SCATTERING OF COMPOSITE PARTICLES AT VERY HIGH ENERGY WITH APPLICATIONS TO QUANTUM ELECTRODYNAMICAL PROCESSES

BY W. CZYŻ

Institute of Nuclear Physics, Cracow*

(Presented at the X^{th} Cracow School of Theoretical Physics, Zakopane, June 12–26, 1970)

It is shown that a straightforward generalization of the multiple diffractive scattering model used in high energy nuclear collisions leads to the correct expressions for the high energy limits of the cross-sections of various quantumelectrodynamical processes.

1. *A simple model and its applications*

We shall start by considering a very simple, well known, and naive model of the high energy multiple scattering at small angles. Let us suppose that a very fast particle scatters from a collection of particles and that we know the individual scattering amplitudes and the spatial distribution of the particles in the target (Fig. 1). Let us assume also that the scattering angles are so small that the process is, to a good approximation, two dimensional. Then, one can immediately construct the scattering amplitude accepting that, *e.g.* the total phase shift equals the sum of the individual phase shifts

$$\chi(\boldsymbol{b}) = \sum_{j=1}^{A} \chi_j(\boldsymbol{b}-\boldsymbol{s}_j) \tag{1}$$

or that the total profile is composed of the individual profiles as follows

$$\Gamma(\boldsymbol{b}, \boldsymbol{s}_1, \dots \boldsymbol{s}_A) = 1-\prod_{j=1}^{A} [1-\gamma_j(\boldsymbol{b}-\boldsymbol{s}_j)]. \tag{2}$$

Eqs (1) and (2) lead to identical results for the total scattering amplitude since the relation between the phase shifts, the profile, and the amplitude is:

$$f = \frac{ik}{2\pi}\int d^2b e^{i\boldsymbol{\delta}\cdot\boldsymbol{b}}\gamma(\boldsymbol{b}) = \frac{ik}{2\pi}\int d^2b e^{i\boldsymbol{\delta}\cdot\boldsymbol{b}}(1-e^{i\chi(\boldsymbol{b})}) \tag{3}$$

* Address: Instytut Fizyki Jądrowej, Kraków 23, Radzikowskiego 152, Poland.

where k is the incident momentum and δ is the momentum transfer. From (1) or (2) and (3) we get for the total elastic amplitude

$$\mathcal{M} = \frac{ik}{2\pi} \int d^2b d^2s_1 \dots d^2s_A e^{i\Delta\cdot\boldsymbol{b}} \varrho(s_1 \dots s_A) \left[1 - \prod_{j=1}^{A} (1-\gamma_j(\boldsymbol{b}-\boldsymbol{s}_j)) \right] \tag{4}$$

where Δ is the total momentum transfer, and the amplitude obtained from (1) or (2) and (3) is averaged over the ground state density $\varrho(s_1 \dots s_A)$ projected on the plane perpendicular to the direction of the incident momentum. We shall discuss below in more detail the

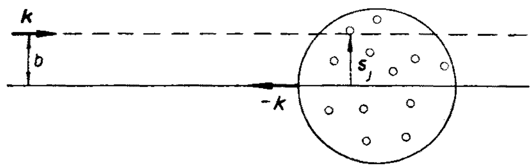


Fig. 1

construction of $\varrho(s_1 \dots s_A)$. Let us also point out here that (4) can easily be generalized to the case of elastic scattering of two composite objects (Fig. 2) $\tilde{A}$ and $\tilde{B}$:

$$\mathcal{M} = \frac{ik}{2\pi} \int d^2b d^2s_1 \dots d^2s_A e^{i\Delta\cdot\boldsymbol{b}} \varrho_A(s_1^A \dots s_A^A) \varrho_B(s_1^B \dots s_B^B) \times$$

$$\times \left[1 - \prod_{j=1}^{A} \prod_{k=1}^{B} (1-\gamma_{jk}(\boldsymbol{b}-\boldsymbol{s}_j^A+\boldsymbol{s}_k^B))\right]. \tag{5}$$

One can give some plausibility arguments for (1) and (2). One may interpret (1) as, *e.g.*, a result of accumulating the phase shift by a fast particle in subsequent collisions with the "subunits" of the target. Eq. (2), on the other hand, expresses precisely the probability

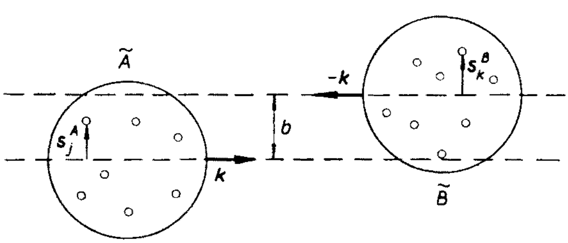


Fig. 2

of hitting the target if the probability of hitting one (j-th) subunit is γ_j. The γ's can indeed be interpreted as probabilities, if the target is purely absorbing (black). We would like to argue that the formulae (4) and (5) — in spite of their "naivete" — describe an enormous wealth of physical phenomena. Strictly speaking, a few additional refinements need to be introduced in (4) and (5) in order to analyze some specific process (*e.g.* spins were neglected so far), but they are not essential. They will be discussed as we go along.

The model described above has been fairly extensively applied to analysis and interpretation of the nuclear scattering at several energies starting from ~1 GeV up to 20 GeV. The projectiles were protons, antiprotons, neutrons, deuterons, and pions and the targets covered a wide range of nuclei (D, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li, Be, C, O, Al, Cu, Pb, U). Some results of calculations are shown, together with the experimental data, in Figs 4–9. One can conclude that the agreement is very impressive.

One should emphasize that the cross-sections shown in Figs 4–9 were computed virtually with no free parameters: the incident hadron-nucleon amplitudes (hence profiles) were taken from experiment and the densities were constructed from the ground state nucleon

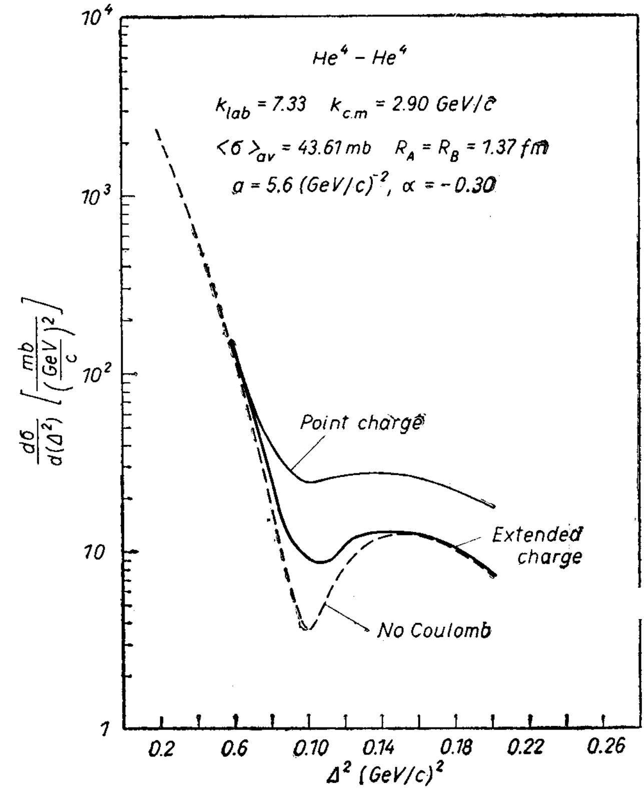


Fig. 3. The role of Coulomb interactions in elastic ^{4}He—^{4}He scattering Gaussian single particle densities were taken for ^{4}He. The three curves show the elastic scattering cross-section for point charges, extended charges (Gaussian distribution) and without Coulomb interaction (Ref. [3])

wave functions which, as far as the spatial distributions of nucleons are concerned, are known well enough for our purposes (the experimental data are still not very accurate). One may find more detailed discussion of this material in Refs [1, 2] and the papers quoted there.

