psi(\tau) = \mathcal{H}\Psi(\tau), \tag{3}$$

but, in fact, is also τ-independent being subject to the constraint [2]

$$\mathcal{H}\Psi = 0. \tag{4}$$

In general, the phenomenological superparticle is not (and need not be) supersymmetrical in four (and in any) dimensions. This is in contrast to the *supersymmetrical superparticle* extensively discussed recently as a zero-mode, pointlike approximation to the supersymmetrical superstring [3]. It may happen that the idea of superstrings is not physically justified, while the concept of phenomenological superparticle is still pretty well applicable. There is an essential point that may make the phenomenological superparticle physically different from a zero-mode, pointlike approximation to the supersymmetrical superstring, namely, the possibility of low-energy excitations of the former with respect to the spin degrees of freedom described by ψ_α. In fact, we conjecture in the present paper the existence of such low-energy spin excitations (cf. Table I).

As it shall be clear from the further context, the concept of superparticle can be viewed as arising — through an act of algebraic abstraction — from the idea of composite particle. In a somewhat analogical way the concept of spin has arisen from the idea of orbital angular momentum. Thus, there is an analogy between the two logical transitions

$$\text{orbital angular momentum} \to \text{spin}$$

and

$$\text{composite particle} \to \text{superparticle}$$

(cf. Ref. [4]).

We will establish the dynamics of our superparticle by assuming that its wave equation given by the constraint (4) has the generalized Dirac form which in the free case is

$$(\bar{\psi}\gamma\psi \cdot p - \hat{m})\Psi = 0, \tag{5}$$

where $(\gamma^\mu) = (\beta, \beta\vec{\alpha})$ are Dirac matrices and $\hat{m}$ denotes an effective mass operator. In the position representation $\Psi = \Psi(x)$ as then $p_\mu = i\partial/\partial x^\mu$. The wave function Ψ can be represented in the "intrinsic Fock space" (cf. Ref. [4]) spanned on the basic vectors

$$\langle 0|,$$

$$\langle \alpha| = \langle 0|\psi_\alpha,$$

$$\langle \alpha_1\alpha_2| = \frac{1}{\sqrt{2!}} \langle 0|\psi_{\alpha_1}\psi_{\alpha_2},$$

$$'\langle\alpha| = \varepsilon_{\alpha\alpha_1\alpha_2\alpha_3}\frac{1}{\sqrt{3!}}\langle 0|\psi_{\alpha_1}\psi_{\alpha_2}\psi_{\alpha_3},$$

$$'\langle 0| = \varepsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\frac{1}{\sqrt{4!}}\langle 0|\psi_{\alpha_1}\psi_{\alpha_2}\psi_{\alpha_3}\psi_{\alpha_4}. \tag{6}$$

These basic bras correspond to the possible consecutive Fermi excitations $n \equiv \bar{\psi}\psi = 0, 1, 2, 3, 4$ (where $\bar{\psi}\psi = \bar{\psi}_\alpha\psi_\alpha$), resulting into spins $s = 0, 1/2, 0, 1/2, 0$, respectively (spin 1 is excluded by antisymmetry of $\langle\alpha_1\alpha_2|$). Note that here $\langle\alpha|\beta\rangle = \langle 0|\psi_\alpha\psi_\beta^+|0\rangle = \beta_{\alpha\beta}$ due to Eq. (2) and $\langle 0|\psi_\beta^+ = 0$.

In the general case when then mass operator $\hat{m}$ may mix the basic vectors (6) of the same spin, the wave equation (5) reduces in the representation (6) to the following set of three nontrivial component wave equations involving the wave-function components $\psi^{(n)}$ related to $n \equiv \bar{\psi}\psi = 1, 2, 3$ and so to $s = 1/2, 0, 1/2$, respectively:

$$(\gamma\cdot p - \hat{m}^{(1)})\psi^{(1)} = \hat{m}^{(13)}\psi^{(3)}, \tag{7}$$

$$[(\gamma_1+\gamma_2)\cdot p - \hat{m}^{(2)}]\psi^{(2)} = 0, \tag{8}$$

$$(-\gamma^{\mathrm{T}}\cdot p - \hat{m}^{(3)})\psi^{(3)} = \hat{m}^{(31)}\psi^{(1)}, \tag{9}$$

where $\hat{m}^{(13)} = \hat{m}^{(31)+}$. Note that for $n = 0$ and $n = 4$ the kinetic term $\bar{\psi}\gamma\psi\cdot p$ in Eq. (5) vanishes (though, in general, not necessarily $\psi^{(0)} = 0$ if $\hat{m}^{(0)} = 0$ as maybe expected; on the other hand, $\psi^{(4)} = 0$ if $\hat{m}^{(4)} \neq 0$). In Eq. (8), (γ_1^μ) and (γ_2^μ) are two commuting sets of Dirac matrices, so that $\beta^\mu = \frac{1}{2}(\gamma_1^\mu+\gamma_2^\mu)$ are the Duffin-Kemmer-Petiau matrices [5] (but spin 1 does not appear because of antisymmetry of $\psi^{(2)}$ in Dirac indices α_1 and α_2). In Eq. (9), $\gamma^{\mu\mathrm{T}} = -C^{-1}\gamma^\mu C$ where C is the charge conjugation matrix ($C^{-1} = C^+$ and $C^{\mathrm{T}} = -C$). Of course, $\{\gamma^{\mu\mathrm{T}}, \gamma^{\nu\mathrm{T}}\} = 2g^{\mu\nu}$ as well as $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

In the present paper we propose to interpret all leptons and quarks as spin-1/2 mass eigenstates of the phenomenological superparticle. To this end we introduce some color and flavor dynamical variables for our superparticle and tentatively propose a form of its effective mass operator $\hat{m}$.

2. *Leptons and quarks as superparticle eigenstates*

We presume that the color and flavor can be described, on the level of quantum mechanics, through the operators c_A, $A = 1, 2, 3$, and ξ_a, $a = 1, 2$, respectively, that satisfy the anticommutation relations

$$\{c_A, c_B^+\} = \delta_{AB}, \quad \{c_A, c_B\} = 0 \tag{10}$$

and

$$\{\xi_a, \xi_b^+\} = \delta_{ab}, \quad \{\xi_a, \xi_b\} = 0. \tag{11}$$

Further, we assume that the color and electric charge of a phenomenological superparticle are given by the operators

$$F_i = \tfrac{1}{2} c^+ \lambda_i c \tag{12}$$

and

$$Q = -\xi_2^+ \xi_2 + \tfrac{1}{3} c^+ c, \tag{13}$$

respectively, where λ_i, $i = 1, \ldots, 8$, are (charge conjugated) Gell-Mann matrices. Other electroweak charges will be identified later (cf. Eqs. (16) and (17)). It follows from Eqs. (10) and (12) that c_A^+ (when acting on the vacuum ket) transforms as a triplet under the color SU(3) generated by F_i. On the other hand, transformation properties of ξ_a^+ (acting on the vacuum ket) under the electroweak SU(2) ⊗ U(1) will be determined later.

Now, we can see that beside the "intrinsic Fock space" given by the basis (6), there are defined for our superparticle two extra "intrinsic Fock spaces" spanned on the basic vectors

$$\begin{array}{lr} & \text{color} \\ \langle 0|, & \mathbf{1} \\ \langle A| = \langle 0|c_A, & \mathbf{3^*} \\ '\langle A| = \langle 0|\varepsilon_{ABC} \frac{1}{\sqrt{2!}} c_B c_C, & \mathbf{3} \\ '\langle 0| = \langle 0|\varepsilon_{ABC} \frac{1}{\sqrt{3!}} c_A c_B c_C & \mathbf{1} \end{array} \tag{14}$$

corresponding consecutively to the Fermi excitations $c^+c = 0, 1, 2, 3$, (where $c^+c = c_A^+ c_A$) and

$$\begin{aligned} &\langle 0|, \\ &\langle a| = \langle 0|\xi_a, \\ &\langle 0| = \langle 0|\varepsilon_{ab} \frac{1}{\sqrt{2!}} \xi_a \xi_b \end{aligned} \tag{15}$$

that correspond consecutively to the Fermi excitations $\xi^+\xi = 0, 1, 2$ (where $\xi^+\xi = \xi_a^+ \xi_a$). In Eqs. (6), (14) and (15) the vacuum bra $\langle 0|$ denotes three different vacuum bras $_\psi\langle 0|$, $_c\langle 0|$ and $_\xi\langle 0|$ with respect to three sets of Fermi annihilation operators $\psi = (\psi_\alpha)$, $c = (c_A)$ and $\xi = (\xi_a)$, respectively: $_\psi\langle 0|\psi_\alpha^+ = 0$, $_c\langle 0|c_A^+ = 0$ and $_\xi\langle 0|\xi_a^+ = 0$. The full "intrinsic Fock space" for our superparticle is spanned on the tensorial product of three bases (6), (14) and (15) (the full vacuum bra is $_\psi\langle 0|\,_c\langle 0|\,_\xi\langle 0|$).

Then, it can be seen that the color and charge content of the "intrinsic Fock subspace" of $n \equiv \bar{\psi}\psi = 1$ is identical with that of four generations of leptons and quarks (including the three familiar generations). In the case of such an identification the remaining spin-1/2 "intrinsic Fock subspace" of $n \equiv \bar{\psi}\psi = 3$ would imply the existence of four extra generations of leptons and quarks that, when charged, should be much heavier than, at least, the three familiar generations. However, in this case, the Cabibbo-Kobayashi-Maskawa mixing of quarks would require a complicated mixing of basic vectors involving $\langle\alpha|\,\langle A|$ and charge conjugate of $\langle\alpha|\,'\langle A|$ (cf. the color assignment of $\langle A|$ and $'\langle A|$ in Eq. (14)). Since such a mixing would be impossible in our formalism, we choose another option, where four generations of "conventional" leptons and quarks (including the familiar three generations) are given by the "intrinsic Fock subspace" of both $n \equiv \bar{\psi}\psi = 1$ and 3, but only with $c^+c = 0$ and 1. The remaining spin-1/2 "intrinsic Fock subspace" of $n \equiv \bar{\psi}\psi = 1$ and 3 with $c^+c = 2$ and 3 predicts the existence of four extra generations of "mirror" leptons and quarks that, when charged, ought to be much heavier than, at least, the familiar three generations. In this case, the Cabibbo-Kobayashi-Maskawa mixing of quarks requires only a simple mixing of the basic vectors involving $\langle\alpha|\,\langle A|$ and $'\langle\alpha|\,\langle A|$ and so corresponding to $n = 1$, $c^+c = 1$ and $n = 3$, $c^+c = 1$, respectively.

