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LEPTON-HADRON PROCESSES AT HIGH ENERGY

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Contents: 1. Phenomenology, 2. Current commutators and light cone behaviour, 3. Dynamical models, 4. Quantum electrodynamics at infinite momentum.

1. Introduction

High energy physics at the present time finds itself at a threshold of great expectations. Not in a decade — perhaps never — has there appeared such a great leap forward as will soon appear in that most basic commodity of the field energy. At present, relatively little has been explored beyond 30 GeV. The Serpukhov machine at 70 GeV, NAL at 200, 400 and eventually 500 GeV, the CERN ISR at lower intensity but a still higher equivalent laboratory energy of over 1500 GeV herald an increase in available energy of between one and two orders of magnitude. Electron-positron colliding beam facilities under construction will reach into a new high-energy regime. From the present region of $s = E^2_{CM} \sim 1$ GeV^2 dominated by the vector-meson production, the new rings will attain an $s \sim 15$–30 GeV^2 with CEA capable of reaching $s \sim 100$ GeV^2. In this high energy region very little theoretical insight exists (other than for the pure electrodynamic processes) on what even the qualitative features will look like.

While the greatest of expectations lies in the anticipation of production of new kinds of particles (W's, quarks, monopoles, heavy leptons, hadronic leptons) or observation of new classes of interactions, or maybe even something present concepts are insufficient to deal with, there are other new classes of phenomena which are still not so unfamiliar as to be impossible for the theoretician to try to discuss. A major area of this nature and the subject of these lectures is that of lepton-induced hadron reactions at high energies. Typical examples are

(a) $e+N \to e+$hadrons

(b) $\mu+N \to \mu+$hadrons

(c) $\nu+N \to \mu^-+$hadrons

(d) $\bar{\nu}+N \to \mu^++$hadrons

(e) $p+p \to \mu^+\mu^-+$hadrons

(f) $e^++e^- \to$ hadrons.

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Also in the same qualitative category are

(g) $\gamma+N \rightarrow \gamma+$hadrons

(h) $\gamma+N \rightarrow \mu^+\mu^-+$hadrons.

At present, we know that reactions (a), (b) and (c) lead to relatively copious number of secondary leptons produced at high transverse momenta $p_\perp$; $p_\perp > 1$ GeV/c. To the extent that high $p_\perp$ correspond to small $\Delta x_\perp$ by the uncertainty principle, these processes should tell us a great deal about the substructure of hadrons at distances $\lesssim 10^{-14}$ cm, an order of magnitude smaller than the size of the nucleon. They will become of increasing importance as the available energy increases. These lectures will be mainly confined to reactions (a) to (d), where there exist data and a considerable amount of theoretical work. The lectures will be organized as follows:

1. Phenomenology: The kinematical description and state of data will be briefly reviewed.

2. Current commutators and light cone behaviour: The cross-sections for these lepton-induced processes are proportional to the Fourier-transform of commutators of the weak and electromagnetic current operators evaluated between nucleon states. Therefore much can be learned about the nature of these commutators.

3. Dynamical models: Very roughly, two classes of models for these highly inelastic processes can be discerned. One class attributes the large observed cross-sections to the incoherent scattering of the lepton from pointlike constituents within the nucleon: this is the parton model. The other class of models views the lepton as possessing a dilute hadronic cloud, which is then absorbed by the target nucleon.

4. Quantum electrodynamics at infinite momentum: Some insight into the nature of the models for "semileptonic" processes can be found by studying the high energy limit of quantum electrodynamics, a program especially vigorously pursued by Cheng and Wu. Their results can be obtained by a formalism which sheds some light on the parton ideas.

2. *Phenomenology*

The formalism for reactions (a) to (d) has been discussed countless numbers of times [1–6], and we here review it in a hopefully brief way.

Let E — energy of incident lepton in laboratory, E' — energy of final lepton in laboratory, θ — scattering angle of lepton, p_μ — four-momentum of incident lepton, p'_μ — four-momentum of final lepton, P_μ — four-momentum of target nucleon. The four-vector $q_\mu = p_\mu - p'_\mu$ plays a special role, as it is the momentum absorbed by the hadron target, *i.e.* the momentum of the "virtual photon" (electrodynamic interaction) or "virtual W" (weak interaction) as in Fig. 1. The invariants formed from q and P play a central role in the description of the hadron dynamics. They are

$$\nu = M^{-1}(q \cdot P) = E - E',$$

$$Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}.$$

If we are interested in high-energy, high transverse-momentum processes for which $Q^2 \gg m_l^2$ it is justifiable to approximate the lepton mass m_l by zero. The lepton-hadron interaction, taken in lowest-order perturbation theory, is

$$H'_{\text{em}} = \frac{e^2}{Q^2}\, \bar{u}(p')\gamma_\mu u(p) J^\mu,$$

$$H'_{\text{weak}} = \frac{G}{\sqrt{2}}\, \bar{u}(p')\gamma_\mu(1-\gamma_5)u(p)\mathcal{J}^\mu \tag{2.1}$$

with J^μ and $\mathcal{J}^\mu$ the hadronic electromagnetic and weak current operators, respectively. The helicity of the leptons is conserved (for $m_1 \approx 0$) and the lepton-current can be computed explicitly [3, 4]. Choosing the z-axis along the direction of $\boldsymbol{q}$, the result is especially simple