Here let us mention only two important extensions of Eqs (4) and (5) which were widely used in computations. First of all the Coulomb phase shift was added to the "strong interaction phase shift" (1). The role of Coulomb scattering is quite important for almost all target nuclei (especially around diffractive minima). Fig. 3 shows $d\sigma_{el}/d\Omega$ computed for $^4He-^4He$ [3] scattering (not measured yet). It illustrates the important role of Coulomb interactions even for such light nuclei. The second extension concerns the so-called poor resolution scattering cross-section (denoted $d\sigma_{sc}/d\Omega$). This is a scattering cross-section which is measured with incident and scattered beams whose particles have energies uncertain

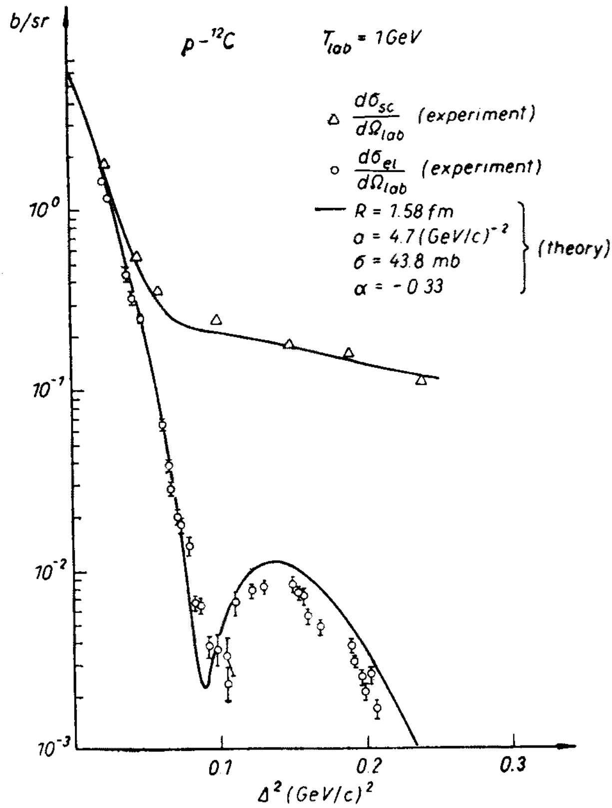


Fig. 4. The relative magnitude of $d\sigma_{el}/d\Omega$ and $d\sigma_{sc}/d\Omega$ for ^{12}C. The curves are taken from Ref. [1], the data from Ref. [19], [20]

to about 50–100 MeV. Such experiments sum over all nuclear excitations but discriminate against meson production. One cannot compare such cross-sections with $d\sigma_{el}/d\Omega = |\mathcal{M}|^2$ of (4) or (5). Instead one has to compute a sum rule (see Refs [1, 2]):

$$\frac{d\sigma_{sc}}{d\Omega} = \left(\frac{k}{2\pi}\right)^2 \int d^2b d^2b' d^2s_1 \ldots d^2s_A e^{i\Delta\cdot(b-b')} \varrho(s_1 \ldots s_A) \Gamma^+(b, s_1 \ldots s_A) \Gamma(b', s_1 \ldots s_A). \tag{6}$$

Fig. 4 shows $d\sigma_{sc}/d\Omega$ and $d\sigma_{el}/d\Omega$ in the case of $p - {}^{12}C$ scattering. One also measures sometimes the inelastic cross-sections defined as follows

$$\frac{d\sigma_{inel}}{d\Omega} = \frac{d\sigma_{sc}}{d\Omega} - \frac{d\sigma_{el}}{d\Omega}\,. \tag{7}$$

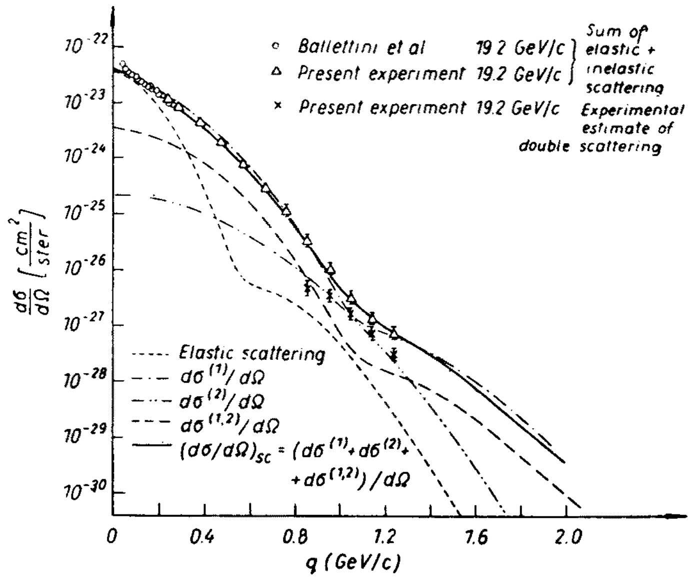


Fig. 5. Poor energy resolution ($d\sigma_{sc}/d\Omega$) p-d cross-section measured and computed by the CERN group (Ref. [21]). The double scattering ($d\sigma^{(2)}/d\Omega$) contribution to the cross-section was experimentally separated from the single scattering contribution ($d\sigma^{(1)}/d\Omega$) and the interference terms ($d\sigma^{(1,2)}/d\Omega$)

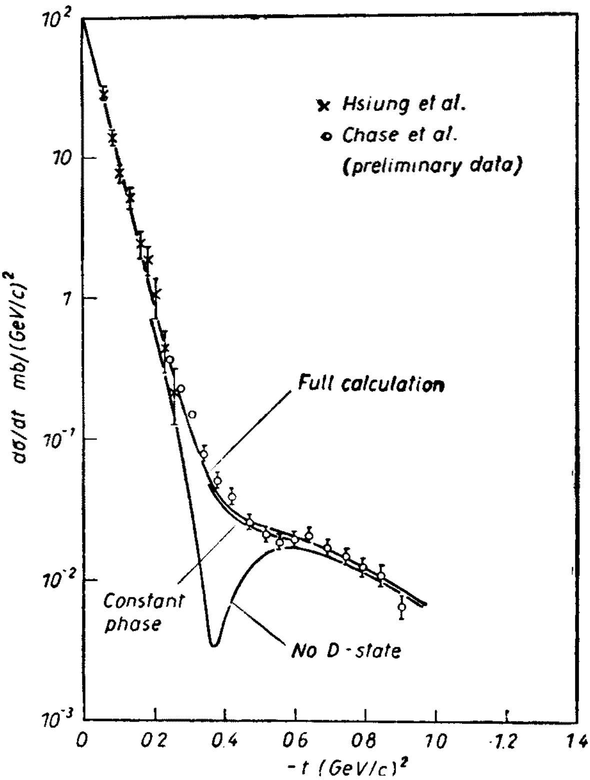


Fig. 6. π-deuteron elastic scattering at 3.65 and 3.75 GeV. Comparison of theoretical calculations and experimental data [22]. The spin dependence of π-nucleon amplitudes was taken into account

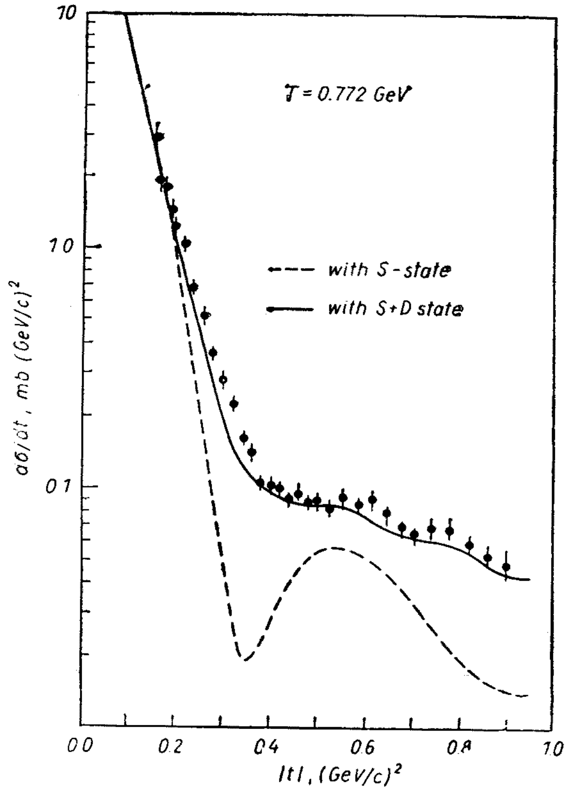


Fig. 7. π-deuteron elastic scattering at 0.77 GeV. Comparison of theoretical calculations and experimental data was taken from Ref. [23]. The spin dependence of π-nucleon amplitude was taken into account

From what has been said so far, one may feel encouraged to look for further applications of the formulae (4), (5) and (6). Before doing so, let us observe that these formulae are in complete harmony with the long ago established description of high energy scattering from composite targets in terms of the optical potential. Without going into any details of this relation let us just point out that Eq. (4) in the limit $A \to \infty$, gives approximately

$$\mathcal{M} = \frac{ik}{2\pi} \int d^2 b e^{i\Delta\cdot b} \left(1 - e^{-A \int d^2 s \varrho^{(1)}(s)\gamma(b-s)}\right). \tag{8}$$

One obtains (8) from (4) assuming

$$\varrho(s_1 \ldots s_A) = \prod_{j=1}^{A} \varrho_j^{(1)}(s_j) \tag{9}$$

and accepting that all single particle densities $\varrho_j^{(1)}$ and all profiles are the same. Figs 10, 11 illustrate how (8) works in the case of nuclear collisions. From Eq. (3) and the relation between the potential and phase shift discussed below we get (in the approximation of γ much narrower than $\varrho^{(1)}$) the well-known relation

$$-\frac{1}{\hbar v} V_{\text{opt}}(r) = A \frac{2\pi}{k} f \varrho^{(1)}(r) \tag{10}$$