Note that the "intrinsic Fock subspace" of $n \equiv \bar{\psi}\psi = 2$ gives a rich spectrum of spin-0 particles that are not intended to be discussed in the present paper.

Going over to details we propose to make for leptons and quarks the identification presented in Table I.

In Table I, ν_ω, ω^- and h, f denote the leptons and quarks of a new fourth generation that appears here beside the familiar generations i.e., the first: ν_e, e^- and u, d, the second: ν_μ, μ^- and c, s and the third: ν_τ, τ^- and t, b. In addition, there are predicted four extra generations of "mirror" leptons and quarks distinguished by the prime-sign from the "conventional" leptons and quarks. In Table I, beside Q, color multiplicity, c^+c, $\xi_1^+\xi_1$ and $\xi_2^+\xi_2$, there is indicated also the difference of baryon and lepton numbers, $B-L$, as defined in Eq. (18). The essential physical conjecture made in Table I is that leptons and quarks of the second and fourth generation are spin excitation $n = 3$ of leptons and quarks

TABLE I

Full list of possible leptons and quarks

	$\langle\alpha|$	$'\langle\alpha|$	Q	Color	$B-L$	c^+c	$\xi_1^+\xi_1$	$\xi_2^+\xi_2$
$\langle 0|\,\langle 0|$	ν_e	ν_μ	0	**1**	-1	0	0	0
$\langle A|\,\langle 0|$	$\bar{d}$	$\bar{s}$	1/3	**3***	$-1/3$	1	0	0
$'\langle A|\,\langle 0|$	u′	c′	2/3	**3**	1/3	2	0	0
$'\langle 0|\,\langle 0|$	$e^{+\prime}$	$\mu^{+\prime}$	1	**1**	1	3	0	0
$\langle 0|\,\langle a|$	ν_τ, e^-	ν_ω, μ^-	0, -1	**1**	-1	0	1, 0	0, 1
$\langle A|\,\langle a|$	$\bar{b}$, $\bar{u}$	$\bar{f}$, $\bar{c}$	1/3, $-2/3$	**3***	$-1/3$	1	1, 0	0, 1
$'\langle A|\,\langle a|$	t′, d′	h′, s′	2/3, $-1/3$	**3**	1/3	2	1, 0	0, 1
$'\langle 0|\,\langle a|$	$\tau^{+\prime}$, $\bar{\nu}_e'$	$\omega^{+\prime}$, $\bar{\nu}_\mu'$	1, 0	**1**	1	3	1, 0	0, 1
$\langle 0|\,'\langle 0|$	τ^-	ω^-	-1	**1**	-1	0	1	1
$\langle A|\,'\langle 0|$	$\bar{t}$	$\bar{h}$	$-2/3$	**3***	$-1/3$	1	1	1
$'\langle A|\,'\langle 0|$	b′	f′	$-1/3$	**3**	1/3	2	1	1
$'\langle 0|\,'\langle 0|$	$\bar{\nu}_\tau'$	$\bar{\nu}_\omega'$	0	**1**	1	3	1	1

For example: the basic vectors $\langle\alpha|\,\langle A|\,\langle a|$ and $'\langle\alpha|\,\langle A|\,\langle a|$ with $a = 1, 2$ correspond to the pairs $\bar{b}$, $\bar{u}$ and $\bar{f}$, $\bar{c}$, respectively.

of the first and third generation, respectively, which in turn are spin excitation $n = 1$ of the vacuum state.

From Table I and Eq. (13) we can read off that in the "intrinsic Fock subspace" of $n \equiv \bar{\psi}\psi = 1$ and 3 the weak isospin $\vec{I}$ and weak hypercharge Y which generate the electroweak SU(2) ⊗ U(1) group are given by the operators

$$
\begin{aligned}
I_1 &= \tfrac{1}{2}(\xi_2+\xi_2^+)\,\tfrac{1}{2}(1-\varepsilon\gamma_5) \equiv \tfrac{1}{2}\tau_1\,\tfrac{1}{2}(1-\varepsilon\gamma_5),\\
I_2 &= \frac{1}{2i}(\xi_2-\xi_2^+)\,\tfrac{1}{2}(1-\varepsilon\gamma_5) \equiv \tfrac{1}{2}\tau_2\,\tfrac{1}{2}(1-\varepsilon\gamma_5),\\
I_3 &= (\tfrac{1}{2}-\xi_2^+\xi_2)\,\tfrac{1}{2}(1-\varepsilon\gamma_5) \equiv \tfrac{1}{2}\tau_3\,\tfrac{1}{2}(1-\varepsilon\gamma_5)
\end{aligned}
\tag{16}
$$

and

$$
\tfrac{1}{2}Y = (\tfrac{1}{2}-\xi_2^+\xi_2)\,\tfrac{1}{2}(1+\varepsilon\gamma_5)+\tfrac{1}{3}c^+c-\tfrac{1}{2}\,. \tag{17}
$$

Here, $\varepsilon = +1$ or -1 when it is acting on basic vectors representing fermions or antifermions, respectively (cf. Table I). Thus,

$$
\begin{aligned}
Q &= -\xi_2^+\xi_2+\tfrac{1}{3}c^+c = I_3+\tfrac{1}{2}Y,\\
\tfrac{1}{2}(B-L) &= \tfrac{1}{3}c^+c-\tfrac{1}{2}\,,
\end{aligned}
\tag{18}
$$

the last formula defining the difference of baryon and lepton numbers, $B-L$. Due to the sign operator ε in the definition (16) of $\vec{I}$ all leptons and quarks, both "conventional" and "mirror", are coupled to $\vec{W}$ weak bosons through lefthanded currents. This is in contrast to the other "mirror" leptons and quarks (having righthanded coupling to $\vec{W}$) that are sometimes introduced in grand unification theories in order to construct — jointly with the "conventional" leptons and quarks — real representations of unifying groups (e.g. of SO(n) groups with $n \neq 4\nu+2$ where only real spinor representations exist).

It is interesting to remark that the operators

$$
\begin{aligned}
H_1 &= \tfrac{1}{2}(\xi_1+\xi_1^+)\,\tfrac{1}{2}(1-\varepsilon\gamma_5) \equiv \tfrac{1}{2}\chi_1\,\tfrac{1}{2}(1-\varepsilon\gamma_5),\\
H_2 &= \frac{1}{2i}(\xi_1-\xi_1^+)\,\tfrac{1}{2}(1-\varepsilon\gamma_5) \equiv \tfrac{1}{2}\chi_2\,\tfrac{1}{2}(1-\varepsilon\gamma_5),\\
H_3 &= (\tfrac{1}{2}-\xi_1^+\xi_1)\,\tfrac{1}{2}(1-\varepsilon\gamma_5) \equiv \tfrac{1}{2}\chi_3\,\tfrac{1}{2}(1-\varepsilon\gamma_5)
\end{aligned}
\tag{19}
$$

generate a lefthanded horizontal SU(2) group that commutes with the second factor of the electroweak SU(2) ⊗ U(1) group, but does not commute with its first factor. In fact, $\{I_k, H_l\} = 0$ for $k, l = 1, 2$, though $[I_k, H_3] = 0$ and $[I_3, H_l] = 0$ for $k, l = 1, 2, 3$. Note that in terms of 2×2 Pauli matrices the operators τ_k and χ_l, $k, l = 1, 2, 3$, introduced in Eqs. (16) and (19) may be represented as follows:

$$
\tau_1 = \sigma_1\otimes\mathbf{1},\quad \tau_2 = \sigma_2\otimes\mathbf{1},\quad \tau_3 = \sigma_3\otimes\mathbf{1} \tag{20}
$$

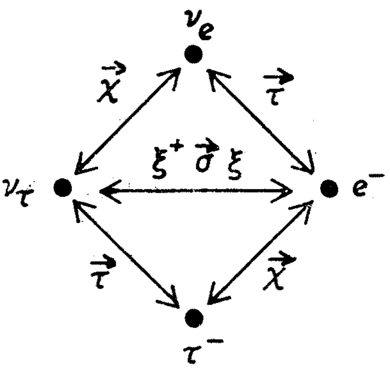
and

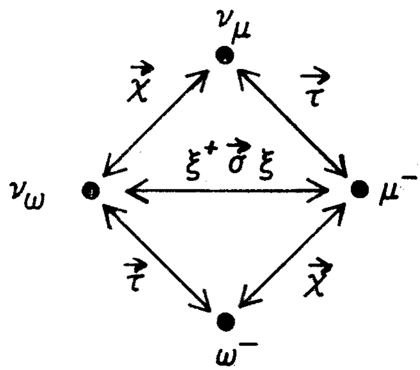
$$\chi_1 = \sigma_3 \otimes \sigma_1, \quad \chi_2 = \sigma_3 \otimes \sigma_2, \quad \chi_3 = 1 \otimes \sigma_3. \tag{21}$$

The new SU(2) is a horizontal group for two pairs of fermion generations separately: for the first and third (where $n = 1$) and for the second and fourth (where $n = 3$).

The minimal compact group containing both the electroweak SU(2) and the horizontal SU(2) as its noncommuting subgroups is the lefthanded SO(5) group generated by the operators: I_k, H_l, $k, l = 1, 2, 3$, and $2iI_kH_l$, $k, l = 1, 2$. It is locally isomorphic with a lefthanded Sp(4) group (for a discussion of the use of simpletic groups Sp(6) and Sp(8) as extensions of the electroweak SU(2) in the case of three and four generations, respectively, treated on the same footing cf. Ref. [6]). Ten matrices $\frac{1}{2}\tau_k$, $\frac{1}{2}\chi_l$, $k, l = 1, 2, 3$, and $\frac{1}{2}i\tau_k\chi_l$, $k, l = 1, 2$, generate a nonchiral SO(5) (its spinor representation) that is the diagonal sum of the previous lefthanded SO(5) and the analogically defined righthanded SO(5). Here, $\tau_k\tau_l = \delta_{kl} + i\varepsilon_{klm}\tau_m$ and $\chi_k\chi_l = \delta_{kl} + i\varepsilon_{klm}\chi_m$, but $\{\tau_k, \chi_l\} = 0$ for $k, l = 1, 2$, though $[\tau_k, \chi_3] = 0$ and $[\tau_3, \chi_l] = 0$ for $k, l = 1, 2, 3$. Thus $\frac{1}{2}\tau_k$, $k = 1, 2, 3$, and $\frac{1}{2}\chi_l$, $l = 1, 2, 3$, generate two nonchiral SU(2) groups, one vertical and one horizontal, respectively, that are noncommuting subgroups of the nonchiral SO(5). Note that the operators $\frac{1}{2}\xi^+\sigma_k\xi$, $k = 1, 2, 3$, where σ_k are 2×2 Pauli matrices, generate another nonchiral SU(2) group which is the third SU(2) subgroup of this SO(5), noncommuting with two previous SU(2) subgroups. Here, $\xi^+\sigma_1\xi = \frac{1}{2}i(\tau_1\chi_2 - \tau_2\chi_1)$, $\xi^+\sigma_2\xi = \frac{1}{2}i(\tau_1\chi_1 + \tau_2\chi_2)$ and $\xi^+\sigma_3\xi = \frac{1}{2}(\tau_3 - \chi_3)$. The operator $1 - \xi^+\xi = \frac{1}{2}(\tau_3 + \chi_3)$ is an invariant of this third SU(2) subgroup.