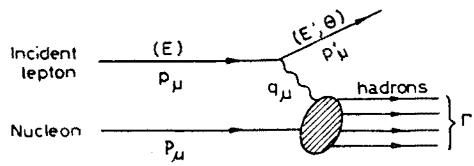

Fig. 1

[5, 6] when the energy transfer is much greater than 1 GeV. In this case the approximation $\nu^2 \gg Q^2$ is justifiable, and one finds, for an incident left-handed (negative helicity) lepton

$$\bar{u}\gamma_\mu u \approx \frac{\sqrt{Q^2}}{\nu}\left\{\sqrt{\frac{E}{2E'}}\,\varepsilon_\mu^L + \sqrt{\frac{E'}{2E}}\,\varepsilon_\mu^R + \varepsilon_\mu^S\right\} \equiv j_\mu^{\text{lept}} \tag{2.2}$$

where we normalize spinors such that $u^+(p)u(p) = 1$, and

$$\varepsilon_L = \frac{1}{\sqrt{2}}\,(0, 1, -i, 0)$$

$$\varepsilon_R = \frac{1}{\sqrt{2}}\,(0, 1, i, 0)$$

$$\varepsilon^S = (Q^2)^{-\frac{1}{2}}\,(\sqrt{\nu^2+Q^2}, 0, 0, \nu) \tag{2.3}$$

are normalized polarization vectors of definite helicity. The lepton electromagnetic current for an incident right-handed particle is obtained by letting $L \leftrightarrow R$ in (2.2). This is also the prescription for antineutrino-induced weak processes. The weak current is twice that given by (2.2) owing to the factor $1-\gamma_5$. The cross-section into a group of final hadron states $d\Gamma$ is then found to be:

$$\frac{d\sigma}{dQ^2 d\nu d\Gamma} = \frac{\pi}{EE'}\,\frac{d\sigma}{d\Omega dE' d\Gamma} = \begin{cases} \dfrac{4\pi\alpha^2}{Q^4}\,\dfrac{E'}{E} \displaystyle\sum_{n\,\text{in}\,d\Gamma} \langle n\,|\,j^{\text{lept}}\cdot J|P\rangle\,(2\pi)^3\delta^4(P_n-P-q) \\ \dfrac{G^2}{2\pi}\,\dfrac{E'}{E} \displaystyle\sum_{n\,\text{in}\,d\Gamma} \langle n\,|\,j^{\text{lept}}\cdot\mathcal{J}|P\rangle(2\pi)^3\delta^4(P_n-P-q). \end{cases} \tag{2.4}$$

In general there will be six terms in (2.4), three diagonal contributions and three interference--terms. To isolate the various terms, consider [7] the effect of a rigid rotation of the hadron system n about the direction of $\boldsymbol{q}$ by azimuthal angle φ. This is equivalent to rotation of the lepton system, for which the current j_μ^{lept} in (2.2) is modified by the replacements

$$\varepsilon_\mu^L \to \varepsilon_\mu^L e^{-i\varphi}, \qquad \varepsilon_\mu^R \to \varepsilon_\mu^R e^{i\varphi}, \qquad \varepsilon_\mu^S \to \varepsilon_\mu^S. \tag{2.5}$$

Upon squaring to obtain the cross-sections, we see that the interference terms between L and S are proportional to $\cos\varphi$ or $\sin\varphi$, while an $L-R$ interference is proportional to $\cos 2\varphi$ or $\sin 2\varphi$. Thus weighted averges with respect to φ will extract the various interference terms; we shall not deal with them further and will not write the expression down.

To eliminate interference-terms, it is only necessary to average over φ. When this is done one obtains

$$\int \frac{d\varphi}{2\pi}\frac{d\sigma}{dQ^2 d\nu d\Gamma}$$
$$= \begin{cases} \dfrac{\alpha}{\pi Q^2 \nu}\dfrac{E'}{E}\left(1-\dfrac{Q^2}{2M\nu}\right)\left[\dfrac{d\sigma_S}{d\Gamma}+\dfrac{E}{2E'}\dfrac{d\sigma_L}{d\Gamma}+\dfrac{E'}{2E}\dfrac{d\sigma_R}{d\Gamma}\right] \text{(electromagnetic)} \\ \dfrac{G^2}{2\pi^2}\dfrac{E'}{E}\dfrac{Q^2}{\nu}\left(1-\dfrac{Q^2}{2M\nu}\right)\left[\dfrac{d\sigma_S}{d\Gamma}+\dfrac{E}{2E'}\dfrac{d\sigma_L}{d\Gamma}+\dfrac{E'}{2E}\dfrac{d\sigma_R}{d\Gamma}\right] (\nu\text{—induced}) \end{cases} \tag{2.6}$$

where for electroproduction

$$\frac{d\sigma_i}{d\Gamma} = \frac{4\pi\alpha^2}{\nu-\dfrac{Q^2}{2M}}\sum_{n\,\text{in}\,d\Gamma}\langle n|\varepsilon_i\cdot J|P_S\rangle(2\pi)^3\delta^4(P_n-P-q) \tag{2.7}$$

and for neutrino-production

$$\frac{d\sigma_i}{d\Gamma} = \frac{\pi}{\nu-\dfrac{Q^2}{2M}}\sum_{n\,\text{in}\,d\Gamma}\langle n|\varepsilon_i\cdot\mathscr{J}|P_S\rangle(2\pi)^3\delta^4(P_n-P-q) \tag{2.8}$$