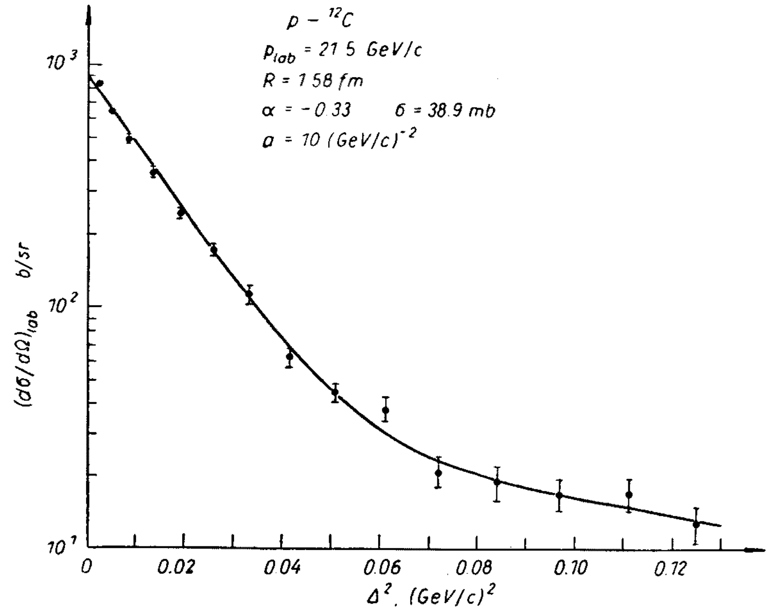


Fig. 8. Poor energy resolution ($d\sigma_{sc}/d\Omega$) p-^{12}C cross-section at 21.5 GeV/c [24] compared with calculations [1]

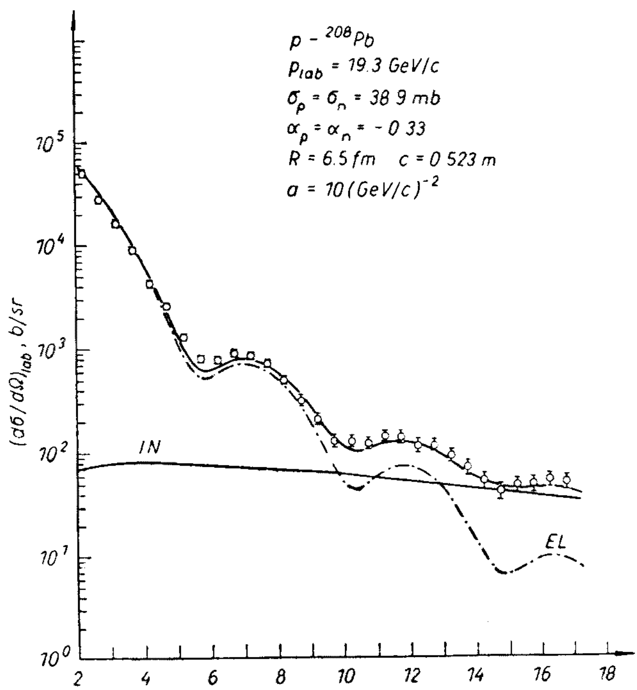


Fig. 9. Poor energy resolution ($d\sigma_{sc}/d\Omega$) p–Pb cross-section at 19.3 GeV/c [24] compared with calculations [1]. The calculations are for pure ^{208}Pb, the measurement are with $^{207.3}$Pb target

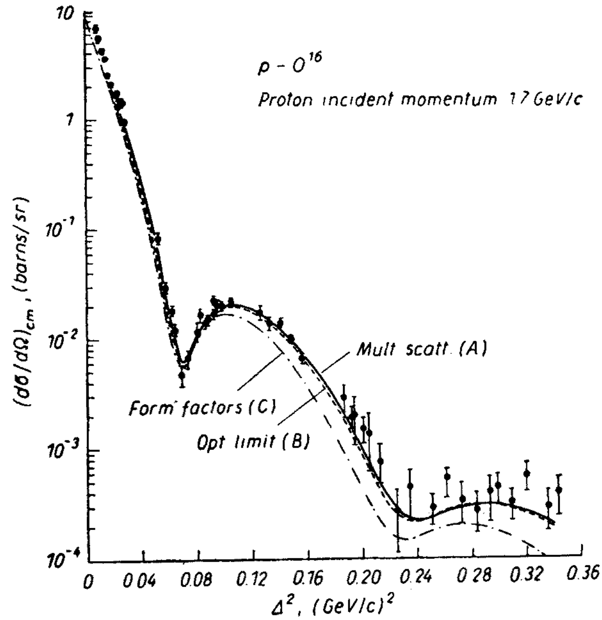


Fig. 10. p–^{16}O elastic scattering cross-section $(d\sigma_{el}/d\Omega)$. Experimental points taken from Ref. [20]. (A) — Multiple scattering cross-section calculated from the single particle densities of the harmonic oscillator potential. (B) The optical limit of (A). (C) — The cross-section calculated from the proton and the ^{16}O charge form factors. The curves are taken from Ref. [6]

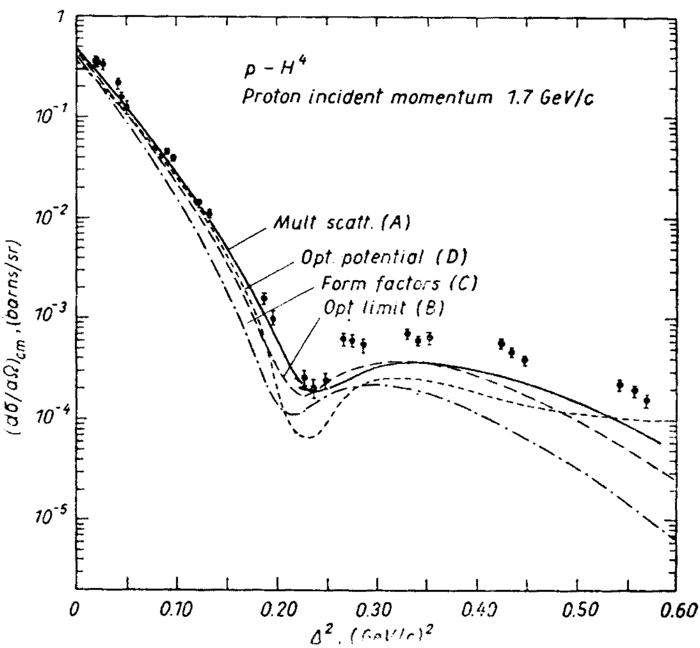


Fig. 11. p — ^{4}He elastic scattering cross-section $(d\sigma_{el}/d\Omega)$.Gaussian (hence not very realistic) single particle densities were used. (A), (B), (C) — the same as in Fig. 10. The curve (D) is the optical potential cross-section curve obtained from Abacus II program (see Ref. [6] for more details)

where f is the elementary forward scattering amplitude. The important conclusion is that, in the high energy limit, the optical potential is uniquely defined in our model and has no free parameters, in principle.

The limiting expressions (called the optical limit henceforth) one obtains when one lets the number of subunits go to infinity ($A, B \to \infty$) is also very interesting in the case of formula (5) [4, 5, 6]. From (5), again assuming factorization of ϱ_A and ϱ_B as in (9), one obtains (more detailed discussion of this limiting procedure is given in Ref. [6]):

$$\mathcal{M} = \frac{ik}{2\pi} \int d^2b e^{i\Delta\cdot b}\left(1 - e^{-AB\int d^2s d^2s' \varrho_A^{(1)}(s)\gamma(b-s+s')\varrho_B^{(1)}(s')}\right). \tag{11}$$

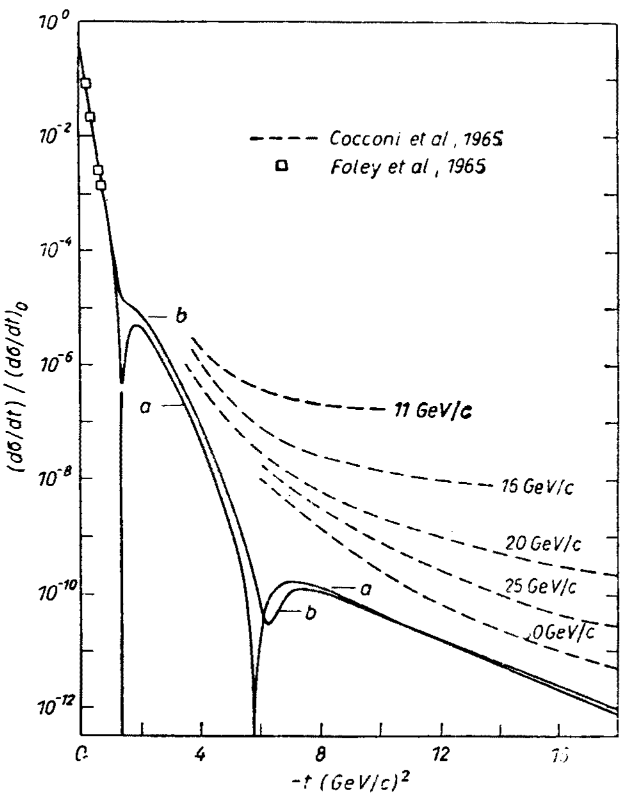


Fig. 12. The p-p elastic scattering cross-section in the optical limit against some experimental data (Ref. [5]). Notice the convergence of experimental curves towards the theoretical

One can apply (11) in a variety of ways and one can also generalize it considerably. The first applications to high energy nucleon-nucleon and meson-nucleon scattering were given in Refs [4] and [5]. Since the profile γ must contain as a factor the subunit A — subunit B total cross-section, one can go over to the limit $A, B \to \infty$ in such a way as to keep $AB\sigma$ = const. Then, one gets a formula which (in the case of γ much narrower than $\varrho_A^{(1)}$ and $\varrho_B^{(1)}$) has only one (complex) free parameter and depends crucially on the density distributions (or the form factors of colliding objects). Fig. 12 shows the elastic nucleon-nucleon cross-section obtained from the nucleon charge form factors: Fig. 13 on the other hand, shows the form factors computed from the elastic nucleon-nucleon cross-section fit to the experimental data. Fig. 14 illustrates the convergence of (5) to (11).

