ON THE RELATIVISTIC CORRECTION TO THE BINDING ENERGY OF ^{9}Be-NUCLEUS

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The ^{9}Be-nucleus is considered as a three-body problem composed of two alpha particles and a neutron. In treating this problem, we follow Faddéev's formalism using the technique proposed by Dąbrowski. The two-body interaction used is assumed to be non-local and separable. The relativistic correction, adopted by Gupta and Mitra, was included. It is found that the relativistic correction improves the results of the calculation of the binding energy of ^{9}Be-nucleus. The results obtained are satisfactory and in better agreement with the experimental value.

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1. Introduction

The ^{9}Be-nucleus is one of the interesting examples of the three-body problem. It can be well described, to a good approximation, to be composed of two-alpha particles and a neutron, since the dissociation energy of ^{9}Be-nucleus into $\alpha+\alpha+n$ is small (1.67 MeV) compared to the energy required to break up an alpha particle ($\sim$20 MeV) [1]. Many attempts have been made to study the structure of ^{9}Be-nucleus [2–8].

As a three-body problem, Grubman and Witten [9] described a new angular momentum decomposition of the Faddéev equations for the general case of three-particles with spin. They applied this formalism to calculate the ground state binding energy of the system consisting of two-alpha particles and a neutron. They represented the n-α and α-α interactions phenomenologically by local potentials and the corresponding two-body T-matrices were approximated by a Sturmian expansion. Fonseca et al. [10] treated the ^{9}Be-nucleus as a three-body problem using the Born Oppenheimer Approximation (BOA) in which the presence of the neutron produces an effective potential added to the original α-α interaction. Fonseca's work is extended by Révai et al. [1] using the adiabatic one-level approximation with the correct angular momentum. They used the non-local separable potential for the n-α interaction, for which the two-center problem is solved exactly [11]. Also for the α-α interaction, the form of Ali-Bodmer [12] potential was used. Their calculated spectrum for ^{9}Be-nucleus agrees only qualitatively with the experimental values.

There are two approaches to improve the results obtained theoretically for the binding

energy of the three-body system. The first is the inclusion of three-body forces which generated much interest in recent years [13–18]. The second is the introduction of the relativistic corrections. Recently there has been much interest in describing the nucleus as a relativistic system [19–22]. The relativistic effect may be significant, since the velocities of nucleons in nuclei can be of the order of one third of the velocity of light. Indeed, there are two approaches to incorporate the relativistic effects into the three-body calculations. One approach stems from the field theory and the Bethe-Salpeter (BS) equations [23]. In the other approach, the relativistic Quantum Mechanics was used and had been discussed extensively [24–27], where one identifies the Hilbert space of a relativistic system as a representation space of the Inhomogenous Lorentz Group (IHLG) and the problem of finding a relativistic theory is equivalent to a search for a set of Hermitian operators satisfying the well known commutation relations for IHLG [28].

In the present work, we have followed Faddéev formalism [29] using the technique proposed by Dąbrowski [30–32]. In this technique, the three-particle system is considered to interact in pairs. The potential of each pair is assumed to be non-local and separable. Accordingly, the three-body wave function will split into three-terms, each describing a pair. The Green's function of the full three-body system is used. For the n-α interaction, we have taken the form used by Révai et al. [1]. For the α-α interaction, we have used the usual form of the non-local separable potential with three-different types of interactions, i.e. Yamaguchi (Y) [33, 34], Gaussian (G) [35] and Tabakin (T) [35] forces. The relativistic correction can be considered using the method given by Gupta et al. [36]. This method is based on the requirements that the Hamiltonian, together with the corrections, remain approximately invariant to the second order in v/c under Lorentz transformations. In Sec. 2, we introduce the different forms of the non-local separable potentials used. In Sec. 3, the three-body non-relativistic Schrödinger equation is considered. In Sec. 4, the relativistic correction is included. In Sec. 5, numerical calculations and results are given. Section 6 is devoted to discussion and conclusion.