As $Q^2 \to 0$, $d\sigma_R/d\Gamma$ and $d\sigma_L/d\Gamma$ approach finite quantities; for electroproduction just the appropriate cross-section for incident circularly polarized photons. For electroproduction, $d\sigma_S/d\Gamma \to 0$ as $Q^2 \to 0$, while for neutrino-production, Adler's theorem [8] gives

$$\frac{d\sigma_S}{d\Gamma} \cong \frac{F_\pi^2}{Q^2}\left(\frac{m_\pi^2}{m_\pi^2+Q^2}\right)^2\frac{d\sigma_\pi}{d\Gamma} \tag{2.9}$$

with $F_\pi \cong 0.9\, m_\pi$ the pion decay constant and $d\sigma_\pi/d\Gamma$ the appropriate pion absorption cross-section. Adler's theorem follows from observing that for small Q^2, $\varepsilon_\mu^S \approx q_\mu/\sqrt{Q^2}$, and PCAC $q_\mu\mathscr{J}^\mu$ may be replaced by the pion field.

Of special importance, both experimental and theoretical, are the cross-sections, differential only in the outgoing lepton momentum. These are obtainable from (2.6) by summing over the $d\Gamma$. A different notation is conventional; this notation may be introduced by considering for fixed ν and Q^2 the limit of (2.6) as E (and E') tend to ∞:

$$\lim_{E\to\infty} \frac{d\delta}{dQ^2 d\nu} = \begin{cases} \dfrac{\alpha}{\pi Q^2 \nu}\left(1-\dfrac{Q^2}{2M\nu}\right)(\sigma_S+\sigma_T) \equiv \dfrac{4\pi\alpha^2}{Q^4}\, W_2(Q^2,\nu) \\ \dfrac{G^2}{2\pi^2}\dfrac{Q^2}{\nu}\left(1-\dfrac{Q^2}{2M\nu}\right)\left(\sigma_S+\dfrac{1}{2}\sigma_R+\dfrac{1}{2}\sigma_L\right) \equiv \dfrac{G^2}{2\pi}\beta(Q^2,\nu) \end{cases} \tag{2.10}$$

$\sigma_T = \sigma_R = \sigma_L$ is, for electroproduction, the cross-section for transverse virtual-photon absorption. $\beta(Q^2, \nu)$ (Adler's notation) is now often denoted by $\mathcal{W}_2(Q^2, \nu)$; the other conventional form-factors W_1, α, γ (or W_1, $\boldsymbol{W}_1$, $\boldsymbol{W}_3$) I find more conveniently expressed as ratios of the σ_i because the positivity restrictions $\sigma_i \geqslant 0$ are more readily comprehended, as well as troublesome minus-signs associated with $\boldsymbol{W}_3$.

From the definitions (2.10) and from (2.6) we may write

$$\frac{d\sigma}{dQ^2 d\nu} \cong \begin{cases} \dfrac{4\pi\alpha^2}{Q^4} W_2(Q^2,\nu)\left\{1-\dfrac{\nu}{E}+\dfrac{\nu^2}{2E^2}\dfrac{\sigma_T}{\sigma_T+\sigma_S}\right\} \\ \dfrac{G^2}{2\pi}\dfrac{E'}{E}\beta(Q^2,\nu)\left\{1+\dfrac{\nu}{E'}(L)-\dfrac{\nu}{E}(R)\right\} \end{cases} \tag{2.11}$$

where

$$(L) = \frac{\sigma_L}{\sigma_L+\sigma_R+2\sigma_S} \leqslant 1, \quad (R) = \frac{\sigma_R}{\sigma_L+\sigma_R+2\sigma_S} \leqslant 1. \tag{2.12}$$

The factors in curly brackets tend to be of order unity. Thus the gross features of the data are shaped by W_2 and β; the finer details by the cross-section ratios.

The relations between the σ_i and W_i are as follows:

$$\begin{aligned} W_1 &\cong \frac{\nu^2}{Q^2} W_2 \left(\frac{\sigma_T}{\sigma_T+\sigma_S}\right), \\ \boldsymbol{W}_1 &\cong \frac{\nu^2}{Q^2}\beta\left(\frac{\sigma_R+\sigma_L}{\sigma_R+\sigma_L+2\sigma_S}\right), \\ \boldsymbol{W}_3 &\cong \frac{2M\nu}{Q^2}\beta\left(\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R+2\sigma_S}\right) \qquad (\nu^2 \geqslant Q^2 \text{ only!}). \end{aligned} \tag{2.13}$$

Turning to the data, one finds it exhibits great simplicity. For electroproduction [9–13] most of the experiments only detect the scattered lepton; one then wishes to explore W_2 and $\sigma_T(\sigma_T+\sigma_S)$ as functions of ν and Q^2.

The range of ν and Q^2 which has been explored is shown in Fig. 2. The behaviour in "deep-inelastic" region $Q^2 > 1\ \mathrm{GeV}^2$, $\nu \gg 1$ GeV is extremely simple; it is shown [13] in Fig. 3.