As we have already mentioned, one may generalize (11), *e.g.*, by considering the densities $\varrho_A^{(1)}$, $\varrho_B^{(1)}$ as density operators $\varrho^{(1)} \sim \psi^+\psi$, where ψ is some second quantization field (fermion field if the subunits are to be fermions) [4]. In such a way one may, with help of our formulae (4) and (5), discuss collisions of composite systems which do not have

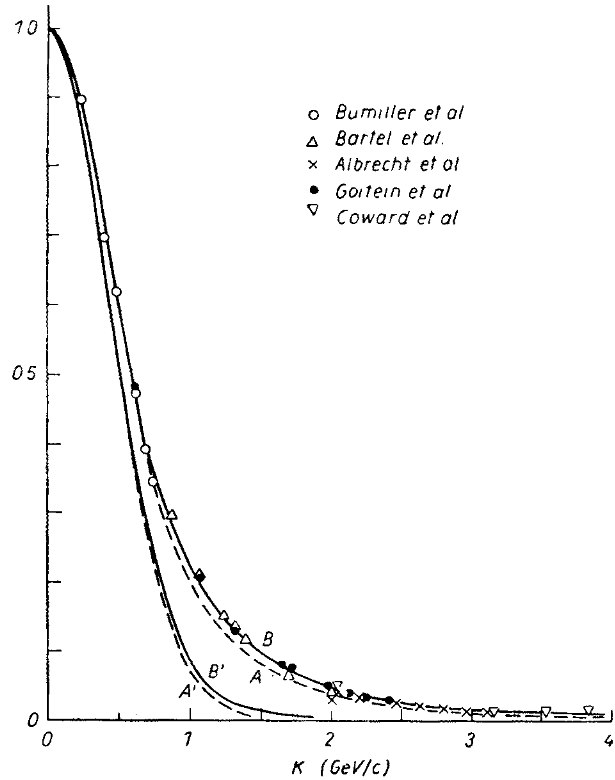


Fig. 13. The proton charge form factor F_1, computed from the elastic proton-proton cross-section by means of Eq. (11) (Ref. [4]). Some experimental points are also shown

a fixed number of pre-existing subunits (in contrast to nuclei or atoms) and whose number of subunits may fluctuate. Let us go over to the problem of collisions of fluctuating composite objects. We shall not follow, however, the generalization just mentioned.

Let us consider an object which is as far remote from nuclei (where our simple model has been successfully tested so far) as it is possible: the photon. It fluctuates, and many different fluctuation are possible, as Fig. 15 shows.

In the limit of very high energy the time dilatation makes the lifetimes of the fluctuations very long (their ratios will stay the same). Hence if a fluctuation hits a target the collision

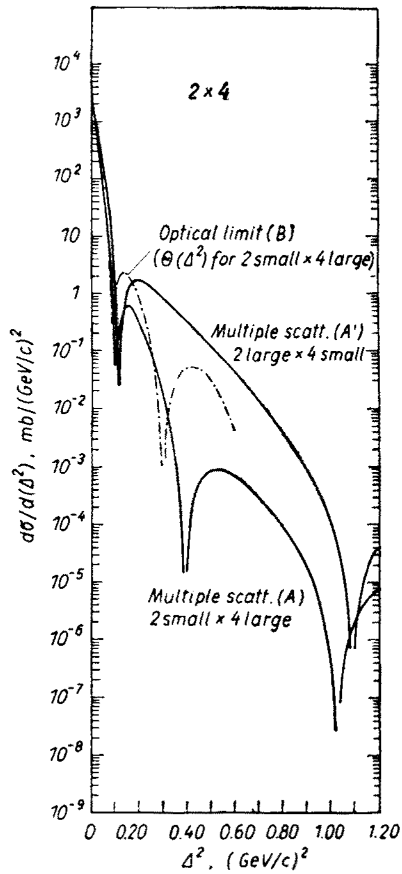


Fig. 14. Example of elastic scattering cross-section for an object with two subunits scattering from an object with four subunits, with two different radii interchanged (A), (A'): $R = 2.28$ fm, $R = 1.00$ fm, and the corresponding optical limit (B). This figure illustrates the point that in the case of two composite objects collision (many subunits against many subunits) the convergence to the optical limit is much slower than in the case one subunits against many. For more details see Ref. [6]

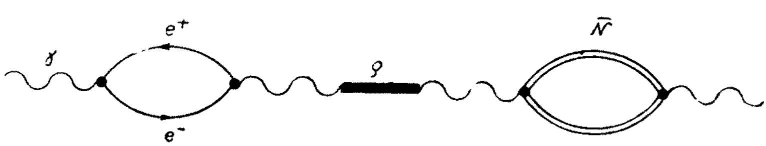


Fig. 15

will be between the subunits of the fluctuation and the target (the probability that the fluctuation will disappear during the collision process is virtually zero, in the limit of very high photon energies). One can also see this effect as follows: If the invariant mass of a fluctuation is m, the energy is not conserved at the fluctuation vertex by an amount

$$\Delta E \approx (m^2+k^2)^{1/2}-k \approx \frac{m^2}{2k} \tag{12}$$

hence we get the time uncertainty

$$\Delta t \approx \frac{2k}{m^2} \tag{13}$$

which contains the time dilatation factor (k/m) which, as discussed above, makes the fluctuation interact with the whole target.

Different targets will "see" different fluctuations. *E.g.*, a nuclear target will "see" "strong fluctuations" of the photon (this indeed seems to be confirmed by the experiments Fig. 16 [7]), a strong Coulomb field instead will have a preference to see the electron-positron fluctuation. Let us consider this last process (Delbrück scattering). In order to apply the

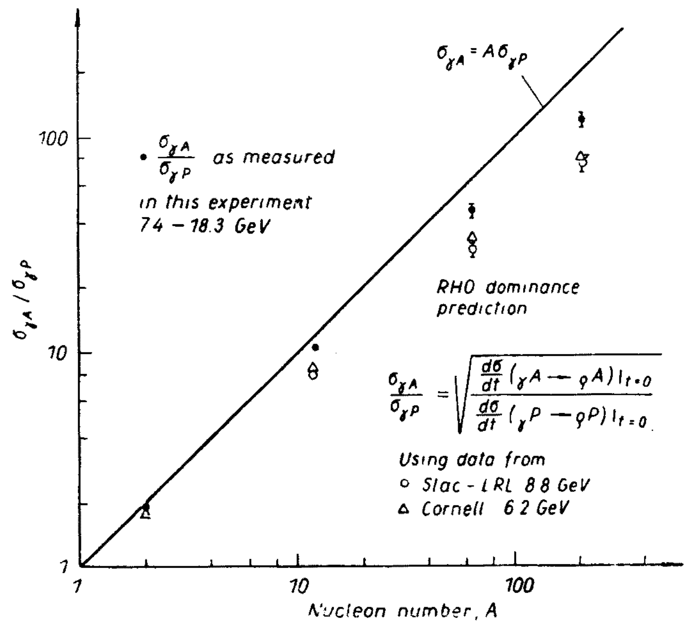
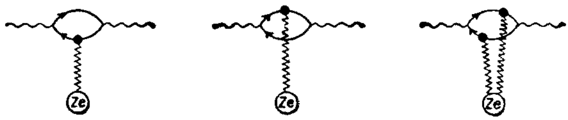
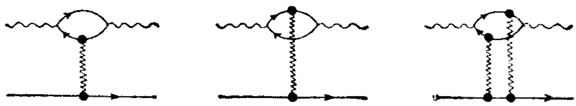


Fig. 16. The ratio of the total photoabsorption cross-section of nucleus A to that of hydrogen as observed (black points) (Ref. [25]), compared with results expected from a purely electromagnetic photon (line) or from a ρ-dominant photon using cornell and SLAC-LRL data (open points)

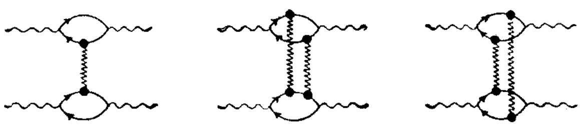
formula (5) we have to know the high energy electron (positron) scattering amplitude on the Coulomb field and the density distribution of the electron-positron fluctuation (let us call it $\varrho^{\gamma}(\mathbf{s}_1, \mathbf{s}_2)$. Before going into any details of the calculations, let us stress the point that by accepting that the high energy Delbrück scattering is, to a very good approximation, a scattering of an electron-positron pair by the Coulomb field, we commit ourselves to a prediction of the amplitudes (again in the limit of high energy) of such processes as Compton scattering and photon-photon scattering. This is so, because again the same elements (ϱ^{γ} and $e^{\pm}-e^{\pm}$ or $e^{\pm}-Ze^{-}$ amplitudes) enter in the expressions for amplitudes of these processes if one uses Eq. (5). Fig. 17 shows all these processes.