Taking for example the "conventional" leptons, the action of three spin-type vectors $\frac{1}{2}\vec{\tau}$, $\frac{1}{2}\vec{\chi}$ and $\frac{1}{2}\xi^+\vec{\sigma}\xi$ may be depicted as follows:





The transitions $\nu_e \leftrightarrow \tau^-$ as well as $\nu_\mu \leftrightarrow \omega^-$ are generated by the operators $\frac{1}{2}i(\tau_1\chi_2 + \tau_2\chi_1) = \xi_2\xi_1 + \xi_1^+\xi_2^+$ and $\frac{1}{2}i(\tau_1\chi_1 - \tau_2\chi_2) = i(\xi_2\xi_1 - \xi_1^+\xi_2^+)$ that together with $\frac{1}{2}(\tau_3 + \chi_3) = 1 - \xi^+\xi$ define the fourth SU(2) subgroup of SO(5), commuting with the third. Of the twelve 4×4 matrices active in the above diagram only ten are linearly independent because of $\xi^+\sigma_3\xi = \frac{1}{2}(\tau_3 - \chi_3)$. These ten generate the nonchiral SO(5) (its quartet representation) which is just a part of the group of our algebraic abstraction leading from the idea of composite particle to the concept of superparticle in an analogical manner as the act of abstraction based on the group of spatial rotations SO(3) has led the way from the orbital angular momentum to the spin (cf. a remark in Introduction).

Of course, the electroweak SU(2) ⊗ U(1) group is embedded in SO(5) ⊗ U(1), where SO(5) is our lefthanded group. Such an SO(5) ⊗ U(1) may be broken down to SU(2) ⊗U(1) ⊗ U′(1) and further to SU(2) ⊗ U(1) and U_{EM} (1), where U′(1) is an extra U(1) generated by H_3. Thus, the group SO(5) ⊗ U(1), if it was a broken gauge group, might be a broken unification group of electroweak and horizontal interactions. In our case, these hypothetical horizontal interactions would operate separately between the first and third generation (where $n = 1$) and between the second and fourth generation (where $n = 3$). Being an orthogonal group, SO(5) is anomaly safe. For each $n = 1, 3$ and $c^+c = 0, 1, 2, 3$ it has one real spinor representation (one real quartet) only. Due to the factor U(1) in SO(5) ⊗ U(1) this quartet becomes a complex quartet for SO(5) ⊗ U(1).

In the case of SO(5) ⊗ U(1) being a broken gauge group there would exist seven massive horizontal vector bosons (four charged and three neutral) in addition to the familiar four electroweak vector bosons W^+, W^-, Z^0 and γ. Six of these seven horizontal vector bosons would be coupled to flavor-changing currents: four to the charged ones corresponding to $2iI_k H_l$, $k, l = 1, 2$, and two to the neutral ones corresponding to H_1 and H_2. The seventh would be coupled to the flavor-conserving neutral current corresponding to H_3, the generator of our extra U(1). Some of these horizontal vector bosons should be much heavier than the weak vector bosons W^+, W^- and Z^0, in order to suppress potentially dangerous processes. It would be certainly true for the horizontal vector bosons coupled to the flavor-changing neutral currents. In contrast, the horizontal vector boson coupled to the flavor-conserving neutral current (cf. Appendix) might be rather light having mass $\gtrsim$ 100–300 GeV because it would be a particular realization of the extra Z^0 (for a general discussion of the extra Z^0 cf. Ref. [7]).

3. Mass operator

Looking at our Table I of leptons and quarks we can see that their spectrum results from an interplay of four types of Fermi excitations: spin excitation $n \equiv \bar{\psi}\psi = 1, 3$, color excitation $N \equiv c^+c = 0, 1, 2, 3$ and flavor excitations $\xi_1^+\xi_1 = 0, 1$ and $\xi_2^+\xi_2 = 0, 1$, the last excitation being in fact the charge excitation $Q = -\xi_2^+\xi_2 + \frac{1}{3}c^+c$ at any fixed c^+c. Observe that $1 - \xi_1^+\xi_1 = \frac{1}{2}(1 + \chi_3)$ and $\xi_1^+\xi_1 = \frac{1}{2}(1 - \chi_3)$ are the projection operators upon states with $\xi_1^+\xi_1 = 0$ and $\xi_1^+\xi_1 = 1$, respectively, while $\xi_1 + \xi_1^+ = \chi_1$, $\frac{1}{i}(\xi_1 - \xi_1^+) = \chi_2$, $1 - 2\xi_1^+\xi_1 = \chi_3$ generate the nonchiral horizontal SU(2) that is a subgroup of the nonchiral SO(5). Note also that $Q = \frac{1}{2}\tau_3 + \frac{1}{3}c^+c - \frac{1}{2}$ and $\xi^+\xi = \xi_1^+\xi_1 + \xi_2^+\xi_2 = 1 - \frac{1}{2}(\tau_3 + \chi_3)$. Thus, in constructing the effective mass operator for the phenomenological superparticle we have to our disposal the following operators not changing its spin, parity, color, electric charge and B–L: 1° scalars built of ψ and $\bar{\psi}$, 2° c^+c, $\xi_2^+\xi_2$ and ξ_1, ξ_1^+ or — in place of the last four — c^+c, τ_3 and χ_1, χ_2, χ_3 of which c^+c, τ_3 and χ_3 are diagonal. In our constructing, the zero mass for neutrinos will be required, what maybe an excellent approximation at least.

Then, after a guesswork, we tentatively propose the following form of effective mass operator for our superparticle that parametrizes masses and mixing angles of all its eigenstates:

$$\hat{m} = G^2[m^{(N)}\tfrac{1}{2}(1+\chi_3)+M^{(N)}\tfrac{1}{2}(1-\chi_3)]\bar{\psi}\psi$$
$$+Q^2[\lambda^{(N)}\tfrac{1}{2}(1+\chi_3)+\Lambda^{(N)}\tfrac{1}{2}(1-\chi_3)]\tfrac{1}{8}:(\bar{\psi}\gamma\psi)^2: \tag{22}$$
$$+QF(\omega_1^{(N)}\chi_1+\omega_2^{(N)}\chi_2+\omega_3^{(N)}\chi_3)\,(\bar{\psi}_\alpha\varepsilon_{\alpha\beta\gamma\delta}\frac{1}{\sqrt{3!}}\psi_\beta\psi_\gamma\psi_\delta+\text{h.c.}),$$

where $m^{(N)}$, $M^{(N)}$, $\lambda^{(N)}$, $\Lambda^{(N)}$ and $\omega_1^{(N)}$, $\omega_2^{(N)}$, $\omega_3^{(N)}$ (with $N = c^+c = 0, 1, 2, 3$) are real constants of mass dimension (that will be chosen as positive) and: $(\bar{\psi}\gamma\psi)^2: = (\bar{\psi}\gamma\psi)^2-4\bar{\psi}\psi$ denotes the normally ordered product. Here,

$$F = F^2 = \tfrac{3}{4}\sum_i F_i^2 = \tfrac{1}{2}N(3-N) = \begin{cases} 0 & \text{for} \quad N = 0, 3 \\ 1 & \text{for} \quad N = 1, 2 \end{cases} \tag{23}$$

is the normalized quadratic Casimir operator of the color SU(3) group and G is conjectured as $G = Q+F(\alpha^{(N)}\tau_3+\beta^{(N)}\chi_3)$ with some real numbers $\alpha^{(N)}$ and $\beta^{(N)}$. In our calculations we will try the assumption that $\alpha^{(N)} = \beta^{(N)} = \frac{1}{2}$. Then

$$G = Q+F\tfrac{1}{2}(\tau_3+\chi_3)$$
$$= \begin{cases} Q & \text{for} \quad N = 0, 3 \quad \text{and} \quad \text{for} \quad N = 1, 2 \text{ with } \xi^+\xi = 1, \\ \{\frac{5}{3}, \frac{4}{3}\} & \text{for} \quad N = 1, 2 \quad \text{with} \quad \xi^+\xi = 0, \\ \{-\frac{5}{3}, -\frac{4}{3}\} & \text{for} \quad N = 1, 2 \quad \text{with} \quad \xi^+\xi = 2, \end{cases} \tag{24}$$

where $\{a, b\} = a$ or b if $|Q| = 2/3$ or $1/3$, respectively. Thus, $G^2 = Q^2$ for leptons and for u, c, b, f and d′, s′, t′, h′, while $G^2 = 25/9$ for u′, c′, t, h and $G^2 = 16/9$ for d, s, b′, f′ (cf. Table I).