The dimensionless quantity νW_2 thus appears to be a function only of $M\nu/Q^2 = q\cdot P/Q^2$, the only dimensionless quantity which can be formed from the kinematical invariants. The major consequence of this result is that the cross-sections at high Q^2 are big; comparable to the scattering from structureless objects. In the cross-hatched region, one can finds resonance-excitation: $N^*(1238)$, $N^*(1520)$, $N^*(1690)$, $N^*(1920)$ have been seen; at large Q^2 the production cross-section is comparable to elastic scattering at the same Q^2. In the other shaded region ($Q^2 < 1$ GeV^2), νW_2 decreases toward zero as $Q^2 \to 0$; $\nu W_2 \to Q^2\sigma_\gamma/4\pi^2\alpha$

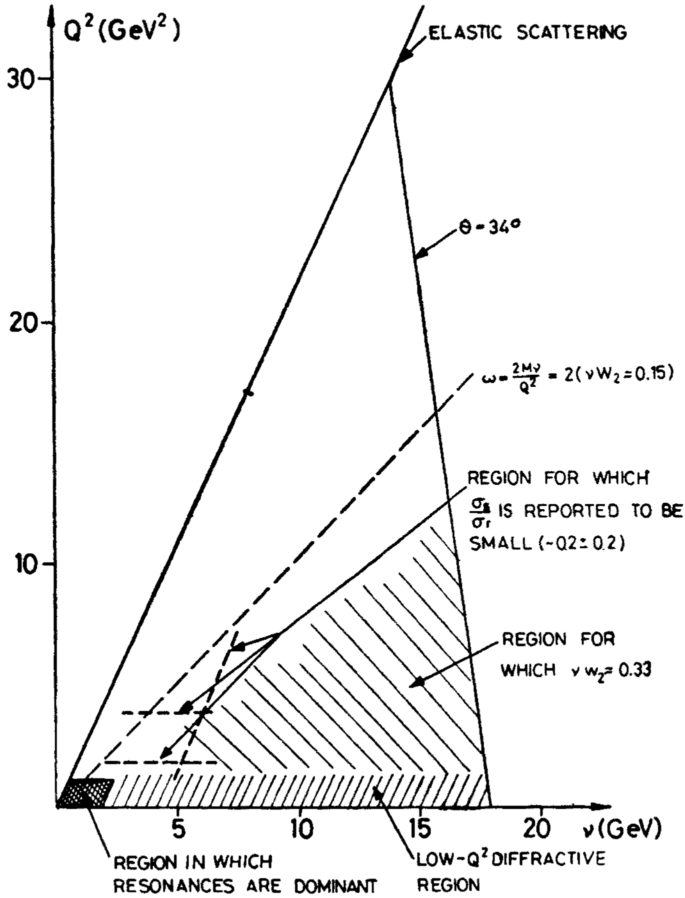

Fig. 2

as $Q^2 \to 0$. The ratio $R = \sigma_S/\sigma_T$ has been measured and reported [9], [12], [13] along the line $s = 8$ (Fig. 3) and for $Q^2 = 2$ and 4; $2 \leqslant \nu \leqslant 6$. In all cases studied R is small, < 0.5 and the SLAC-MIT group quotes $R = 0.2 \pm 0.2$ with the error mainly systematic.

Data on neutrino-induced reactions come from the CERN heavy-liquid bubble chamber [14]. The total cross-section appears to rise linearly up to $E_\nu \sim 10$ GeV; they find

$$\sigma_{tot} \cong \frac{G^2ME}{\pi}(0.6 \pm 0.15) = (0.8 \pm 0.2)E \times 10^{-38}\ \text{cm}^2. \tag{2.14}$$

Rough features of the muon energy distribution also support the scaling property. Especially significant is the observation of events for which $\frac{E'}{E}$ is small. From (2.11) and the data it follows that $\sigma_L \neq 0$; all other terms give vanishing cross-section as $E' \to 0$. The CERN group quotes $\langle\sigma_S/\sigma_T\rangle < 1$, assuming $\sigma_L \approx \sigma_R$.

Most of the features of the neutrino data can (in retrospect) be understood semiquantitatively from the electroproduction data and some "reasonable" assumptions, notably the

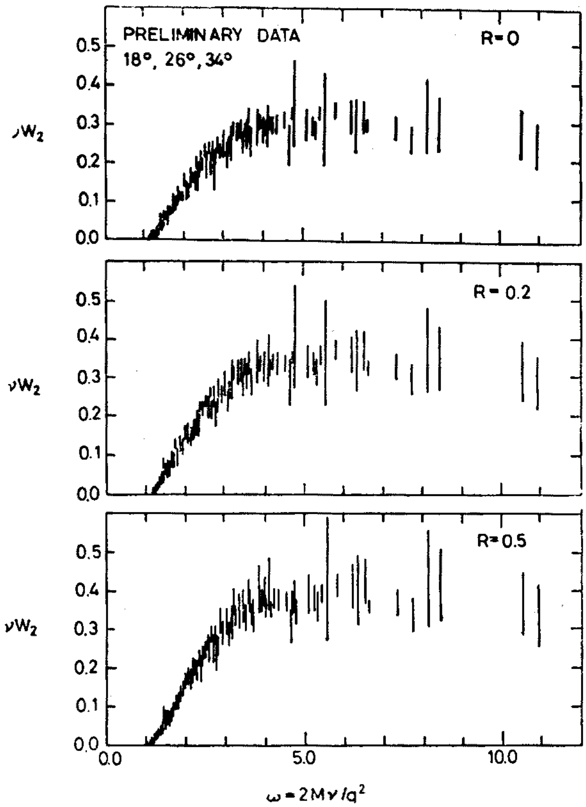

Fig. 3. W_2 for preliminary MIT-SLAC data at 18°, 26°, 34°, for various values of $R = \sigma_L/\sigma_T$