(a) Delbrück scattering (all possible scatterings of subunits)



(b) Compton scattering (all possible scatterings of subunits)



(c) Photon-photon scattering (some of the possible scatterings of subunits)

Fig. 17

Let us suppose that we know the two-dimensional density of the electron-positron fluctuation of the photon

$$\varrho^{\gamma}(\mathbf{s}_1, \mathbf{s}_2) = \frac{1}{(2\pi)^4} \int d^2\delta_1 d^2\delta_2 \tilde{\varrho}^{\gamma}(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2) e^{-i\boldsymbol{\delta}_1\cdot\mathbf{s}_1 - i\boldsymbol{\delta}_2\cdot\mathbf{s}_2} \tag{14}$$

and the electron (positron) — Coulomb field high-energy scattering amplitude. Then, from Eqs (4) and (5) we immediately get the amplitudes:

For Delbrück scattering

$$\mathcal{M}_{\mathrm{Del}} = \frac{ik}{(2\pi)^3} \int d^2\delta_1 d^2\delta_2 \delta^{(2)}(\boldsymbol{\Delta} - \boldsymbol{\delta}_1 - \boldsymbol{\delta}_2) \tilde{\varrho}^{\gamma}_{\mu\nu}(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2) \times$$
$$\times \{(2\pi)^4 \delta^{(2)}(\boldsymbol{\delta}_1)\delta^{(2)}(\boldsymbol{\delta}_2) - \Sigma_1^z(\boldsymbol{\delta}_1)\Sigma_2^z(\boldsymbol{\delta}_2\} \tag{15}$$

where the indices μ, ν indicate polarizations of ingoing and outgoing photons, and

$$\Sigma_{1,2}^z(\boldsymbol{\delta}) = \int d^2t e^{i\boldsymbol{\delta}\cdot\boldsymbol{t}}(1 - \gamma_{1,2}^z(\boldsymbol{t})) \tag{16}$$

$$= \int d^2t e^{i\boldsymbol{\delta}\cdot\boldsymbol{t}} e^{i\chi_{1,2}^z(t)}. \tag{17}$$

For Compton scattering

$$\mathcal{M}_{\mathrm{Compt}} = \frac{ik}{(2\pi)^3} \int d^2\delta_1 d^2\delta_2 \delta^{(2)}(\boldsymbol{\Delta} - \boldsymbol{\delta}_1 - \boldsymbol{\delta}_2) \tilde{\varrho}^{\gamma}_{\mu\nu}(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2) \times \{(2\pi)^4 \delta^{(2)}(\boldsymbol{\delta}_1)\delta^{(2)}(\boldsymbol{\delta}_2) - \Sigma_1(\boldsymbol{\delta}_1)\Sigma_2(\boldsymbol{\delta}_2)\} \tag{18}$$

where

$$\Sigma_{1,2}(\boldsymbol{\delta}) = \Sigma_{1,2}^{z=1}(\boldsymbol{\delta}).$$

For photon-photon amplitude

$$\mathscr{M}^{\gamma\gamma} = \frac{ik}{(2\pi)^7} \int d^2\delta_1 d^2\delta_2 d^2\delta_3 d^2\delta_4 \delta^{(2)}(\boldsymbol{\Delta}-\boldsymbol{\delta}_1-\boldsymbol{\delta}_2-\boldsymbol{\delta}_3-\boldsymbol{\delta}_4)\times$$
$$\times \tilde{\varrho}^{\gamma}_{\mu\nu}(\boldsymbol{\delta}_1+\boldsymbol{\delta}_2,\ \boldsymbol{\delta}_3+\boldsymbol{\delta}_4)\tilde{\varrho}^{\gamma}_{\mu'\nu'}(\boldsymbol{\delta}_1+\boldsymbol{\delta}_3,\ \boldsymbol{\delta}_2+\boldsymbol{\delta}_4)\times$$
$$\times[(2\pi)^8\delta^{(2)}(\boldsymbol{\delta}_1)\delta^{(2)}(\boldsymbol{\delta}_2)\delta^{(2)}(\boldsymbol{\delta}_3)\delta^{(2)}(\boldsymbol{\delta}_4)-\Sigma_1(\boldsymbol{\delta}_1)\Sigma_2(\boldsymbol{\delta}_2)\Sigma_3(\boldsymbol{\delta}_3)\Sigma_4(\boldsymbol{\delta}_4)]. \tag{19}$$

Note that k is the CM momentum everywhere. This is so because our fundamental formula $\mathscr{M} = \frac{ik}{2\pi}\int d^2b e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}}(1-e^{i\chi(b)})$ was obtained from the partial wave expansion valid in the c.m. system.

2. *Some details of computations of Delbrück scattering*

Now let us discuss some details of computing Σ's and $\tilde{\varrho}^{\gamma}$. First let us consider the scattering of a Dirac particle from a Coulomb potential in the limit of very high energy. The equation to be solved is ($\hbar = c = 1$):

$$\left(i\frac{\partial}{\partial t}+i\left(\alpha_1\frac{\partial}{\partial x}+\alpha_2\frac{\partial}{\partial y}\right)+i\alpha_3\frac{\partial}{\partial z}+\beta m\right)\Psi = eV(b,z)\Psi, \tag{20}$$

where we explicitely split the transverse and longitudinal degrees of freedom: the incident electron moves along z-axis with a very large momentum, the components of the electron coordinate $\boldsymbol{r}$ are $\boldsymbol{b}$ (transverse) and z (longitudinal). One can convice oneself[1] that in the limit $E\to\infty$ [11]

$$(1-\alpha_3)\Psi\to 0, \quad \text{hence} \quad (1+\alpha_3)\Psi\to 2\Psi. \tag{21}$$

Multiplying (20) from the left by $(1+\alpha_3)$ we get

$$\left(i\frac{\partial}{\partial t}+i\alpha_3\frac{\partial}{\partial z}\right)(1+\alpha_3)\Psi+\left(i\alpha_1\frac{\partial}{\partial x}+i\alpha_2\frac{\partial}{\partial y}+\beta m\right)(1-\alpha_3)\Psi = eV(b,z)(1+\alpha_3)\Psi. \tag{22}$$

Hence, in the limit $E\to\infty$, we have the equation

$$i\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)\Psi = eV(b,z)\Psi, \tag{23}$$

whose solution is

$$\Psi = u(p)e^{-iE(t-z)}e^{-ie\int_{-\infty}^{z}V(b,z)dz'} \tag{24}$$

[1] It follows directly from (20) or from any standard spinor algebra see *e. g.* Ref. [8].

where $u(p)$ is the incident wave spinor. The scattering amplitude is therefore

$$\mathcal{M} = -\frac{m}{2\pi}\int d^3r\bar{\psi}_f\gamma_0 eV(b,z)\Psi, \tag{25}$$

where Ψ is the solution (24), and ψ_f is the plane wave final state (in the high energy limit $E \approx p'$):

$$\psi_f = u(p')e^{-iE(t-z)+i\mathbf{\Delta}\cdot\mathbf{b}} \tag{26}$$

$$\Delta = \mathbf{p}'-\mathbf{p}. \tag{27}$$

After few manipulations we get

$$\mathcal{M} = \frac{m}{2\pi}\,\bar{u}(p')\gamma_0 u(p) i\int d^2b e^{-i\mathbf{\Delta}\cdot\mathbf{b}}[1-e^{-ie\int_{-\infty}^{+\infty}dzV(b,z)}]. \tag{28}$$

Notice that if the transverse momentum transfer is much smaller than p ($\Delta \ll p$) we have virtually only the non-spin flip scattering, because ($\gamma_0 = \beta$):

$$\bar{u}(p')\gamma_0 u(p) \approx \frac{p}{m} \qquad \text{(no spin flip)}$$

and

$$\bar{u}(p')\gamma_0 u(p) \approx \frac{p}{2m}\,O\left(\frac{\Delta}{p}\right) \qquad \text{(spin flip)}.$$

So, for non-spin flip transitions we can write

$$\mathcal{M} = \frac{ip}{2\pi}\int d^2b e^{i\mathbf{\Delta}\cdot\mathbf{b}}[1-e^{i\chi(b)}] \tag{29}$$

where

$$\chi(b) = -e\int_{-\infty}^{+\infty}dzV(b,z).$$

Notice that, following our terminology introduced before, $1-e^{i\chi(b)}$ is the profile of the electron (positron)-Coulomb field elastic scattering.