When neglecting for the moment the mixing of $n = 1$ and $n = 3$ (for quarks) by the third term in Eq. (22), we obtain in the case of four generations of "conventional" leptons and quarks i.e.,

1. $n = 1, \quad \xi_1^+\xi_1 = 0 : \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ and $\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$,

2. $n = 3, \quad \xi_1^+\xi_1 = 0 : \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$ and $\begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix}$,

3. $n = 1, \quad \xi_1^+\xi_1 = 1 : \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$ and $\begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}$,

4. $n = 3, \quad \xi_1^+\xi_1 = 1 : \begin{pmatrix} \nu_\omega^\omega \\ \omega^- \end{pmatrix}$ and $\begin{pmatrix} \bar{f} \\ \bar{h} \end{pmatrix}$,

the following masses parametrized by $m^{(N)}$, $M^{(N)}$, $\lambda^{(N)}$, $\Lambda^{(N)}$ with $N \equiv c^+c = 0, 1$:

$$1. \qquad \begin{pmatrix} 0 \\ m^{(0)} \end{pmatrix}, \qquad \begin{pmatrix} \frac{16}{9} m^{(1)} \\ \frac{4}{9} m^{(1)} \end{pmatrix}, \tag{25}$$

$$2. \qquad \begin{pmatrix} 0 \\ |3m^{(0)} - \lambda^{(0)}| \end{pmatrix}, \qquad \begin{pmatrix} |\frac{16}{3} m^{(1)} - \frac{1}{9} \lambda^{(1)}| \\ |\frac{4}{3} m^{(1)} - \frac{4}{9} \lambda^{(1)}| \end{pmatrix}, \tag{26}$$

$$3. \qquad \begin{pmatrix} 0 \\ M^{(0)} \end{pmatrix}, \qquad \begin{pmatrix} \frac{1}{9} M^{(1)} \\ \frac{25}{9} M^{(1)} \end{pmatrix}, \tag{27}$$

$$4. \qquad \begin{pmatrix} 0 \\ |3M^{(0)} - \Lambda^{(0)}| \end{pmatrix}, \qquad \begin{pmatrix} |\frac{1}{3} M^{(1)} - \frac{1}{9} \Lambda^{(1)}| \\ |\frac{25}{3} M^{(1)} - \frac{4}{9} \Lambda^{(1)}| \end{pmatrix}, \tag{28}$$

respectively. Thus, our predictions based on formulae (22)-(24) are in this case

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = m_{\nu_\omega} = 0 \tag{29}$$

and

$$\begin{aligned} m_u : m_d &= 1:4, \qquad m_c : m_s \simeq 4:1, \\ m_t : m_b &= 25:1, \qquad m_h : m_f \simeq 4:1 \end{aligned} \tag{30}$$

because $\lambda^{(1)} \gg m^{(1)}$ and $\Lambda^{(1)} \gg M^{(1)}$ (as here $\lambda^{(1)} = \frac{9}{4} m_c + \frac{27}{4} m_u$ and $\Lambda^{(1)} = 9m_f + 27m_b$, while $m^{(1)} = \frac{9}{4} m_u = \frac{9}{16} m_d$ and $M^{(1)} = \frac{9}{25} m_t = 9m_b$). So, putting $m_c \simeq 1.5$ GeV and $m_b \simeq 5$ GeV we predict $m_s \simeq 0.37$ GeV and $m_t \simeq 125$ GeV. Note that in the prediction (30) the inequality of up and down quark masses for the first generation is correctly reversed in comparison with that for higher generations, though the ratio $m_d/m_u = 4$ is too high (this ratio will be changed when the mixing term in Eq. (22) is switched on).

Masses for the predicted "mirror" leptons and quarks are also easily obtained if we neglect for the moment the mixing of $n = 1$ and $n = 3$ (for quarks). For their four generations i.e.,

$$1. \qquad n = 1, \quad \xi_1^+ \xi_1 = 0 : \begin{pmatrix} e^{+\prime} \\ \bar{\nu}'_e \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} u' \\ d' \end{pmatrix},$$

$$2. \qquad n = 3, \quad \xi_1^+ \xi_1 = 0 : \begin{pmatrix} \mu^{+\prime} \\ \bar{\nu}'_\mu \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} c' \\ s' \end{pmatrix},$$

$$3. \qquad n = 1, \quad \xi_1^+ \xi_1 = 1 : \begin{pmatrix} \tau^{+\prime} \\ \bar{\nu}'_\tau \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} t' \\ b' \end{pmatrix},$$

$$4. \qquad n = 3, \quad \xi_1^+ \xi_1 = 1 : \begin{pmatrix} \omega^{+\prime} \\ \bar{\nu}'_\omega \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} h' \\ f' \end{pmatrix},$$

we get the following masses parametrized by $m^{(N)}$, $M^{(N)}$, $\lambda^{(N)}$, $\Lambda^{(N)}$ with $N \equiv c^+c = 2, 3$:

$$1. \qquad \begin{pmatrix} m^{(3)} \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} \frac{25}{9} m^{(2)} \\ \frac{1}{9} m^{(2)} \end{pmatrix}, \tag{31}$$

2. $$\begin{pmatrix} |3m^{(3)}-\lambda^{(3)}| \\ 0 \end{pmatrix}, \quad \begin{pmatrix} |\frac{25}{3}m^{(2)}-\frac{4}{9}\lambda^{(2)}| \\ |\frac{1}{3}m^{(2)}-\frac{1}{9}\lambda^{(2)}| \end{pmatrix}, \tag{32}$$

3. $$\begin{pmatrix} M^{(3)} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \frac{4}{9}M^{(2)} \\ \frac{16}{9}M^{(2)} \end{pmatrix}, \tag{33}$$

4. $$\begin{pmatrix} |3M^{(3)}-\Lambda^{(3)}| \\ 0 \end{pmatrix}, \quad \begin{pmatrix} |\frac{4}{3}M^{(2)}-\frac{4}{9}\Lambda^{(2)}| \\ |\frac{16}{3}M^{(2)}-\frac{1}{9}\Lambda^{(2)}| \end{pmatrix}, \tag{34}$$

respectively. In particular there are four "mirror" neutrinos (with $N \equiv c^+c = 3$) of mass zero, giving together with four "conventional" neutrinos (with $N \equiv c^+c = 0$) the total number of eight massless neutrinos. All of them are neutral spin-1/2 eigenstates of our superparticle.

4. *Quark mixing*

Switching on the third term in the effective mass operator (22) we introduce for quarks the well known phenomenon of generation mixing. In the case of Eq. (22) no such phenomenon could appear for leptons (even if neutrino masses were nonzero and nonequal) since for them $F = 0$. Note that even without the factor F in the mixing term there would be no effective lepton mixing because neutrinos would be still massless, and thus degenerate, due to the factor Q in the mixing term.

Then, labelling rows and columns of the appropriate mass submatrix $\hat{m}^{(u)}$ or $\hat{m}^{(d)}$ in the up and down sector of "conventional" quarks by u, c, t, h or d, s, b, f, respectively, we obtain from Eqs. (22)–(24) (suppressing the superscript $N = 1$ in the parameters):

$$m^{(u)} = \begin{pmatrix} \frac{4}{9}m & -\frac{2}{3}\omega_3 & 0 & -\frac{2}{3}\omega \\ -\frac{2}{3}\omega_3^* & \frac{4}{3}m-\frac{4}{9}\lambda & -\frac{2}{3}\omega & 0 \\ 0 & -\frac{2}{3}\omega^* & \frac{25}{9}M & \frac{2}{3}\omega_3 \\ -\frac{2}{3}\omega^* & 0 & \frac{2}{3}\omega_3^* & \frac{25}{3}M-\frac{4}{9}\Lambda \end{pmatrix}, \tag{35}$$

$$m^{(d)} = \begin{pmatrix} \frac{16}{9}m & \frac{1}{3}\omega_3 & 0 & \frac{1}{3}\omega \\ \frac{1}{3}\omega_3^* & \frac{16}{3}m-\frac{1}{9}\lambda & \frac{1}{3}\omega & 0 \\ 0 & \frac{1}{3}\omega^* & \frac{1}{9}M & -\frac{1}{3}\omega_3 \\ \frac{1}{3}\omega^* & 0 & -\frac{1}{3}\omega_3^* & \frac{1}{3}M-\frac{1}{9}\Lambda \end{pmatrix},$$

where $\omega = \omega_1 + i\omega_2$ and so $\omega^* = \omega_1 - i\omega_2$. Here, $\omega_3^* = \omega_3$. If m and λ were zero, the mass matrices (35) would be of the Fritzsch type [8] with a trivial choice of phases. In our approach, λ is expected to be quite large, larger than $|\omega|$.

Denoting by $U^{(u)}$ and $U^{(d)}$ the unitary matrices that diagonalize the Hermitian mass matrices $\hat{m}^{(u)}$ and $\hat{m}^{(d)}$,

$$U^{(u,d)+}\hat{m}^{(u,d)}U^{(u,d)} = \begin{pmatrix} m_{u,d} & 0 & 0 & 0 \\ 0 & -m_{c,s} & 0 & 0 \\ 0 & 0 & m_{t,b} & 0 \\ 0 & 0 & 0 & -m_{hf} \end{pmatrix}, \tag{36}$$

we can write the four-generation counterpart of the Kobayashi-Maskawa mixing matrix [9] as $V = U^{(u)+}U^{(d)}$. It transforms the down states d, s, b, f into the mixed down states d_W, s_W, b_W, f_W appearing in weak interactions along with the up states u, c, t, h. Thus,

$$\begin{pmatrix} d_W \\ s_W \\ b_W \\ f_W \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{uf} \\ V_{cd} & V_{cs} & V_{cb} & V_{cf} \\ V_{td} & V_{ts} & V_{tb} & V_{tf} \\ V_{hd} & V_{hs} & V_{hb} & V_{hf} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \\ f \end{pmatrix} \tag{37}$$

or $q_W^{(d)} = Vq^{(d)}$ for short. The minus sign at $m_{c,s}$ and $m_{h,f}$ in Eq. (36) is consistent with the minus sign at $\gamma^T \cdot p$ in Eq. (9), though sign at the mass in a Dirac equation is irrelevant.

In order to calculate for our superparticle the generalized Kobayashi-Maskawa matrix V we split the mass matrices (35) into two parts $\hat{m}^{(u,d)} = \hat{m}_0^{(u,d)} + \hat{m}_1^{(u,d)}$, where

$$\hat{m}_1^{(u,d)} = -\left\{\tfrac{2}{3}, -\tfrac{1}{3}\right\} \begin{pmatrix} 0 & 0 & 0 & \omega \\ 0 & 0 & \omega & 0 \\ 0 & \omega^* & 0 & 0 \\ \omega^* & 0 & 0 & 0 \end{pmatrix}. \tag{38}$$

Here, $\{a, b\} = a$ or b for u or d, respectively. Then, in the first step, we diagonalize $\hat{m}_0^{(u)}$ and $\hat{m}_0^{(d)}$ exactly. After some calculations we find the following formulae for the eigenvalues:

$$\begin{aligned} m_{0u,d} &= \tfrac{1}{9}\left[4\{1,4\}m + (\{4,1\}\lambda - 8\{1,4\}m)\tfrac{1}{2}\left(\sqrt{1+\tan^2 2\theta^{(u,d)}} - 1\right)\right], \\ -m_{0c,s} &= \tfrac{1}{9}\left[4\{1,4\}m - (\{4,1\}\lambda - 8\{1,4\}m)\tfrac{1}{2}\left(\sqrt{1+\tan^2 2\theta^{(u,d)}} + 1\right)\right], \\ m_{0t,b} &= \tfrac{1}{9}\left[\{25,1\}M + (\{4,1\}\Lambda - 2\{25,1\}M)\tfrac{1}{2}\left(\sqrt{1+\tan^2 2\vartheta^{(u,d)}} - 1\right)\right], \\ -m_{0h,f} &= \tfrac{1}{9}\left[\{25,1\}M - (\{4,1\}\Lambda - 2\{25,1\}M)\tfrac{1}{2}\left(\sqrt{1+\tan^2 2\vartheta^{(u,d)}} + 1\right)\right], \end{aligned} \tag{39}$$

where

$$\begin{aligned} \tan 2\theta^{(u,d)} &= \{1, -2\}\frac{3\omega_3}{\lambda - 2\{1, 16\}m}, \\ \tan 2\vartheta^{(u,d)} &= -\{1, -2\}\frac{3\omega_3}{\Lambda - \frac{1}{2}\{25, 1\}M}, \end{aligned} \tag{40}$$

while the diagonalizing matrices are

$$U_0^{(u,d)} = \begin{pmatrix} \cos\theta^{(u,d)} & \sin\theta^{(u,d)} & 0 & 0 \\ -\sin\theta^{(u,d)} & \cos\theta^{(u,d)} & 0 & 0 \\ 0 & 0 & \cos\vartheta^{(u,d)} & \sin\vartheta^{(u,d)} \\ 0 & 0 & -\sin\vartheta^{(u,d)} & \cos\vartheta^{(u,d)} \end{pmatrix}. \tag{41}$$