2. *Non-local separable potentials*

In the analysis of the three-body problem, separable potentials were found to be very useful in studying the three-nucleon system. This reduces the computational work by almost an order of magnitude.

For the n-α interaction we can use the non-local separable potential that is spin and angular momentum dependent whose parameters are fitted to the low energy $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ phase shift analysis [1, 11]. This potential is given by the form

$$V_{n\alpha}(\vec{q}, \vec{q}') = \sum_{nljm} g_{nlj}(q) g^*_{nlj}(q') \lambda_{nlj}$$

$$\sum_{\mu\sigma} (l\mu_{1/2\sigma}|jm)^2 Y_{l\mu}(\hat{q}) Y^*_{l\mu}(\hat{q}') \chi_{1/2\sigma}(\hat{s}) \chi^+_{1/2\sigma}(\hat{s}'), \tag{1}$$

where

$$g_{nlj}(q) = \frac{q^l}{(q^2 + \beta^2_{nlj})^{l+1}} . \tag{2}$$

TABLE I

The parameters of the n-α interaction [1]

	$s_{1/2}$ (attractive)	$s_{1/2}$ (repulsive)	$p_{1/2}$	$p_{3/2}$
β (fm^{-1})	1.494	0.700	1.177	1.449
λ^{-1}	0.0616842	−0.151978	0.0412992	0.00942794

The values of the parameters of the n-α interaction are given in Table I.

For the α-α interaction we used the separable potential of the form

$$V_{\alpha\alpha}(\vec{q}, \vec{q}') = \lambda_\beta V_\beta(q) V_\beta(q'). \tag{3}$$

The used S-wave α-α interactions can be chosen to fit the scattering length and the effective range, which can be determined experimentally neglecting Coulomb effects. These potentials are suggested in a way that both attraction and repulsion can be given. For the separable potential between α-particles each of mass m_α, we have

$$V_{\alpha\alpha}(q, q') = -\frac{\hbar^2}{2\pi^2 m_\alpha} f(q) f(q'). \tag{4}$$

In this work, three types of non-local separable α-α potentials are used. These are Yamaguchi (Y) [33, 34], Gaussian (G) [35] and Tabakin (T) [35] potentials. The Yamaguchi (Y) potential has the form

$$f_Y(q) = \frac{\alpha}{(q^2+b^2)}. \tag{5}$$

The Gaussian (G) potential has the form

$$f_G(q) = \alpha(q_c^2 - q^2)(a + bq^2) \exp(-dq^2). \tag{6}$$

The form of Tabakin (T) potential is

$$f_T(q) = \alpha(q_c^2 - q^2)[(q^2+d^2)/(q^2+b^2)](q^4+a^4)^{-1}. \tag{7}$$

The values of the parameters for the Yamaguchi potential are those suggested by Harrington [33, 34]. Those of Gaussian and Tabakin potentials chosen to fit the S-wave α-α nuclear scattering phase shifts are listed in Table II, where $q_c = 1.70046$ for all interactions.

TABLE II

Parameters of the α-α interactions

Interaction	a	b	d	α^2
Yamaguchi (Ref. [33, 34])		0.736 fm^{-1}		2.3600 fm^{-6}
Tabakin (Ref. [35])	1.9 fm^{-1}	1.300 fm^{-1}	5.0 fm^{-1}	3.1865 fm^{-6}
Gaussian (Ref. [35])	1.0 fm^{-1}	0.050 fm^{-1}	0.5 fm^2	4.4320 fm^2

3. Non relativistic three-body Schrödinger equation

Dąbrowski [30–32] treated the bound-state problem of three-particles of equal and non-equal masses by solving Faddéev type equations for a simple but non-trivial two-body interaction. In this technique the set of integral equations is obtained by rearranging the Schrödinger equation of the three-particle system, and defining the Green's function $D = E - T$ of the full three-body system, in the barycentric subspace, where T is the kinetic energy of three-particle system.