Let us compute $\tilde{\varrho}^{\gamma}(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2)$. In the formulae (4) and (5) we have some ground state densities introduced without specifying in detail the methods to obtain them. In computing $\tilde{\varrho}^{\gamma}(\boldsymbol{\delta}_1, \boldsymbol{\delta}_2)$ we shall follow very closely the analogy with the hadron (*e.g.* pion) — deuteron scattering.

First of all let us notice that the amplitude (4) or (5) should be considered to be a matrix element of the amplitude where all the subunits are frozen in certain positions in space:

$$\mathcal{M} = \frac{ik}{2\pi}\int d^2b e^{i\mathbf{\Delta}\cdot\mathbf{b}}\langle\Psi_f|\Gamma(b, s_1\ldots s_A)|\Psi_i\rangle. \tag{30}$$

For instance, in the case of hadron-deuteron scattering we should take

$$\Psi_i^D(\mathbf{r}_1\,\mathbf{r}_2) = \mathcal{N}e^{i\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)\cdot\mathbf{K}_i}\varphi(\mathbf{r}_1-\mathbf{r}_2) \tag{31}$$

$$\Psi_f^D(\mathbf{r}_1\,\mathbf{r}_2) = \mathcal{N}e^{i\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)\cdot\mathbf{K}_f}\varphi(\mathbf{r}_1-\mathbf{r}_2) \tag{32}$$

where $\mathcal{N}$ is a normalization constant, and

$$\Gamma = 1-(1-\gamma_1(\boldsymbol{b}-\boldsymbol{s}_1))\,(1-\gamma_2(\boldsymbol{b}-\boldsymbol{s}_2)). \tag{33}$$

$\boldsymbol{K}_i$ and $\boldsymbol{K}_f$ are the deuteron momenta before and after collision, $(\boldsymbol{r}_1+\boldsymbol{r}_2)/2$ and $\boldsymbol{r}_1-\boldsymbol{r}_2$ are the CM coordinate and the relative coordinate, respectively.

It is more customary to describe the electron-positron fluctuation in the momentum space, not in the coordinate space, hence we shall write (30) in momentum space. Let us denote

$$\tilde{M} = \frac{ik}{2\pi}\int d^2b e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\Gamma(b, s_1, s_2). \tag{34}$$

Then the scattering amplitude is $\langle\Psi_f|\hat{M}|\Psi_i\rangle$. In the case of Delbrück scattering $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the initial and final states, respectively, of a fluctuation photon. If we denote them by $|\gamma_i\rangle$ and $|\gamma_f\rangle$, respectively, we can write the amplitude as follows:

$$\langle\Psi_f|\hat{M}|\Psi_i\rangle = \langle\gamma_f|\hat{M}|\gamma_i\rangle$$
$$= \int d^3k_1 d^3k_2 d^3q_1 d^3q_2 \langle\gamma_f|\boldsymbol{k}_1\boldsymbol{k}_2\rangle\,\langle\boldsymbol{k}_1\boldsymbol{k}_2|\hat{M}|\boldsymbol{q}_1\boldsymbol{q}_2\rangle\,\langle\boldsymbol{q}_1\boldsymbol{q}_2|\gamma_i\rangle, \tag{35}$$

provided we restrict ourselves to the electron-positron fluctuation (which is, presumably, by far the leading process). In order to compute $|\gamma_i\rangle$ and $|\gamma_f\rangle$ we employ the standard perturbation theory and take the lowest order expression containing the fluctuation in question. This is in accordance with the "infinite momentum" frame technique (see *e.g.* Refs [9] and [10] and the lectures by J. D. Bjorken given at this school).

$$|\gamma\rangle = Z^{1/2}|\gamma_0\rangle + \sum \frac{\langle k_1k_2|H'|\gamma_0\rangle}{\omega-E_1(k_1)-E_2(k_2)}\,|\boldsymbol{k}_1\boldsymbol{k}_2\rangle \tag{36}$$

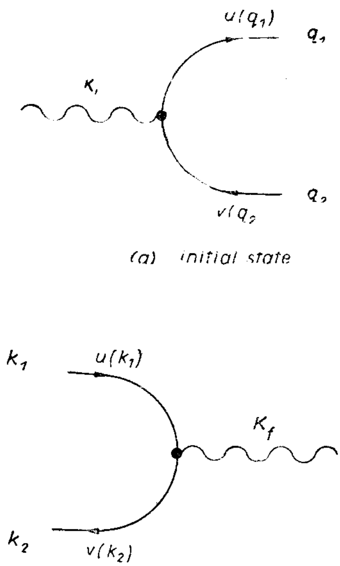
where $|\gamma_0\rangle$ is the bare photon state and $Z^{1/2}$ is the renormalization constant (which assures the right normalization of the dressed photon state $|\gamma\rangle$), ω is the incident photon energy, E_1 and E_2 are the energies of the electron and the positron and k_1 and k_2 their momenta. From (36) we get the initial and final photon "wave functions":

$$\langle\boldsymbol{q}_1\boldsymbol{q}_2|\gamma_i\rangle_\mu = \frac{\langle\boldsymbol{q}_1\boldsymbol{q}_2|H'|\gamma_{0i}\rangle}{\omega-E_1(q_1)-E_2(q_2)}$$
$$= \frac{e}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_1(q_1)}}\sqrt{\frac{m}{E_2(q_2)}}\sqrt{\frac{1}{2\omega}}\,\frac{[\bar{u}(q_1)\gamma_\mu v(q_2)]}{\omega-E_1(q_1)-E_2(q_2)}\,\delta^{(3)}(\boldsymbol{K}_i-\boldsymbol{q}_1-\boldsymbol{q}_2),$$
$$\langle\gamma_f|\boldsymbol{k}_1\boldsymbol{k}_2\rangle_\nu = \frac{\langle\gamma_{0f}|H'|\boldsymbol{k}_1\boldsymbol{k}_2\rangle}{\omega-E_1(k_1)-E_2(k_2)} \tag{37}$$
$$= \frac{e}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_1(k_1)}}\sqrt{\frac{m}{E_2(k_2)}}\sqrt{\frac{1}{2\omega}}\,\frac{[\bar{v}(k_2)\gamma_\nu u(k_1)]}{\omega-E_1(k_1)-E_2(k_2)}\,\delta^{(3)}(\boldsymbol{K}_f-\boldsymbol{k}_1-\boldsymbol{k}_2). \tag{38}$$

In these formulae K_i and K_f are the initial and final photon momenta,

$$H' = e\int d^3x\bar{\psi}(x,0)\gamma_\mu\psi(x,0)A_\mu(x,0) \tag{39}$$

is the interaction Hamiltonian (ψ — spinor field, A_μ — photon field, the normalizations of the electron and positron and photon plane waves are the same as in Ref. [8]). We used only the electron-positron component of (36) because the first term of r.h.s. of (36) does not, according to our model, contribute to the Delbrück scattering amplitude. The "wave



(b) final state

Fig. 18

functions" (37) and (38) can be graphically presented as shown in Fig. 18. Incidentally, notice that the deuteron initial and final states (31), (32), have, in the momentum space, analogous structure to (37), (38):

$$\langle \boldsymbol{k}_1\boldsymbol{k}_2|\Psi_i^D\rangle = \mathcal{N}\tilde{\varphi}(\tfrac{1}{2}(\boldsymbol{k}_1-\boldsymbol{k}_2))\,\delta^{(3)}(\boldsymbol{K}_i-\boldsymbol{k}_1-\boldsymbol{k}_2), \tag{40}$$

$$\langle \Psi_f^D|\boldsymbol{q}_1\boldsymbol{q}_2\rangle = \mathcal{N}\tilde{\varphi}(\tfrac{1}{2}(\boldsymbol{q}_1-\boldsymbol{q}_2))\,\delta^{(3)}(\boldsymbol{K}_i-\boldsymbol{q}_1-\boldsymbol{q}_2). \tag{41}$$