Note that in Eq. (40) we can approximately pass to ∞ with the mass scale Λ of the fourth generation quarks h and f, since we expect that $m_h \gg m_t$ and $m_f \gg m_b$ (then $\Lambda \gg M \gg \lambda$

$\gtrsim \omega_3$ as $m_f \gg m_b \gg m_s \gtrsim \omega_3$). So, $M/\Lambda \to 0$ and $\omega_3/\Lambda \to 0$ approximately. Thus, we can put $\vartheta^{(u,d)} \to 0$ and then get from Eqs. (39)

$$m_{0t,b} \to \tfrac{1}{9}\{25, 1\}M,$$

$$m_{0h,f} \to \tfrac{1}{9}(-3\{25, 1\}M + \{4, 1\}\Lambda) \to \tfrac{1}{9}\{4, 1\}\Lambda \tag{42}$$

in consistency with Eqs. (27) and (28).

Next, in the second step, we treat $U_0^{(u)+}\hat{m}_1^{(u)}U_0^{(u)}$ and $U_0^{(d)+}\hat{m}_1^{(d)}U_0^{(d)}$ as small perturbations of the diagonal matrices $U_0^{(u)}\hat{m}_0^{(u)}U_0^{(u)}$ and $U_0^{(d)+}\hat{m}_0^{(d)}U_0^{(d)}$, respectively. This assumption will be justified a posteriori (cf. Eq. (80)). Carrying out perturbation calculations of the first order with respect to $\omega = \omega_1 + i\omega_2$ and $\omega^* = \omega_1 - i\omega_2$ divided by m_{0t} or m_{0b} we find up to $O(|\omega|)$ the additional unitary matrices $U_1^{(u)}$ and $U_1^{(d)}$ such that $U^{(u)} = U_0^{(u)}U_1^{(u)}$ and $U^{(d)} = U_0^{(d)}U_1^{(d)}$ have the property (36). It results up to $O(|\omega|)$ into the unchanged mass formulae $m_{u,d} = m_{0u,d}$ plus $O(|\omega|^2)$, etc., while the diagonalizing matrices in the approximation of $\Lambda \to \infty$ are

$$U^{(u,d)} = \begin{pmatrix} c^{(u,d)} & s^{(u,d)} & 0 & 0 \\ -s^{(u,d)} & c^{(u,d)} & \{\frac{2}{3}, -\frac{1}{3}\}\dfrac{\omega}{m_{t,b}} & 0 \\ \{\frac{2}{3}, -\frac{1}{3}\}\dfrac{\omega^*}{m_{t,b}} s^{u,d} & -\{\frac{2}{3}, -\frac{1}{3}\}\dfrac{\omega^*}{m_{t,b}} c^{(u,d)} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{43}$$

plus $O(|\omega|^2)$, where $s^{(u,d)} = \sin\theta^{(u,d)}$ and $c^{(u,d)} = \cos\theta^{(u,d)}$. Here, in the denominators we neglected $m_{c,s}/m_{t,b}$ (and $m_{u,b}/m_{t,b}$) for simplicity. Thus, up to $O(|\omega|)$ we can identify the masses given in Eq. (39) with the physical masses $m_{u,d}$, $m_{c,s}$, $m_{t,b}$, $m_{h,f}$.

Finally, from Eqs. (43) we calculate the generalized Kobayashi-Maskawa matrix $V = U^{(u)+}U^{(d)}$ in the approximation of $\Lambda \to \infty$, finding

$$V_{ud} = V_{cs} = c,$$

$$V_{us} = -V_{cd} = s,$$

$$V_{ub} = \left(\tfrac{1}{3}\frac{\omega}{m_b} + \tfrac{2}{3}\frac{\omega}{m_t}\right)s^{(u)},$$

$$V_{cb} = -\left(\tfrac{1}{3}\frac{\omega}{m_b} + \tfrac{2}{3}\frac{\omega}{m_t}\right)c^{(u)},$$

$$V_{td} = -\left(\tfrac{1}{3}\frac{\omega^*}{m_b} + \tfrac{2}{3}\frac{\omega^*}{m_t}\right)s^{(d)},$$

$$V_{ts} = \left(\tfrac{1}{3}\frac{\omega^*}{m_b} + \tfrac{2}{3}\frac{\omega^*}{m_t}\right)c^{(d)},$$

$$V_{tb} = V_{hf} = 1, \quad \text{others} = 0 \tag{44}$$

plus $O(|\omega|^2)$, where $s = \sin(\theta^{(d)} - \theta^{(u)})$ and $c = \cos(\theta^{(d)} - \theta^{(u)})$. Hence, $\theta_C \simeq \theta^{(d)} - \theta^{(u)}$, where θ_C is the Cabibbo angle. Of course, due to the approximation of $\Lambda \to \infty$ we got here formulae for the ordinary 3×3 Kobayashi-Maskawa matrix since in this approximation the fourth generation does not mix with the three others.

From Eqs. (44) we readily deduce up to $O(|\omega|)$ the following relationships

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* = -V_{ub}, \; V_{cd}V_{td}^* + V_{cs}V_{ts}^* = -V_{cb} \tag{45}$$

and

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* = -V_{td}, \quad V_{us}V_{ub}^* + V_{cs}V_{cb}^* = -V_{ts}. \tag{46}$$

They are a part of unitary conditions for the V matrix because of $V_{tb}^* = 1$. The rest of these conditions is fulfilled trivially up to $O(|\omega|)$, in particular $|V_{ud}|^2 + |V_{us}|^2 = 1$ and $|V_{tb}|^2 = 1$. Two nontrivial relations following from Eqs. (44) are

$$|V_{ub}|^2 + |V_{cb}|^2 = |V_{td}|^2 + |V_{ts}|^2 = \left(\tfrac{1}{3}\frac{1}{m_b} + \tfrac{2}{3}\frac{1}{m_t}\right)^2 |\omega|^2 \tag{47}$$

plus $O(|\omega|^3)$. They may enable us to estimate $|\omega| = \sqrt{\omega_1^2 + \omega_2^2}$ from experimental values of m_b, m_t, $|V_{ub}|$, $|V_{cb}|$ and predict $|V_{td}|^2 + |V_{ts}|^2$ up to $O(|\omega|^2)$. Taking $m_b \simeq 5$ GeV, $m_t \simeq 125$ GeV and $|V_{ub}| = 0-0.008$, $|V_{cb}| = 0.037-0.053$ [10] we obtain

$$|\omega| \simeq 0.51 - 0.74 \text{ GeV} \tag{48}$$

and $\sqrt{|V_{td}|^2 + |V_{ts}|^2} = 0.037$–$0.054$. Of course, this estimation for $|\omega|$ is valid only if the inserted experimental limits for $|V_{ub}|$ and $|V_{cb}|$ are both consistent with the present theory (cf. Eqs. (77) and (81)).

We can see that through Eqs. (39), (40) and (44) seven parameters m, M, λ, Λ and ω_1, ω_2, ω_3 determine (in the first order with respect to $\omega = \omega_1 + i\omega_2$ and $\omega^* = \omega_1 - i\omega_2$) eight masses $m_{u,d}$, $m_{c,s}$, $m_{t,b}$, $m_{h,f}$ and all elements of the generalized Kobayashi-Maskawa matrix V (the latter in the approximation of $\Lambda \to \infty$). Alternatively, we may use as parameters the other seven: $m_{c,s} - m_{u,d}$, $m_{h,f} - m_{t,b}$ and ω_1, ω_2, ω_3. In the second case we can calculate m, M, λ, Λ, $m_{c,s} + m_{u,d}$, $m_{h,f} + m_{t,b}$ and all elements of V. In particular, from the first two Eqs. (39) we get $m_{c,s} - m_{u,d} = \frac{1}{9}(\{4, 1\}\lambda - 16\{1, 4\}m)$ and have

$$m = \tfrac{3}{80}\left[m_c - m_u - 4(m_s - m_d)\right] \tag{49}$$

and

$$\lambda = \tfrac{3}{5}\left[4(m_c - m_u) - (m_s - m_d)\right]. \tag{50}$$

Similarly, $m_{c,s} + m_{u,d} = \frac{1}{9}(\{4, 1\}\lambda - 8\{1, 4\}m)\sqrt{1 + \tan^2 2\theta^{(u,d)}}$. If $\{4, 1\}\lambda - 8\{1, 4\}m \neq 0$, we also deduce that

$$\left(m_{u,d} - \tfrac{4}{9}\{1, 4\}m\right)\left(m_{c,s} + \tfrac{4}{9}\{1, 4\}m\right) = \tfrac{1}{9}\{4, 1\}\omega_3^2 \tag{51}$$

and

$$\frac{m_{u,d} - \frac{4}{9}\{1, 4\}m}{m_{c,s} + \frac{4}{9}\{1, 4\}m} = \tan^2 \theta^{(u,d)}, \tag{52}$$

where we used additionally Eq. (40) in the first formula and the identity $(\sqrt{1+\tan^2 2x}-1)/$ $/(\sqrt{1+\tan^2 2x}+1) = \tan^2 x$ in the second. In the case of $m = 0$ but $\lambda \neq 0$ we get from Eqs. (51) and (52) the relations

$$m_u m_c = 4 m_d m_s = \tfrac{4}{9}\,\omega_3^2 \tag{53}$$

and

$$\theta^{(d)} - \theta^{(u)} = \operatorname{sgn}\theta^{(d)} \arctan\sqrt{\frac{m_d}{m_s}} - \operatorname{sgn}\theta^{(u)} \arctan\sqrt{\frac{m_u}{m_c}}\,, \tag{54}$$

respectively, the second being the Fritzsch formula [8] for the Cabibbo angle $\theta_C \simeq \theta^{(d)} - \theta^{(u)}$.