conserved vector current hypothesis. The vector $\Delta S = 0$ part of the heavy-liquid $\beta(\nu, Q^2)$ is related to the isovector part of W_2 as follows:

$$\beta_{\nu,\Delta S=0}(\nu, Q^2) \cong \frac{1}{2}(\beta_p+\beta_n)_{\nu,\Delta S=0} = (W_{2p}+W_{2n})_{\text{isovector}}. \tag{2.15}$$

Because νW_{2p} is scale-invariant, it is necessary that $\nu\beta$ is bounded below by a scale-invariant function, unless only isoscalar photons contribute to W_2 (very unlikely on grounds of SU(3)-symmetry). It is therefore natural to suppose $\nu\beta$ is scale invariant

$$\nu\beta = F(Q^2/2M\nu). \tag{2.16}$$

Given that assumption alone, one finds from (2.11)

$$\sigma_{\text{tot}} = \frac{G^2ME}{\pi}\int_0^1 dxF(x)\left\{\frac{1}{2}+\frac{1}{2}\langle L\rangle-\frac{1}{6}\langle R\rangle\right\}. \tag{2.17}$$

The factor in curly brackets must lie between 1/3 and 1, and thus σ_{tot}, on the average, increases linearly with neutrino energy. To estimate numerical magnitudes, assume:

1) All electroproduction goes *via* the isovector current.
2) $W_{2n} \approx W_{2p}$.
3) All ν absorption goes *via* $\Delta S = 0$ processes.
4) For the neutrino process, axial absorption = vector absorption; *i.e.* $\beta_A = \beta_V$.
5) $\sigma_T \gg \sigma_S$.
6) $\sigma_R = \sigma_L$.

These hypotheses imply $\beta = 4W_{2p}$, and from measurements one finds using (2.17)

$$\sigma_{tot} = \frac{2}{3}\frac{G^2ME}{\pi}\int_0^1 dxF(x) \cong 0.48\left(\frac{G^2ME}{\pi}\right). \tag{2.18}$$

This comparison is meant only to illustrate the magnitudes involved, in particular a realistic modification of hypothesis 1) will widen the discrepancy between (2.14) and (2.18). But the above argument does show that almost any theory of electroproduction will give about the right magnitude for the neutrino process.

However, it would appear that a somewhat smaller slope for the linear rise in the neutrino total cross-section than claimed experimentally would be more comfortable from the theoretical point of view. It also would make it easier to reconcile the observed flux of neutrino-induced muons deep underground [15, 16], which is claimed to be lower than that obtained by assumption of an indefinitely rising cross-section with the slope given by (2.14).

While phenomenologically it is best to deal with W_2 and β, in the theoretical discussions to follow it is most convenient to deal with W_1 and $\boldsymbol{W}_1$, which are connected to the transverse cross-sections, which have a minimum of kinematical complications. Hereafter we will confine ourselves to this, and recall from (2.13) that

$$W_1 \approx \frac{\nu^2}{Q^2}W_2 = \frac{1}{2M}\left(\frac{2M\nu}{Q^2}\right)\nu W_2. \tag{2.19}$$

Thus W_1 is scale-invariant and tends to 0.3 $\omega/2M$ as $\omega = 2M\nu/Q^2 \to \infty$.

3. *Current commutators*

The structure-function W_1 is directly related to a Furier transform of a product of currents. From (2.7), (2.10), (2.13) and some routine spin considerations

$$\begin{aligned}\boldsymbol{W}_1 &= \frac{P_0}{M}\int\frac{d^4xe^{iq\cdot x}}{2\pi}\langle P|\mathcal{J}_x(x)\mathcal{J}_x^{+}(0)|P\rangle\\ &= \frac{P_0}{M}\int\frac{d^4xe^{iq\cdot x}}{2\pi}\langle P|[\mathcal{J}_x(x),\mathcal{J}_x^{+}(0)]|P\rangle.\end{aligned} \tag{3.1}$$

We choose $\boldsymbol{q}$ and $\boldsymbol{P}$ in the z-direction. We shall also routinely change from W_1 to $\boldsymbol{W}_1$ when the difference is of no consequence. In the customary way, for q spacelike, the operator-

-product may be replaced by commutator. Just as the total photoabsorption cross-section for real photons is related to the forward Compton amplitude, so also is W_1 related to the virtual Compton amplitude $T_1^*(\nu, Q^2)$ by the optical theorem

$$\mathrm{Im}\, T_1^*(\nu, Q^2) = W_1(\nu, Q^2). \tag{3.2}$$

T_1^* is in principle an observable, being obtainable from the forward scattering amplitude for a lepton pair from a nucleon according to Fig. 4. T_1^* satisfies for fixed spacelike q^2, a dispersion relation in ν with one subtraction, because empirically $W_1 \underset{\sim}{\propto} \nu$ as $\nu \to \infty$ for fixed Q^2. Thus

$$T_1^*(\nu, Q^2) = T_1^*(0, Q^2) + \frac{\nu}{\pi} \int_{-\infty}^{\infty} d\nu' \, \frac{W_1(\nu', Q^2)}{\nu'(\nu'-\nu)} \,. \tag{3.3}$$

For electroproduction, the crossing-properties ensure the convergence of (3.3): $T_1(-\nu, Q^2) = T_1(\nu, Q^2)$. For the neutrino process, $W_1(-\nu, Q^2) = -\overline{W}_1(\nu, Q^2)$ where $\overline{W}_1$ is the structure-function for the corresponding antineutrino-induced process. One must assume the Pomeranchuk theorem holds in this case in order to write down (3.3).