Now back to Delbrück scattering. Since we limit ourselves to the electron-positron fluctuations, we have

$$\langle \boldsymbol{k}_1\boldsymbol{k}_2|\hat{M}|\boldsymbol{q}_1\boldsymbol{q}_2\rangle$$
$$= \frac{i\omega}{2\pi}\left\langle \boldsymbol{k}_1\,\boldsymbol{k}_2\right|\int d^2b e^{i\Delta\cdot\boldsymbol{b}}\{1-[1-\gamma_1(\boldsymbol{b}-\boldsymbol{s}_1)][1-\gamma_2(\boldsymbol{b}-\boldsymbol{s}_2)]\}\left|\boldsymbol{q}_1\boldsymbol{q}_2\right\rangle. \tag{42}$$

Introducing the $\Sigma(\delta)$ functions (compare Eq. (16)) and changing the variables, we get (we drop 1 in the curly bracket of (42) because it contributes to the forward, $\Delta = 0$, scattering amplitude, and it can be easily included afterwards):

$$\langle \boldsymbol{k}_1\boldsymbol{k}_2|\hat{M}(\Delta\neq 0)|\boldsymbol{q}_1\boldsymbol{q}_2\rangle$$
$$= \frac{i\omega}{2\pi}\frac{N^2}{(2\pi)^6}\,\delta(q_{1z}-k_{1z})\,\delta(q_{2z}-k_{2z})\,\delta^{(2)}(\Delta-\boldsymbol{k}_1-\boldsymbol{k}_2+\boldsymbol{q}_1+\boldsymbol{q}_2)\sum\nolimits_1(\boldsymbol{k}_1-\boldsymbol{q}_1)\sum\nolimits_2(\boldsymbol{k}_2-\boldsymbol{q}_2). \tag{43}$$

In the r.h.s. of (43) all the vectors are two dimensional (in the plane perpendicular to the direction of the incident photon). Notice also that functions $\Sigma_{1,2}$, as defined in (16), do not have any spin dependence. On the other hand, the electron (positron)-Coulomb field scattering amplitude (28) does have the spin dependent coefficient $\bar{u}(p')\gamma_0 u(p)$ (or $\bar{v}(p')\gamma_0 v(p)$ in the case of positrons). So, to have all spin effects correctly taken care of in our amplitude, we replace in the final expression for the amplitude

$$\sum\nolimits_1 (\boldsymbol{k}_1-\boldsymbol{q}_1) \to \sqrt{\frac{m^2}{E_1(k_1)E_1(q_1)}}\,\bar{u}(k_1)\gamma_0 u(q_1) \sum\nolimits_1 (\boldsymbol{k}_1-\boldsymbol{q}_1),$$
$$\sum\nolimits_2 (\boldsymbol{k}_2-\boldsymbol{q}_2) \to \sqrt{\frac{m^2}{E_2(k_2)E_2(q_2)}}\,\bar{v}(q_2)\gamma_0 v(k_2) \sum\nolimits_2 (\boldsymbol{k}_2-\boldsymbol{q}_2). \tag{44}$$

The coefficients $(m^2/E(k)E(q))^{1/2}$ give the correct normalization because for forward scattering there is no difference between the l.h.s. and the r.h.s. of (44). Now we are in a position to write the complete expression for the Delbrück scattering amplitude. Summing over all spin directions we get

$$\langle\gamma_f|\hat{M}(\Delta \neq 0)|\gamma_i\rangle \sim \frac{e^2}{(2\pi)^3}\frac{\mathcal{N}^2}{(2\pi)^6}\frac{i}{4\pi}\int d^3k_1 d^3k_2 d^3q_1 d^3q_2 \times$$
$$\times\ \delta(q_{1z}-k_{1z})\delta(q_{2z}-k_{2z})\delta^{(3)}(\boldsymbol{K}_i-\boldsymbol{q}_1-\boldsymbol{q}_2)\delta^{(3)}(\boldsymbol{K}_f-\boldsymbol{k}_1-\boldsymbol{k}_2) \times$$
$$\times\ \delta^{(2)}[\Delta-(\boldsymbol{k}_1-\boldsymbol{q}_1)-(\boldsymbol{k}_2-\boldsymbol{q}_2)] \sum\nolimits_1 (\boldsymbol{k}_1-\boldsymbol{q}_1) \sum\nolimits_2 (\boldsymbol{k}_2-\boldsymbol{q}_2) \times$$
$$\times\ \frac{m_4}{E_1(k_1)E_2(k_2)E_1(q_1)E_2(q_2)} \sum_{\text{spins}} \times$$
$$\times\ \frac{(\bar{u}(q_1)\gamma_\mu v(q_2))(\bar{v}(q_2)\gamma_0 v(k_2)(\bar{v}(k_2)\gamma_\nu u(k_1))(\bar{u}(k_1)\gamma_0 u(q_1))}{[\omega-E_1(q_1)-E_2(q_2)][\omega-E_1(k_1)-E_2(k_2)]}. \tag{45}$$

This expression is, to within a constant factor, identical with the *e.g.* Cheng and Wu expression [11] for the high energy Delbrück scattering amplitude. In order to reduce it to (15) one should introduce the variables

$$\delta_1 = \boldsymbol{k}_1-\boldsymbol{q}_2,\ \ \delta_2 = \boldsymbol{k}_2-\boldsymbol{q}_2, \tag{46}$$

and perform all integrations except over the transverse $\boldsymbol{\delta}_1$, and $\boldsymbol{\delta}_2$.

The electron-positron fluctuation density $\tilde{\varrho}^\gamma(\boldsymbol{\delta}_1\boldsymbol{\delta}_2)$ of Eqs (15), (18), (19) extracted from such a formula takes the form [10] (we shall skip the computations):

$$\tilde{\varrho}^\gamma_{\mu\nu}(\boldsymbol{\delta}_1\boldsymbol{\delta}_2) \sim \frac{\alpha}{\pi}\int_0^1 d\beta d\beta' dx\,\delta(1-\beta-\beta')\times$$
$$\times\ \frac{[4K_\mu K_\nu\beta\beta' x(1-x)-\frac{1}{2}\,\delta_{\mu\nu}\boldsymbol{K}^2(1-8\beta\beta'(x-\frac{1}{2})^2)]}{[m^2+x(1-x)\boldsymbol{K}^2]}, \tag{47}$$

where

$$K = \beta'\boldsymbol{\delta}_1-\beta\boldsymbol{\delta}_2. \tag{48}$$

The function $\delta(1-\beta-\beta')$ in the integrand of (47) expresses conservation of the longitudinal momenta and comes from integrations over some longitudinal momenta in (45). Hence $\beta = \frac{k_{1z}}{k_z}$ and $\beta' = \frac{k_{2z}}{k_z}$ are proportional to the longitudinal momenta of the pair (hence, in the high energy limit, to their energies). The fact that $\tilde{\varrho}^\gamma$ is a function of K only means physically that the coordinates $\mathbf{s}_1$ and $\mathbf{s}_2$ of the pair are not completely independent. Taking the Fourier transform of $\tilde{\varrho}^\gamma_{\mu\nu}(\boldsymbol{\delta}_1\boldsymbol{\delta}_2)$

$$\varrho^\gamma_{\mu\nu}(\mathbf{s}_1, \mathbf{s}_2) = \int \frac{d^2\delta_1}{(2\pi)^2}\, \frac{d^2\delta_2}{(2\pi)^2}\, e^{-i\boldsymbol{\delta}_1\cdot\mathbf{s}_1 - i\boldsymbol{\delta}_2\cdot\mathbf{s}_2}\tilde{\varrho}^\gamma_{\mu\nu}(\boldsymbol{\delta}_1\boldsymbol{\delta}_2) \tag{49}$$

one gets (under the integral sign of (47)) a factor

$$\delta^{(2)}(\beta\mathbf{s}_1 + \beta'\mathbf{s}_2) \tag{50}$$

which expresses the constraint of the type of the c. m. motion conservation. This constraint is different from the one one gets in the case of "nonrelativistic" wave functions (40), (41). There, the density is a function of 1/2 $(\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2)$ (see (40) and (41)) and the CM constraint comes out to be

$$\delta^{(2)}(\mathbf{s}_1 + \mathbf{s}_2). \tag{51}$$

So, it is not involved in the internal structure of the wave function as in the case of the photon "wave function" which fact is clearly demonstrated by the existence of the factor (50).

One can apply this method of calculating the high energy limits to the other electrodynamical cross-sections: $\gamma - e^-(e^+)$, $\gamma - \gamma$. The results were already given (Eqs (18), (19)). Once one computes the density of a particular fluctuation one may then compute all processes which contain the same fluctuation, without going into all the details of the detailed computations, by employing this density and the formula (4) or (5), as was done in computing (18) and (19).

3. *Tests of the model developed in 1 and 2*

Let us talk about "theoretical" tests of the model. Quantum electrodynamics is the only relativistic theory in existence (to the best of this author's knowledge) which can test the model. This test, which one can perform by comparing the results discussed here, and in the lectures by J. D. Bjorken given at this school (see also the results published *e. g.* in Refs [10, 11]) strongly confirms the soundness of our model. The results of *e. g.* Ref. [10] where one obtains the high energy cross-sections for $e^+ - e^\pm$, $\gamma - e$ and $\gamma - \gamma$ scattering by summing certain classes of diagrams agree with the results produced by our model. One may question, in principle, the completeness of the results quoted in the sense that the proof that they differ little from the exact electrodynamical expressions seems to be lacking, but as things stand now, the complete agreement between our model and the summation of some sets of Feynman graphs is striking. It is very amusing that all off-shell effects, which are so important at lower energies, disappear in the high energy limit, and only the on-shell effects are present.