Taking the reasonable bounds

$$4 \leqslant \frac{m_c - m_u}{m_s - m_d} \leqslant 16 \tag{55}$$

we obtain from Eqs. (49) and (50) that

$$0 \leqslant \frac{m}{\lambda} \leqslant \tfrac{1}{84}\,, \tag{56}$$

respectively. Then, Eq. (40) gives

$$3\{1, -2\}\left(\frac{\omega_3}{\lambda}\right)_0 \leqslant \tan 2\theta^{(u,d)} \leqslant 3\{\tfrac{42}{41}, -\tfrac{42}{13}\}\left(\frac{\omega_3}{\lambda}\right)_{1/84} \tag{57}$$

and

$$-2 \geqslant c \geqslant -\tfrac{41}{13}\,, \tag{58}$$

where $c = \tan 2\theta^{(d)}/\tan 2\theta^{(u)}$, while $(\omega_3/\lambda)_0$ and $(\omega_3/\lambda)_{1/84}$ correspond to the bounds (56) of m/λ. In general, Eq. (40) and Eqs. (49) and (50) imply that

$$c = -2\,\frac{\lambda - 2m}{\lambda - 32m} = -\frac{1}{4}\,\frac{31(m_c - m_u) - 4(m_s - m_u)}{2(m_c - m_u) + 2(m_s - m_u)}\,. \tag{59}$$

5. *Estimation of mixing parameters*

Making use of the first two Eqs. (44) we infer that

$$\tan 2(\theta^{(d)} - \theta^{(u)}) = \frac{2 V_{ud} V_{us}}{V_{ud}^2 - V_{us}^2}\,, \tag{60}$$

where $V_{ud}^2 + V_{us}^2 = 1$. Taking as the experimental value [10]

$$|V_{us}| = 0.219 \text{ to } 0.225 \tag{61}$$

implying

$$|V_{ud}| = 0.976 \text{ to } 0.974 \tag{62}$$

we calculate from Eq. (60) that

$$|\tan 2(\theta^{(d)} - \theta^{(u)})| = 0.473 \text{ to } 0.488. \tag{63}$$

With the value (63) as the input for the absolute magnitude of $t = \tan 2(\theta^{(d)} - \theta^{(u)})$ we evaluate $t^{(u,d)} = \tan 2\theta^{(u,d)}$ from the system of two equations

$$\frac{t^{(d)} - t^{(u)}}{1 + t^{(d)} t^{(u)}} = t, \qquad \frac{t^{(d)}}{t^{(u)}} = c \tag{64}$$

where c is taken from the range (58) corresponding to the interval (56) of m/λ. Writing $t = \eta|t|$ with $\eta = \operatorname{sgn} t$ and choosing for the equation

$$t^{(d)2} + \frac{1}{t}(1 - c)t^{(d)} + c = 0 \tag{65}$$

resulting from the system (64) the root of smaller absolute magnitude, we obtain

$$t^{(d)} = \eta\left[-\frac{|c|+1}{2|t|} + \sqrt{\left(\frac{|c|+1}{2|t|}\right)^2 + |c|}\right] \tag*{(6)}$$

as $c < 0$. Hence, $\operatorname{sgn} t^{(d)} = \operatorname{sgn} t$. On the other hand, Eq. (40) shows that $\operatorname{sgn} t^{(d)} = -\operatorname{sgn} \omega_3$. Thus, choosing ω_3 as positive we get $\eta = -1$ and so $t^{(d)} < 0$ and $t < 0$.

Then, from Eq. (66) and the relation $t^{(u)} = c^{-1} t^{(d)}$ we estimate $t^{(d)}$ and $t^{(u)}$ as follows:

$$-0.301 \geqslant t^{(d)} \geqslant -0.346 \text{ to } -0.310 \geqslant t^{(d)} \geqslant -0.356 \tag{67}$$

and

$$0.151 \geqslant t^{(u)} \geqslant 0.110 \text{ to } 0.155 \geqslant t^{(u)} \geqslant 0.113, \tag{68}$$

where we inserted the respective bounds (58) for c and the respective limits (63) for $|t|$. Hence

$$-16.7^\circ \geqslant 2\theta^{(d)} \geqslant -19.1^\circ \text{ to } -17.2^\circ \geqslant 2\theta^{(d)} \geqslant -19.6^\circ \tag{69}$$

and

$$8.56^\circ \geqslant 2\theta^{(u)} \geqslant \ .25^\circ \text{ to } 8.80^\circ \geqslant 2\theta^{(u)} \geqslant 6.43^\circ. \tag{70}$$

Of course for both bounds of c we obtain the same

$$\theta_C \simeq 2(\theta^{(d)} - \theta^{(u)}) = -25.3^\circ \text{ to } -26.0^\circ \tag{71}$$

in consistency with the input (63).

Now, from Eqs. (57) and (68) we can estimate the ratio ω_3/λ by evaluating its bounds $(\omega_3/\lambda)_0$ and $(\omega_3/\lambda)_{1/84}$:

$$0.0502 \geqslant \frac{\omega_3}{\lambda} \geqslant 0.0357 \text{ to } 0.0516 \geqslant \frac{\omega_3}{\lambda} \geqslant 0.0367. \tag{72}$$

The mass scale λ may be estimated through Eqs. (50) and (55) that give

$$3.4 \text{ GeV} \lesssim \lambda \lesssim 3.5 \text{ GeV} \tag{73}$$

where we put $m_c - m_u \simeq 1.5$ GeV. Hence

$$0.17 \text{ GeV} \gtrsim \omega_3 \gtrsim 0.13 \text{ GeV} \text{ to } 0.17 \text{ GeV} \gtrsim \omega_3 \gtrsim 0.13 \text{ GeV} \tag{74}$$

(here, exactly the same bounds appearing for two limits (63) of $|t|$ are due to poor accuracy).

Finally, we come to estimating the elements of V matrix. To this end we may use the proportion

$$|V_{cb}|:|V_{ub}|:|V_{ts}|:|V_{td}| = 1:|\tan\theta^{(u)}|:\left|\frac{\cos\theta^{(d)}}{\cos\theta^{(u)}}\right|:\left|\frac{\sin\theta^{(d)}}{\cos\theta^{(u)}}\right| \tag{75}$$

following from Eq. (44). It enables us to determine $|V_{ub}|$, $|V_{ts}|$ and $|V_{td}|$ if we take as the experimental value [10]

$$|V_{cb}| = 0.037-0.053 \tag{76}$$

and apply Eqs. (69) and (70) estimating the mixing angles $\theta^{(d)}$ and $\theta^{(u)}$. Then,

$$(0.0028-0.0040) \geqslant |V_{ub}| \geqslant (0.0020-0.0029)$$

$$\text{to } (0.0018-0.0041) \geqslant |V_{ub}| \geqslant (0.0021-0.0030), \tag{77}$$

while

$$|V_{ts}| \simeq |V_{cb}|, \quad |V_{td}| = |V_{ts}|\,|\tan\theta^{(d)}| \simeq |c|V_{ub}| \tag{78}$$

with $-2 \leqslant c \leqslant -41/13$. The prediction (77) is consistent with the experimental estimate [10]

$$|V_{ub}| = 0-0.008. \tag{79}$$

Note that $|V_{ub}/V_{cb}| = \tan\theta^{(u)}$ independently of the experimental value (76). Here $0.0748 \geqslant \tan\theta^{(u)} \geqslant 0.0546$ to $0.0770 \geqslant \tan\theta^{(u)} \geqslant 0.0562$ due to Eq. (70).

Making use of Eq. (44) for V_{cb} and its experimental value (76) as well as the estimation (70) for $\theta^{(u)}$ we can evaluate $|\omega|/m_b$, obtaining in all cases

$$\frac{|\omega|}{m_b} = 0.10-0.15, \tag{80}$$

where we took $m_t = 25m_b$. Hence, in consistency with our previous estimation (48), we get in all cases

$$|\omega| \simeq 0.51-0.74 \text{ GeV}, \tag{81}$$

where we put $m_b \simeq 5$ GeV. Note that, if all ω_1, ω_2 and ω_3 were equal, the relation $|\omega| = \omega_3\sqrt{2}$ would hold. In the real case, Eqs. (74) and (81) show for the ratio $x = |\omega|/\omega_3\sqrt{2}$ that

$$\begin{aligned} &(2.2-3.1) \lesssim x \lesssim (2.9-4.1) \\ \text{to } &(\_.1-3.0) \lesssim x \lesssim (2.8-4.0). \end{aligned} \tag{82}$$

The reasonable smallness of $|\omega|/m_b$ and so $|\omega|/m_t$ following from Eq. (80) justifies a posteriori the use of our first-order perturbative calculation for the quark mass matrices (35). Here, $O(|\omega|/m_b) = O(0.1)$. It may be worthwhile to point out that in the first step of our calculations (that was exact) there also exists a virtual expansion parameter of the order $O(3\omega_3\sqrt{2}/\lambda) = O(4\omega_3/\lambda) = O(2|\omega|/m_b)$. In fact, from Eqs. (40) and (64) we can deduce that

$$\tan 2(\theta^{(d)} - \theta^{(u)}) = -\frac{3\sqrt{2}}{2}\,\frac{\left(1-6\dfrac{2m}{\lambda}\right)\dfrac{3\omega_3\sqrt{2}}{\lambda}}{1-17\dfrac{2m}{\lambda}+16\left(\dfrac{2m}{\lambda}\right)^2-\left(\dfrac{3\omega_3\sqrt{2}}{\lambda}\right)^2}, \tag{83}$$

where

$$1 \geqslant 1-17\frac{2m}{\lambda}+16\left(\frac{2m}{\lambda}\right)^2 \geqslant \frac{1}{1.35} \tag{84}$$

for the bounds (56) of m/λ, respectively. In the first order in this parameter

$$2(\theta^{(d)} - \theta^{(u)}) = -\frac{1-6\dfrac{2m}{\lambda}}{1-17\dfrac{2m}{\lambda}+16\left(\dfrac{2m}{\lambda}\right)^2}\,\frac{9\omega_3}{\lambda}\text{ rad} \tag{85}$$

plus $O[(3\omega_3\sqrt{2}/\lambda)^3]$. For instance, taking $m/\lambda = 0$ in the case of $|V_{ud}| = 0.219$ we have $\omega_3/\lambda = 0.0502$ from Eq. (72), and then the first-order formula (85) gives $2(\theta^{(d)} - \theta^{(u)}) = -0.452$ rad $= -25.9°$, while the exact value in this case is $2(\theta^{(d)} - \theta^{(u)}) = -25.3°$ from Eq. (71).