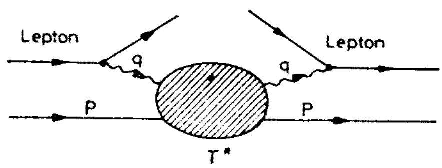

Fig. 4

The asymptotic behaviour of T_1^* as $q \to \infty$ is of particular interest. If it is sufficiently well-behaved, then T_1^* is a retarded commutator [17]

$$T_1^* = \frac{2iP_0}{M} \int \frac{d^4x}{2\pi} \, e^{iq\cdot x}\vartheta(x_0)\langle P|[\mathcal{J}_x(x), \mathcal{J}_x^+(0)]|P\rangle. \tag{3.4}$$

Suppose for a moment this is the case. Then let [18] $q_0 \to i\,\infty$ (so that $q^2 \to -\infty$, real spacelike). The exponential contains a factor $e^{-|q_0|t}$, so that provided the operator-product in the commutator is smooth enough, we may expand the commutator in terms of equal-time commutators:

$$[\mathcal{J}_x(t, \boldsymbol{x}), \mathcal{J}_x^+(0)] = [\mathcal{J}_x(0, \boldsymbol{x}), \mathcal{J}_x^+(0)] + t\,[\dot{\mathcal{J}}_x(0, \boldsymbol{x}), \mathcal{J}_x^+(0)] + \dots$$

and

$$T_1^* \to \frac{2iP_0}{M} \int \frac{d^3x}{2\pi} \, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \left\{ \frac{[\mathcal{J}_x(0, \boldsymbol{x}), \mathcal{J}_x^+(0)]}{|q_0|} + \frac{[\dot{\mathcal{J}}_x(0, \boldsymbol{x}), \mathcal{J}_x^+(0)]}{|q_0|^2} + \dots \right\}. \tag{3.5}$$

Conversely, given the data for W_1 we may empirically examine the degree of singularity of the operator-product and determine the existence of such equal-time commutators.

We go back to (3.3) and let $q_0 \to i\infty$. For $|\nu'| < Q^2/2M$, W_1 vanishes. Theorefore in the limit $|q_0|P_0| \cong |\nu| \ll |q_0|^2/2M \leqslant |\nu'|$ and the denominator may be developed:

$$T_1^*(\nu, Q^2) = T_1^*(0, Q^2) + \sum_{k=1}^{\infty} \nu^k I_k(Q^2) \tag{3.6}$$

with

$$I_k(Q^2) = \frac{1}{\pi} \int_{Q^2/2M}^{\infty} \frac{d\nu'}{(\nu')^{k+1}} [W_1(\nu', Q^2) + (-1)^k \overline{W}_1(\nu', Q^2)]. \tag{3.7}$$

From the data W_1 appears to be scale-invariant for $Q^2 \to \infty$:

$$W_1 = W_1(\lambda),\ \lambda = Q^2/2M\nu.$$

Given this, the asymptotic behaviour of the coefficient I_k can be deduced

$$I_k \to \frac{1}{\pi}\left(\frac{2M}{Q^2}\right)^k \int_0^1 d\lambda\, \lambda^{k-1}[W_1(\lambda) + (-1)^k \overline{W}_1(\lambda)]. \tag{3.8}$$

Through order q_0^{-2}, we need only keep I_1 and I_2:

$$T_1^* \xrightarrow[q_0 \to i\infty]{} T_1^*(0, Q^2) + \frac{2iP_0}{\pi|q_0|} \int_0^1 d\lambda [W_1(\lambda) - \overline{W}_1(\lambda)] -$$

$$- \frac{4P_0^2}{\pi|q_0|^2} \int_0^1 d\lambda\lambda [W_1(\lambda) + \overline{W}_1(\lambda)]. \tag{3.9}$$

The crossing-odd part of T_1^* and crossing-even part may be considered separately; thus the $[\mathcal{J}_x, \mathcal{J}_x^+]$ exists and is given as follows

$$\int d^3x e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \langle P|[\mathcal{J}_x(0, \boldsymbol{x}), \mathcal{J}_x^+(0)]|P\rangle$$

$$= 2M \int_0^1 d\lambda [W_1(\lambda) - \overline{W}_1(\lambda)]. \tag{3.10}$$

For the crossing-even part of the amplitude, we find, in a similar way [19, 20]

$$-i \int d^3x e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \langle P|[\dot{J}_x(0, \boldsymbol{x}), J_x(0)]|P\rangle$$

$$= \frac{2M}{P_0} \lim_{q^2 \to -\infty} \pi q^2 T_1^*(0, Q^2) + 4MP_0 \int_0^1 d\lambda\lambda [W_1(\lambda) + \overline{W}_1(\lambda)] \tag{3.11}$$

provided that the limit exists.