Following Dąbrowski [30–32], the Schrödinger equation of the three-particle system is given by

$$\phi_i(\vec{q}\vec{k}) = (1/2\pi)^3 \int d\vec{k}' \mathscr{D}^{-1}(qk')_i \langle \vec{k}|\hat{V}_i|\vec{k}'\rangle_i \phi_i(\vec{q}\vec{k}')$$

$$+(1/2\pi)^3 \sum_{k\neq i} \int d\vec{q}' D^{-1}(qq')_i \left\langle \vec{k}|\hat{V}_i| \pm \left(\frac{m_i}{m_i+m_j}\vec{q}+\vec{q}'\right)\right\rangle_i$$

$$\phi_k\left(\vec{q}'; \mp\left(\frac{m_k}{m_k+m_j}\vec{q}'+\vec{q}\right)\right); \quad i = 1, 2, 3, \tag{8}$$

where

$$\mathscr{D}_i(qk') = E - \frac{k'^2}{2\mu_i^2} - \frac{q'^2}{2M_i^2};$$

$$D_i(qq') = E - \frac{1}{2m_j}\left[\frac{m_j+m_k}{m_k} q^2 + 2\vec{q}\cdot\vec{q}' + \frac{m_i+m_j}{m_i} q'^2\right];$$

$$\mu_i = \frac{m_j m_k}{m_j+m_k}; \quad M_i = \frac{m_i(m_j+m_k)}{m_i+m_j+m_k}. \tag{9}$$

Equation (8) represents a set of linear homogeneous integral equations for the unknown wave function ϕ_i. In the case of the ^{9}Be-nucleus, we have a system of three-particles, where $m_1 = m_2 =$ the mass of alpha-particle m_α and m_3 is the mass of neutron m_n. The set of equations (8) is simplified greatly by introducing the non-local separable potential. Using these types of the two-body potentials, we see that ϕ_i depends only on $\vec{k}$, through the factors $V_{i\alpha}$. So, we can write

$$\phi_i(\vec{q}\vec{k}) = \sum_\alpha V_{i\alpha}(k)\chi_{i\alpha}(\vec{q}), \quad i = 1, 2$$

$$= V_{3\alpha}(k)\chi_{3\alpha}(\vec{q}), \quad i = 3. \tag{10}$$

Using equation (10) together with equations (1), (2) and (3), the system of equations (8) becomes

$$\sum_\beta \{(y_i^{\alpha\beta}(q) - \lambda_{i\beta}^{-1}\delta_{\beta\alpha})\chi_{i\beta}(q) + \sum_{k\neq i} \int_0^\infty dq' K_{ik}^{\alpha\beta}(qq')\chi_{k\beta}(q')\} = 0, \tag{11}$$

where

$$K_{ik}(\vec{q}\vec{k};\vec{q}'\vec{k}') = {}_i\langle\vec{q}\vec{k}|V_iD^{-1}|\vec{q}'\vec{k}'\rangle_k$$
$$= D^{-1}(qk)_i\langle\vec{q}\vec{k}|V_i|\vec{q}'\vec{k}'\rangle_k,$$

and

$$y_i^{\alpha\beta}(q) = (2\pi)^{-3}\int d\vec{k}'\mathscr{D}^{-1}(qk')V_{i\alpha}(k')V_{i\beta}(k'). \tag{12}$$

4. *Relativistic correction*

In this section we intend to introduce the relativistic correction adopted by Gupta and Mitra [36]. An expression for the second order relativistic correction (ΔV) to the two-body potential $\hat{V}$ is effectively given by [24–27, 36]

$$\langle\vec{q}\vec{k}|\Delta\hat{V}_i|\vec{q}'\vec{k}'\rangle = -\frac{1}{(m_j+m_k)^2}\left[q^2+1/2\,(\vec{q}\cdot\vec{k})\left(\vec{q}\cdot\frac{\partial}{\partial\vec{k}}\right)+1/2\,(\vec{q}'\cdot\vec{k}')\left(\vec{q}'\cdot\frac{\partial}{\partial\vec{k}'}\right)\right]$$
$$\times\langle\vec{k}|\hat{V}_i|\vec{k}'\rangle. \tag{13}$$