In conclusion, our analysis of quark masses and mixing angles in terms of the tentatively proposed form (22)–(24) of superparticle effective mass operator is summarized in Table II. The listed values of parameters for u, d, c, s quarks correspond to the end points of ranges $0 \leqslant m/\lambda \leqslant 1/84$ and $0.219 \leqslant |V_{us}| \leqslant 0.225$. Additional inputs are $m_c \simeq 1.5$ GeV, $m_b \simeq 5$ GeV and $0.037 \leqslant |V_{cb}| \leqslant 0.053$. For t, b quarks the approximation of $\Lambda \to \infty$ is used, where $M/\Lambda \to 0$ and $\omega_3/\Lambda \to 0$.

TABLE II

Quark masses and mixing parameters analysed in terms of the effective mass operator (22)–(24)

	$m_c - m_u = 4(m_s - m_d)$		$m_c - m_u = 16(m_s - m_d)$	
	$\|V_{us}\| = 0.219$	$\|V_{us}\| = 0.225$	$\|V_{us}\| = 0.219$	$\|V_{us}\| = 0.225$
c	-2		$-41/13$	
ω_3/λ	0.0502	0.0516	0.0357	0.0367
λ (GeV) [a]	3.4		3.5	
ω_3 (GeV) [a]	0.17	0.17	0.13	0.13
m (GeV) [a]	0		0.042	
m_u (MeV) [a]	8.5 [b]	9.0	23	24
m_d (MeV) [a]	8.3 [b]	8.8	82	82
m_c (GeV) [a]	1.5 [b]	1.5	1.5	1.5
m_s (GeV) [a]	0.38 [b]	0.38	0.18	0.18
m_t (GeV) [c]	125			
m_b (GeV) [c]	5			
$\|V_{ub}\|$ (10^{-4}) [d]	28–40	28–41	20–29	21–30
$\|V_{cb}\|$ [d]	0.037–0.053			
$\|\omega\|$ (GeV) [d]	0.52–0.74	0.52–0.74	0.51–0.74	0.51–0.74
$\|\omega\|/\omega_3\sqrt{2}$ [d]	2.2–3.1	2.1–3.0	2.9–4.1	2.8–4.0

[a] if $m_c - m_u = 1.5$ GeV (input).

[b] $m_u = 9.6$ MeV, $m_d = 14$ MeV, $m_c = 1.5$ GeV, $m_s = 0.37$ GeV, when m begins to grow giving $m_c - m_u = 4.25\,(m_s - m_u)$ i.e., $m/\lambda = 1/1024$.

[c] if $M/\Lambda \to 0$, $\omega_3/\Lambda \to 0$ and $m_b = 5$ GeV (input).

[d] if $|V_{cb}| = 0.037$–0.053 (input).

We can see from Table II that our tentatively proposed form (22)–(24) of the effective mass operator reasonably organizes the experimental data on quark masses and mixing angles, except for light quark mases if one wants to interpret them as the current masses. In particular, the masses m_u and m_d are typically larger than the values $m_u \simeq 4$–6 GeV and $m_d \simeq 7$–9 GeV usually referred to as the current masses for u and d quarks, though their order of magnitude is the same when $m/\lambda \to 0$. Also the mass m_s is too large in comparison with the current mass $m_s = 0.033\,m_b$ for s quark [13] that is $m_s \simeq 0.165$ GeV if $m_b \simeq 5$ GeV. It is possible that our m_u, m_d and m_s are rather some effective masses. The same is probably true for our m_c, m_b and m_t, but in the case of heavy quark masses differences between effective masses and current masses become less important.

6. *Mixing phase*

Our calculated Kobayashi-Maskawa matrix V_{KM} as given in Eqs. (44) with Eqs. (46) invoked, is of the form generally discussed by Gronau and Schechter [11] and also by Fritzsch and others [12]. Namely, it can be rewritten as

$$V_{ud} = V_{cs} = c_{12}, \quad V_{tb} = 1,$$
$$V_{us} = -V_{cd}^{*} = s_{12}e^{i\varphi_{12}},$$

$$
\begin{aligned}
V_{\mathrm{ub}} &= s_{13}e^{i\varphi_{13}}, \\
V_{\mathrm{cb}} &= s_{23}e^{i\varphi_{23}}, \\
V_{\mathrm{td}} &= -c_{12}s_{13}e^{-i\varphi_{13}}+s_{12}s_{23}e^{-i(\varphi_{12}+\varphi_{23})}, \\
V_{\mathrm{ts}} &= -c_{12}s_{23}e^{-i\varphi_{23}}-s_{12}s_{13}e^{i(\varphi_{12}-\varphi_{13})}
\end{aligned}
\tag{86}
$$

plus $O(|\omega|^2)$, where

$$
\begin{aligned}
c_{12} &= c, \quad s_{12} = s, \\
s_{13} &= \left(\tfrac{1}{3}\frac{|\omega|}{m_{\mathrm{b}}}+\tfrac{2}{3}\frac{|\omega|}{m_{\mathrm{t}}}\right)s^{(\mathrm{u})}, \\
s_{23} &= \left(\tfrac{1}{3}\frac{|\omega|}{m_{\mathrm{b}}}+\tfrac{2}{3}\frac{|\omega|}{m_{\mathrm{t}}}\right)c^{(\mathrm{u})},
\end{aligned}
\tag{87}
$$

and $\varphi_{12} = 0$, $\varphi_{13} = \delta$ and $\varphi_{23} = \delta+\pi$ with $\delta = \arg\omega$. Here, $c_{13} = 1$ plus $O(|\omega|^2)$ and $c_{23} = 1$ plus $O(|\omega|^2)$. Then, our V_{KM} has really the Gronau-Schechter form, where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ with θ_{ij} denoting the mixing angle between the generation i and generation j. The Gronau-Schechter rephasing-invariant mixing phase $\varphi = \varphi_{12}+\varphi_{23}-\varphi_{13}$ is here zero (modulo π), showing that in our case the PC conservation holds though $\omega = \omega_1+i\omega_2 = |\omega|\exp(i\delta)$ is complex. Consequently, through the rephasing transformation of up and down quark states of three generations $j = 1, 2, 3$,

$$
|q_j^{(\mathrm{u,d})}\rangle \to |q_j^{(\mathrm{u,d})}\rangle\, e^{i\varphi_j}
\tag{88}
$$

with $\varphi_1-\varphi_2 = 0$ and $\varphi_2-\varphi_3 = \varphi_1-\varphi_3 = \delta$, we eliminate from Eq. (86) the phase δ.

Since the PC conservation is experimentally violated, a nontrivial mixing phase should be introduced to our effective mass operator (22). To this end let us replace the factor $\left(\bar{\psi}_\alpha\varepsilon_{\alpha\beta\gamma\delta}\frac{1}{\sqrt{3!}}\psi_\beta\psi_\gamma\psi_\delta+\mathrm{h.c.}\right)$ in the third term of this operator by the new factor

$$
\left(\hat{f}\bar{\psi}_\alpha\varepsilon_{\alpha\beta\gamma\delta}\frac{1}{\sqrt{3!}}\psi_\beta\psi_\gamma\psi_\delta+\mathrm{h.c.}\right),
\tag{89}
$$

where

$$
\hat{f} = \tfrac{1}{2}(1+\tau_3)f^{(\mathrm{u})}+\tfrac{1}{2}(1-\tau_3)f^{(\mathrm{d})}
\tag{90}
$$

with $f^{(\mathrm{u,d})} = \exp(i\alpha^{(\mathrm{u,d})})$ is a phase operator. Then, the elements of quark mass matrices (35) get the phase factors $f^{(\mathrm{u,d})}$ at ω_3 and ω as well as $f^{(\mathrm{u,d})*}$ at ω_3^* and ω^*. Consequently, in the diagonalizing matrices (41) and (43) there appear the phase factors $f^{(\mathrm{u,d})}$ at $s^{(\mathrm{u,d})}$ and ω as well as $f^{(\mathrm{u,d})*}$ at $-s^{(\mathrm{u,d})}$ and ω^*. Of course, mass formulae (39) remain unchanged. So, our substitution implies that, now, Eqs. (44) for the generalized Kobayashi-Maskawa matrix $V = U^{(u)+}U^{(d)}$ are to be replaced by

$$
\begin{aligned}
V_{ud} &= V_{cs}^* = c^{(d)}c^{(u)} + f^{(d)*}f^{(u)}s^{(d)}s^{(u)} \simeq 1,\\
V_{us} &= -V_{cd}^* = f^{(d)}s^{(d)}c^{(u)} - f^{(u)(}s^{(u)}c^{(d)},\\
V_{ub} &= \left(\tfrac{1}{3}\frac{\omega}{m_b}f^{(d)} + \tfrac{2}{3}\frac{\omega}{m_t}f^{(u)}\right)f^{(u)}s^{(u)},\\
V_{cb} &= -\left(\tfrac{1}{3}\frac{\omega}{m_b}f^{(d)} + \tfrac{2}{3}\frac{\omega}{m_t}f^{(u)}\right)c^{(u)},\\
V_{td} &= -\left(\tfrac{1}{3}\frac{\omega^*}{m_b}f^{(d)*} + \tfrac{2}{3}\frac{\omega^*}{m_t}f^{(u)*}\right)f^{(d)*}s^{(d)},\\
V_{ts} &= \left(\tfrac{1}{3}\frac{\omega^*}{m_b}f^{(d)*} + \tfrac{2}{3}\frac{\omega^*}{m_t}f^{(u)*}\right)c^{(d)},\\
V_{tb} &= V_{hf} = 1, \quad \text{others} = 0
\end{aligned} \tag{91}
$$

plus $O(|\omega|^2)$. Obviously, the unitary relationships (45) and (46) do not change. Also $|V_{ud}|^2 + |V_{us}|^2 = 1$ does not.