The Lorentz-transformation properties of the two terms in (3.11) differ. The coefficient of the last term is finite and (formally) survives as $P_0 \to \infty$. To be a little more careful let

$q_0 \to i\,\infty$, $P_z \to \infty$; q_0/P_0 fixed [21]. Then, provided only that $\boldsymbol{T}_1^*(0, Q^2) \to 0$ as $Q^2 \to \infty$

$$\lim_{P_z\to\infty} \frac{-i}{4MP_0} \int d^3x e^{-iq\cdot x} \langle P|[\dot{\mathscr{J}}_x(0, \boldsymbol{x}), \mathscr{J}_x^+(0)]|P\rangle = \int_0^1 d\lambda\lambda[\boldsymbol{W}_1 + \overline{\boldsymbol{W}}_1]. \tag{3.12}$$

This provision also suffices to show that T_1^* is a retarded commutator. Using the above method, a large number of sum-rules can be derived. In terms of the phenomenological notation, we catalogue them as well as a few others

$$\int_0^\infty d\nu[\bar{\beta}(\nu, Q^2) - \beta(\nu, Q^2)] = J_{00} \tag{3.13}$$

$$\lim_{Q^2\to\infty} \int_0^\infty d\nu[\bar{\beta}(\nu, Q^2)\,(\bar{R}+\bar{L}) - \beta(\nu, Q^2)\,(R+L)] = J_{xx} \tag{3.14}$$

$$\lim_{Q^2\to\infty} \int_0^\infty d\nu[\bar{\beta}(\nu, Q^2)\,(\bar{L}-\bar{R}) + \beta(\nu, Q^2)\,(L-R)] = iJ_{xy} \tag{3.15}$$

where

$$J_{\mu\nu} = \lim_{P_z\to\infty} \int d^3x\, \langle P_z|[\mathscr{J}_\mu^+(0, \boldsymbol{x}), \mathscr{J}_\nu(0)]|P_z\rangle. \tag{3.16}$$

Equation (3.13) in the classical Adler sum-rule [22] and depends upon a reliable current--commutator J_{00} but has not a reliable derivation. (3.14) is the "backward" asymptotic sum rule [23]. (3.15) was derived by Gross and Llewellyn-Smith [20].

The right hand sides of the last two sum rules are model-dependent. If one postulates the $U(6) \otimes U(6)$ current algebra of Feynman, Gell-Mann and Zweig [24], one finds the following table [25]

TABLE I

Proton target		Neutron target	
$\Delta S = 0$	$\Delta S = 1$	$\Delta S = 0$	$\Delta S = 1$
$2\cos^2\theta_c$	$4\sin^2\theta_c$	$-2\cos^2\theta_c$	$2\sin^2\theta_c$
$2\cos^2\theta_c$	$4\sin^2\theta_c$	$-2\cos^2\theta_c$	$2\sin^2\theta_c$
$6\cos^2\theta_c$	$4\sin^2\theta_c$	$6\cos^2\theta_c$	$2\sin^2\theta_c$

For electroproduction, a byproduct of the Adler sum-rule (3.13) is an inequality [26] obtainable by isospin rotation:

$$\int_{Q^2/2M}^{\tilde{\nu}(Q^2)} d\nu\ \ W_{2p}(\nu, Q^2) + W_{2n}(\nu, Q^2)] \geqslant \tfrac{1}{2} \tag{3.17}$$

$\tilde{\nu}(Q^2)$ is that value of ν for which the Adler sum rule has converged. From the data, $\int_0^{\tilde{\nu}} d\nu W_{2p} \gtrsim \frac{1}{4}$ provided $\tilde{\nu} > 3Q^2$, and $\int_0^{\tilde{\nu}} d\nu W_{2p} \geqslant \frac{1}{2}$ for $\tilde{\nu} > 7Q^2$. However, there is not much in the inequality to spare, and the derivation is rather inefficient.

For electroproduction, $J_{xx} = 0$ and there is no analogue to (3.14) except for an inequality similar to (3.17). But iJ_{xy}, given the $U(6)\otimes U(6)$ algebra, is nonzero and expressible in terms of the axial current. The isotopic-vector part of this axial current is related to the β-decay coupling g_A/g_V by conserved vector current hypothesis. One gets the sum rule [18], for target nucleons polarized along the direction of $\boldsymbol{q}$:

$$\lim_{Q^2\to\infty}\int_0^\infty d\nu W_2(\nu, Q^2)\left(\frac{\sigma_A-\sigma_P}{\sigma_A+\sigma_P+2\sigma_S}\right)=\begin{cases}\bar{z}+\dfrac{1}{6}\left|\dfrac{g_A}{g_V}\right| & \text{proton target}\\[2ex] \bar{z}-\dfrac{1}{6}\left|\dfrac{g_A}{g_V}\right| & \text{neutron target.}\end{cases}\tag{3.18}$$