The kinetic energy correction (ΔT_i) for a particle of momentum $\vec{P}$ is given by [24–27, 36]

$$\Delta T_i = -\frac{1}{8m_i^3}P_i^4. \tag{14}$$

After performing all the operations of partial differentiation appearing in equation (13) we are left with the following correction terms

$$\Delta F \equiv \sum_\beta\{\Delta y_i^{\alpha\beta}(q)\chi_{i\beta}(q)+\sum_{k\neq i}\int q'^2dq'\Delta K_{ik}^{\alpha\beta}(qq')\chi_{k\beta}(q)\}, \tag{15}$$

where

$$\Delta y_i^{\alpha\beta}(q) = (2\pi)^{-3}\int d\vec{k}'\left\{\frac{1}{(m_j+m_k)^2}[q^2-(\vec{q}\cdot\vec{k})^2-(\vec{q}\cdot\vec{k}')^2]\right\}$$
$$\times\mathscr{D}^{-1}(qk')V_{i\alpha}(k')\sum_\beta V_{i\beta}(k'), \tag{16}$$

and

$$\Delta K_{ik}^{\alpha\beta}(qq') = (2\pi)^{-3}\int dq'D^{-1}(qq')\left\{\frac{-1}{(m_j+m_k)^2}[q^2-(\vec{q}\cdot\vec{k})^2-(\vec{q}\cdot\vec{k}')^2]\right\}$$
$$\times V_{i\alpha}\left(\left|\frac{m_i}{m_i+m_j}\vec{q}+\vec{q}'\right|\right)\sum_\beta V_{k\beta}\left(\left|\frac{m_j}{m_j+m_k}q'+q\right|\right). \tag{17}$$

We may notice that the correction for the kinetic energy part is included in the operators $\mathscr{D}$ and D defined above. These terms are incorporated into the original system of equations (17) and the resulting equations are solved numerically.

5. *Numerical calculations and results*

The numerical calculation of the ground state binding energy of the ^{9}Be-nucleus has been performed for the two-body potentials described in Sec. 2. The two main quantities of the non-relativistic three-body Schrödinger equations (11) are $y_i^{\alpha\beta}(q)$ and $K_{ik}^{\alpha\beta}(qq')$ given by equation (12). The quantities $y_i^{\alpha\beta}(q)$ may be calculated analytically. The integrals over q' in the system of integral equations (11) are approximated by summations. Thus, after some algebraic manipulations the system of integral equations (11) is transformed into a system of linear equations. This system of equations is solved numerically for the binding energy of ^{9}Be-nucleus in two cases, one of them is without the relativistic correction and the other with it. It is to be noted that the Coulomb energy between the two-alphas is not included explicitly in the system of equations (11). So the calculated values of the ground state binding energy of ^{9}Be-nucleus "E_g" are corrected for the effect of Coulomb force [37] between the two-alphas. Table III shows the calculated values of "E_g" in the two cases considered.

6. *Discussion and conclusion*

The aim of this work is to find out the role of the relativistic correction on the calculated ground state binding energy of ^{9}Be-nucleus which is treated as a three-body problem. To see this effect we have calculated the binding energy with and without the relativistic correction. The results of these calculations together with the experimental value [38] are shown in Table III. We may notice from Table III that the results obtained using the

TABLE III

Ground state binding energy of ^{9}Be-nucleus "E_g" in MeV

		E_g Non-relativistic	E_g With relativistic correction
Present work	G	−1.890	−1.309
	Y	−2.020	−1.215
	T	−1.980	−1.226
Experimental value		−1.571	

Gaussian and Tabakin potentials are closer to the experimental value than those obtained using the Yamaguchi potential, in both cases of inclusion of relativistic correction or not. This result has been obtained before by Osman [39] and it was attributed to the fact that both the Gaussian and Tabakin potentials contain attraction and repulsion while the Yamaguchi form is only attractive.

In conclusion, it can be seen that the relativistic correction plays a good role in improving the calculated binding energy of ^{9}Be-nucleus as a three-body problem.

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