Defining c_{12}^2, s_{12}^2 and R^2 by the formulae

$$
\begin{aligned}
c_{12}^2 - \cos^2(\theta^{(d)} - \theta^{(u)}) &= \sin^2(\theta^{(d)} - \theta^{(u)}) - s_{12}^2\\
&= -4c^{(d)}c^{(u)}s^{(d)}s^{(u)}\sin^2\tfrac{1}{2}(\alpha^{(d)} - \alpha^{(u)}),\\
R^2 &= \left(\tfrac{1}{3}\frac{|\omega|}{m_b} + \tfrac{2}{3}\frac{|\omega|}{m_t}\right)^2 - \tfrac{8}{9}\frac{|\omega|^2}{m_b m_t}\sin^2\tfrac{1}{2}(\alpha^{(d)} - \alpha^{(u)})
\end{aligned} \tag{92}
$$

where $\alpha^{(u,d)} = \arg f^{(u,d)}$ and $\delta = \arg\omega$, we readily calculate from Eqs. (91) that

$$
\begin{aligned}
|V_{ud}|^2 &= c_{12}^2 \simeq \cos^2(\theta^{(d)} - \theta^{(u)}) \simeq 1,\\
\varphi_{11} &= \arg V_{ud} = -\arctan\frac{s^{(d)}s^{(u)}\sin(\alpha^{(d)} - \alpha^{(u)})}{c^{(d)}c^{(u)} + s^{(d)}s^{(u)}\cos(\alpha^{(d)} - \alpha^{(u)})} \simeq 0
\end{aligned} \tag{93}
$$

and

$$
\begin{aligned}
|V_{us}|^2 &= s_{12}^2,\\
\varphi_{12} &= \arg V_{us} = \arctan\frac{s^{(d)}c^{(u)}\sin\alpha^{(d)} - c^{(d)}s^{(u)}\sin\alpha^{(u)}}{s^{(d)}c^{(u)}\cos\alpha^{(d)} - c^{(d)}s^{(u)}\cos\alpha^{(u)}};
\end{aligned} \tag{94}
$$

as well as

$$
\begin{aligned}
|V_{ub}|^2 &= s_{13}^2 = R^2 s^{(u)2},\\
\varphi_{13} &= \arg V_{ub} = \varphi_{23} + \alpha^{(u)} + \pi
\end{aligned} \tag{95}
$$

and

$$|V_{cb}|^2 = s_{23}^2 = R^2 c^{(u)},$$

$$\varphi_{23} = \arg V_{cb} = \arctan \frac{m_t \sin(\alpha^{(d)}+\delta)+2m_b \sin(\alpha^{(u)}+\delta)}{m_t \cos(\alpha^{(d)}+\delta)+2m_b \cos(\alpha^{(u)}+\delta)} + \pi. \tag{96}$$

Due to Eq. (95) the rephasing-invariant mixing phase $\varphi = \varphi_{12}+\varphi_{23}-\varphi_{13}$ is equal to

$$\varphi = \varphi_{12}-\alpha^{(u)}-\pi. \tag{97}$$

In the degenerate case of $\alpha^{(u)} = \alpha^{(d)}$ we obtain $\varphi_{11} = 0$, $\varphi_{12} = \alpha^{(u)}$, $\varphi_{23} = \alpha^{(u)}+\delta+\pi$ and $\varphi_{13} = 2\alpha^{(u)}+\delta+\pi$, so that $\varphi = 0$ (modulo π). The zero values φ_{11} and φ imply that in this degenerate case PC is conserved. In general, however, $\varphi \neq 0$ and the PC conservation is violated.

Carrying out the rephasing transformation (88) with $\varphi_1-\varphi_2 = \varphi_{12}$, $\varphi_2-\varphi_3 = \varphi_{23}$ and $\varphi_1-\varphi_3 = \varphi_{12}+\varphi_{23}$ and invoking Eqs. (46), we get the V matrix in the form

$$V \simeq \begin{pmatrix} 1 & s_{12} & Rs^{(u)}e^{-i\varphi} & 0 \\ -s_{12} & 1 & Rc^{(u)} & 0 \\ R(c^{(u)}s_{12}-s^{(u)}e^{i\varphi}) & -Rc^{(u)} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{98}$$

where $s_{12} = \sqrt{s_{12}^2}$, $R = \sqrt{R^2}$, and $\varphi = \varphi_{12}-\alpha^{(u)}-\pi$ with s_{12}^2, R^2 and φ_{12} as given in Eqs. (92) and (94). Note that $s_{13} = Rs^{(u)}$ and $s_{23} = Rc^{(u)}$, while (after the transformation) $\varphi_{12} = 0$, $\varphi_{23} = 0$ and $\varphi_{13} = -\varphi$. From Eq. (98) we have $|V_{ub}|+|V_{cb}|^2 = R^2$ and $|V_{ub}/V_{cb}| = \tan\theta^{(u)}$.

An especially simple case of PC violation would correspond to $\varphi_{12} = 0$ (before the transformation). Then, from Eq. (97) $\varphi = -\alpha^{(u)}-\pi$ and from Eq. (94)

$$\sin\alpha^{(u)}:\sin\alpha^{(d)} = \cot\theta^{(u)}:\cot\theta^{(d)}. \tag{99}$$

In this case, under the additional simplifying hypothesis (not contradicting Eq. (99)) that $\alpha^{(u)}+\alpha^{(d)} = 90°$, we should get

$$\tan\alpha^{(d)} = \frac{\tan\theta^{(u)}}{\tan\theta^{(d)}}. \tag{100}$$

Thus, modulo π

$$-117° \leqslant \varphi \leqslant -108° \text{ to } -117° \leqslant \varphi \leqslant -108° \tag{101}$$

when using our estimation (69) and (70) for $\theta^{(d)}$ and $\theta^{(u)}$.

To summarize, our tentative form of the effective mass operator, (22)–(24) with the new factor (89), predicts in the quark sector that

$$0.38 \text{ GeV} \gtrsim m_s \gtrsim 0.18 \text{ GeV}, \quad m_t \simeq 125 \text{ GeV}$$

and

$$0.0748 \geqslant |V_{ub}/V_{cb}| \geqslant 0.0546 \text{ to } 0.0770 \geqslant |V_{ub}/V_{cb}| \geqslant 0.0562$$

and, under some simplifying assumptions,

$$117^\circ \geqslant |\varphi| \geqslant 108^\circ.$$

Here, the left and right bounds correspond to $m_c - m_u = 4(m_s - m_d)$ and $m_c - m_u = 16(m_s - m_d)$, respectively, while $m_c \simeq 1.5$ GeV and $m_b \simeq 5$ GeV.

To compare, a recent analysis of PC-violation and $B_d - \bar{B}_d$-mixing data within the three-generation standard model [14] suggests the limits

$$43 \text{ GeV} \leqslant m_t \leqslant 180 \text{ GeV},$$

$$0.02 \leqslant |V_{ub}/V_{cb}| \leqslant 0.22 \tag{102}$$

with the preference to the range

$$83 \text{ GeV} \leqslant m_t \leqslant 180 \text{ GeV},$$

$$0.04 \leqslant |V_{ub}/V_{cb}| \leqslant 0.22, \tag{103}$$

and probably

$$108^\circ \leqslant |\varphi| \leqslant 175^\circ. \tag{104}$$

These numbers seem to be consistent with our predictions.

Of course, these predictions depend crucially on the tentative form (22)–(24). For instance, if in place of the conjecture (24), $G^2 = [Q + F\frac{1}{2}(\tau_3 + \chi_3)]^2$, we take the non-coherent sum $G^2 = Q^2 + [F\frac{1}{2}(\tau_3 + \chi_3)]^2$, we get $G^2 = 13/9$ for u′, c′, t, h and $G^2 = 10/9$ for d, s, b′, f′ instead of $G^2 = 25/9$ and $G^2 = 16/9$ and then predict $m_t \simeq 65$ GeV instead of $m_t \simeq 125$ GeV when $m_b \simeq 5$ GeV. The experiment will tell us more about the structure of the effective mass operator when the top quark is found.

APPENDIX

After the generation mixing caused by the nondiagonal mass matrices (35) is switched on, the quark part of the previously flavor-conserving neutral current of the generator H_3, coupled to our extra Z^0, takes the form

$$(U^{(u)*}_{1i} U^{(u)}_{1j} + U^{(u)*}_{2i} U^{(u)}_{2j} - U^{(u)*}_{3i} U^{(u)}_{3j} - U^{(u)*}_{4i} U^{(u)}_{4j}) \bar{q}^{(u)}_i \gamma^\mu (1-\gamma_5) q^{(u)}_j$$

$$+ (U^{(d)*}_{1i} U^{(d)}_{1j} + U^{(d)*}_{2i} U^{(d)}_{2j} - U^{(d)*}_{3i} U^{(d)}_{3j} - U^{(d)*}_{4i} U^{(d)}_{4j}) \bar{q}^{(d)}_i \gamma^\mu (1-\gamma_5) q^{(d)}_j \tag{A.1}$$

which includes small flavor-changing corrections.

In fact, making use of the diagonalizing matrices (43) (and suppressing the matrix $\gamma^\mu(1-\gamma_5)$ between quark operators) we can rewrite the last form as

$$\bar{u}u + \bar{d}d + \bar{c}c + \bar{s}s - \bar{t}t - \bar{b}b - \bar{h}h - \bar{f}f$$

$$+\tfrac{4}{3}\left(\frac{\omega}{m_t}\, s^{(u)}\bar{u}t + \text{h.c.}\right) - \tfrac{2}{3}\left(\frac{\omega}{m_b}\, s^{(d)}\bar{d}b + \text{h.c.}\right)$$

$$-\tfrac{4}{3}\left(\frac{\omega}{m_t}\, c^{(u)}\bar{c}t + \text{h.c.}\right) + \tfrac{2}{3}\left(\frac{\omega}{m_b}\, c^{(d)}\bar{s}b + \text{h.c.}\right) \tag{A.2}$$

plus $O(|\omega|^2)$. It follows from Eqs. (44) that here $\frac{1}{3}\frac{|\omega|}{m_t}|s^{(u)}| \simeq \frac{m_b}{m_t}|V_{ub}|$, $\frac{1}{3}\frac{|\omega|}{m_t}c^{(u)} \simeq \frac{m_b}{m_t}|V_{cb}|$ and $\frac{1}{3}\frac{|\omega|}{m_b}|s^{(d)}| \simeq |V_{td}|$, $\frac{1}{3}\frac{|\omega|}{m_b}c^{(d)} \simeq |V_{ts}| \simeq |V_{cb}|$ with $|V_{td}| \simeq (2\div 3)|V_{ub}|$. Thus, in the order $O(|\omega|)$, the coupling of quarks to our extra Z^0 induces for them slow, Kobayashi-Maskawa-depressed, flavor-changing neutral transitions $t \leftrightarrow u$, $b \leftrightarrow d$ and $t \leftrightarrow c$, $b \leftrightarrow s$ (but not those between the first and second generation, $c \leftrightarrow u$, $s \leftrightarrow d$, responsible for the potentially dangerous processes like e.g. $K^0 \to \mu^+\mu^-$ or $D^0 \to \mu^+\mu^-$). The absence of terms with $\bar{u}c$, $\bar{c}u$, $\bar{d}s$, $\bar{s}d$ in the order $O(|\omega|)$ is due to a partial GIM mechanism that is working in our case.

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