Notice that our model can be extended to all field theoretic models once we know that a certain set of fluctuations (it may be just one fluctuation — as in the examples quoted above) dominates a scattering. The fundamental trouble in applying the model to strong interactions (large coupling constant) seems to be the lack of a clear criterium of limiting the complexity of fluctuations relevant in the processes in question.

The second "theoretical" test is the potential theory scattering. An example of the high energy limit of a potential scattering amplitude was worked out above explicitly for the case of the Dirac equation. If we start from the Schrödinger equation we get [12, 13] essentially the same formula as (29) (we use $\hbar = c = 1$ units)

$$\mathcal{M} = \frac{ik}{2\pi} \int d^2b e^{i\Delta \cdot b} (1 - e^{-\frac{i}{v} \int_{-\infty}^{+\infty} dz V(b, z)}) \tag{52}$$

where V is the interaction potential (equal to eV in the case of Coulomb scattering). We have explicitly exhibited the velocity v in the exponent, because in the case of the Schrödinger equation, velocity may become infinite. Notice that if we accept that $v \leqslant 1$, Eq. (52) is identical with (29). Let us accept the "proper" high energy limit of (52) which tells us to put $v = 1$. Let us also suppose that

$$V(\boldsymbol{r}) = \sum_i V_i(\boldsymbol{r} - \boldsymbol{r}_i) \tag{53}$$

where V_i are the interaction potentials between the incoming particle and the target particles (which sit at the points $\boldsymbol{r}_i$). From (52) and (53) we get formula (4) because, due to (53), we have additivity of phase shifts. So, in the case when the positions of subunits of the target are fixed in space, the total scattering amplitude can be constructed from the individual "on-shell" amplitudes. Hence a similar situation occurs as in the case of quantum electrodynamics. One may argue that in the case of potential scattering from a collection of fixed in space scattering centres this is understandable, but if the scattering centres are some bound subunits of the target (with some definite binding energy) it is hard to believe that only "on-shell" elements enter into calculations: the subunits are "off-shell" from the start, after all. This and related problems have been discussed many times. Some more recent contributions to this subject are contained in Refs [14] and [15] which represent opposite tendencies (Ref. [14] gives arguments supporting the view that in the high potential scattering the off-shell contributions cancel each other, instead Ref. [15] argues that it is probably not so). In any case it would seem that one should expect some considerable cancellations to occur if one uses a multiple scattering series, which for *e. g.* hadron-deuteron scattering looks as follows [14]

$$T = T^{(1)} + T^{(2)} + T^{(3)} + T^{(4)} + \ldots \tag{54}$$

where

$$\begin{aligned} T^{(1)} &= t_n + t_p \\ T^{(2)} &= t_p G_0 t_n + t_n G_0 t_p \\ T^{(3)} &= t_p G_0 t_n G_0 t_p + t_n G_0 t_p G_0 t_n \\ &\ldots \end{aligned} \tag{55}$$

$t_{n,p}$ are scattering matrices on neutron and proton and G_0 are Green's functions. One gets the scattering amplitude from T by evaluating it between states of a deuteron plus a plane wave representing the incident (or outgoing) hadron. One may construct such operators $t_{n,p}$ and the Green function G_0 that, on the one hand $T^{(1)}+T^{(2)}$ taken on-shell are identical to the Glauber model hadron-deuteron scattering amplitude, but, on the other, be sure that the complete T is also completely equivalent to Glauber model (for explicit construction see Ref. [14]). In this case the off-shell contribution to $T^{(2)}$ must be cancelled exactly by the remaining (infinitely many) terms. One can see it explicitly by computing various contributions of the series (55) (compare [14]). This example is a warning to anybody who wants to compute corrections to the Glauber model starting from any commonly used multiple scattering series: a warning that one should, in any case, watch out for substantial concellations between various terms of the expansion.

One should stress, however, that in the case of nuclear collisions there are presumably some "off-shell" effects present. For instance, in the case of hadron-deuteron scattering, the contribution given by production of *e. g.* some excited states of hadrons on one nucleon and reabsorption of them on the other so the net results is elastic scattering, is such an "off-shell" effect which should, in principle, exist [17, 18] although it has not been observed so far. One may propose the following understanding of the "off-shell" effects. As long as a theory is "complete" in the sense that all degrees of freedom are explicitly introduced in the multiple scattering calculations (as is done in the case of quantum electrodynamics and potential scattering) the off-shell effects tend to disappear. But if we introduce explicitly into calculations only part of all degrees of freedom which are active in a high energy scattering process, and the rest is taken into account by adopting some "effective" elastic scattering amplitudes on subunits of the colliding objects, then the off-shell effects may become important under some circumstances. Such a situation exists in nuclear collisions (in the version described in the first section): the coordinates of nucleons are introduced explicitly into calculations. The mesonic degrees of freedom are included indirectly in the effective hadron-nucleon elastic amplitudes.

REFERENCES

[1] W. Czyż, L. Leśniak, H. Wołek, *Nuclear Phys.*, **B19**, 125 (1970).

[2] W. Czyż, *High Energy Scattering from Nuclei*, Advances in Nuclear Physics, Vol. 4, ed. M. Baranger and E. Vogt.

[3] W. Czyż, L. C. Maximon, Brookhaven National Laboratory, *Report BNL 14052*, 1969; *Ann. Phys.* (USA), **60**, 484 (1970).

[4] T. T. Chou, C. N. Yang, *Phys. Rev. Letters*, **20**, 1213 (1968); *Phys. Rev.*, **170**, 159 (1968); *Phys. Rev.*, **175**, 1832 (1968).

[5] L. Durand, R. Lipes, *Phys. Rev. Letters*, **20**, 637 (1968).

[6] W. Czyż, L. C. Maximon, *Ann. Phys.* (USA), **52**, 59 (1969).

[7] D. O. Caldwell, V. B. Elings, W. P. Hesse, G. E. Jahn, R. J. Morrison, F. V. Murphy, D. E. Yount, *Phys. Rev. Letters*, **23**, 1256 (1969).

[8] J. D. Bjorken, S. D. Drell, *Relativistic Quantum Mechanics, and Relativistic Quantum Fields*, Mc Graw-Hill, 1964–65.

[9] S. D. Drell, D. J. Levy, Tung-Mew Yan, *SLAC-PUB-645*, September 1969.

[10] S. J. Chang, S. K. Ma, *Phys. Rev. Letters*, **22**, 1334 (1969).
[11] H. Cheng, T. T. Wu, in *High Energy Collisions*, Gordon and Breach, p. 329, 1969.
[12] T. T. Wu, *Phys. Rev.*, **108**, 466 (1957).
[13] L. I. Schiff, *Phys. Rev.*, **103**, 443 (1956); D. S. Saxon, L. I. Schiff, *Nuovo Cimento*, **6**, 614 (1957).
[14] D. R. Harrington, *Phys. Rev.*, **184**, 1745 (1969).
[15] P. B. Kantor, *Phys. Rev. Letters*, **24**, 1081 (1970).
[16] L. L. Foldy, *Phys. Rev.*, **67**, 107 (1945).
[17] J. Pumplin, M. Ross, *Phys. Rev. Letters*, **21**, 1778 (1968).
[18] D. Harrington, *Phys. Rev.* D, **1**, 260 (1970).
[19] J. Friedes, H. Palevsky, R. J. Sutter, G. W. Bennet, G. J. Igo, W. D. Simpson, D. M. Corley, *Nuclear Phys.*, **A104**, 294 (1967).
[20] H. Palevsky, J. L. Friedes, R. J. Sutter, D. M. Corley, N. S. Wall, R. L. Stearns, B. Gottschalk, *Phys. Rev. Letters*, **18**, 1200 (1967).
[21] J. V. Allaby, A. N. Diddens, R. J. Glauber, A. Klovning, O. Kofoed-Hansen, E. J. Sacharidis, K. Schlüpmann, A. M. Thorndike, A. M. Wetherell, *CERN preprint*, November 1969.
[22] C. Michael, C. Wilkin, *Nuclear Phys.*, **B11**, 99 (1969).
[23] G. Alberi, L. Bertocchi, *Nuovo Cimento*, **63A**, 285 (1969).
[24] G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillethun, G. Matthiae, J. P. Scanlon, A. M. Wetherell, *Nuclear Phys.*, **79**, 609 (1966).
[25] D. O. Caldwell, V. B. Elings, W. P. Hesse, G. E. Jahn, R. J. Morrison, F. V. Murphy, D. E. Yount, *Phys. Rev. Letters*, **23**, 1256 (1969).