Here A and P stand for antiparallel and parallel configurations of nucleon-photon spins. Eq. (3.18) implies [27] a mean polarization asymmetry for electroproduction, for either proton or neutron, greater than 20% for large ν and Q^2 (unless (3.18) converges extremely slowly). Sum rules for $[\dot{J}, J]$ as in (3.12) are of importance in electroproduction, because the sum is measured. However, the commutator is extremely model-dependent. Callan and Gross [19] and Cornwall and Norton [28] derived the sum rules

$$\lim_{Q^2\to\infty}\int_0^1 d\lambda\nu W_2\left(\frac{\sigma_T}{\sigma_T+\sigma_S}\right)=\lim_{P_z\to\infty}\frac{-i}{P_0}\int d^3x\langle P_z|[\dot{J}_x(0,\boldsymbol{x}), J_x(0)]|P_z\rangle,\tag{3.19}$$

$$\lim_{Q^2\to\infty}\int_0^1 d\lambda\nu W_2\left(\frac{\sigma_S}{\sigma_T+\sigma_S}\right)=\lim_{P_z\to\infty}\frac{i}{P_0}\int d^3x\langle P_z|[\dot{J}_z(0,\boldsymbol{x}), J_z(0)]|P_z\rangle.\tag{3.20}$$

To evaluate the right-hand side, Callan and Gross used Lagrangian models and canonical commutators, concluding that for the "gluon model" (quarks + neutral vector meson coupled to baryon number) $\sigma_S/\sigma_T \to 0$ as $Q^2 \to \infty$, while for field algebra or the Sugawara model $\sigma_T/\sigma_S \to 0$. However, Adler and Tung [29], Vainstein and Ioffe [30], and Jackiw and Preparata [31] argue that equal-time commutators computed from Feynman diagrams, *e. g. via* $q_0 \to i\,\infty$ method, do not agree with those expected by "naive" canonical manipulations of the field-operators. This occurs because of the singular nature of the products of local operators at the same specetime point. Thus considerable doubt is shed on the validity of the formal manipulation of canonical fields, and one is put again in the position of trying to postulate, in a sensible way, commutators such as appear in (3.19) and (3.20). The recent approaches along this line relate the commutator $[\dot{\mathcal{J}}_i^\alpha, \mathcal{J}_j^\beta]$ to a local operator $\theta_{ij}^{\alpha\beta}$ which is a piece of a second rank Lorenz tensor. α and β are internal $SU(3)\otimes SU(3)$ indices. Brandt and Preparata [32, 33] and also Cornwall [34] relate $\theta_{ij}^{\alpha\beta}$ to an assumed nonet of tensor fields, which act as interpolating fields for the 2^+ nonet of tensor mesons [$f, f', K^*(1405), A2$]. They are able, with some assumptions of exchange degeneracy with the vector nonet, to estimate the matrix-element between proton states. Differences between ep and en scattering, and between νp and νn total cross-sections are therefore anticipated in this approach. Another approach is taken by Wilson [35], Ciccariello *et al.* [36], and Mack [37] who assert, by a quite different and deep line of argument, that $\theta_{ij}^{\alpha\beta} = \delta^{\alpha\beta}\theta_{ij}$, where θ_{ij} is a piece of the conserved energy-momentum tensor. The Callan-Gross integral becomes universal, because $\langle P|\theta_{ij}|P\rangle \sim \frac{P_iP_j}{P_0}$ can be evaluated. What is predicted is therefore

$$\sigma^{\nu p}_{\text{tot}} = \sigma^{\nu n}_{\text{tot}} = \sigma^{\bar{\nu} p}_{\text{tot}} = \sigma^{\bar{\nu} n}_{\text{tot}}, \tag{3.21}$$

$$\int_0^1 d\lambda \nu W_{2p} = \int_0^1 d\lambda \nu W_{2n}. \tag{3.22}$$

The relation between the Callan-Cross integral (3.22) and the neutrino total cross-section cannot be completed without an additional assumption about σ_S/σ_T. Assuming σ_S/σ_T to be zero or small, one satisfies all assumptions 1) to 6) in Section 2 except the $|\Delta S| = 1$ correction, of order $\sin^2 \theta_c$ and the first, which should be

$$\int_0^1 d\lambda\nu(W_{2p}+W_{2n})_{\text{isovector}} = \tfrac{3}{4}\int_0^1 d\lambda\nu(W_{2p}+W_{2n}). \tag{3.23}$$

With this, the prediction for $\sigma^{\nu p}_{\text{tot}}$ is

$$\sigma^{\nu p}_{\text{tot}} \cong \frac{G^2ME}{\pi}(0.36) \tag{3.24}$$

somewhat smaller than the observed value but at present not seriously smaller.

This latter line of argument stems from the work of Wilson [35], who proposed that given the set $O_i(x)$ of all meaningful local operators in a theory, then

1. $As\, x \to 0$ $$O_i(x)O_j(0) \underset{x\to 0}{\to} \sum_k f_{ijk}(x)O_k(0). \tag{3.25}$$

That is, the operator product can be expanded in a series of local operators for sufficiently small x.

2. The operators $O_i(x)$ have well-defined dimension; under a scale-transformation $x \to sx$, $O_i(x) \to s^{d_i}O_i(sx)$ with d_i the dimension of the operator. The energy-momentum tensor has dimension $+4$, and the currents $\mathscr{I}_\mu(x)$ have dimension 3 in each case because of the nonlinear commutation relations of current algebra; *e. g.*

$$[\mathscr{I}^\alpha_0(sx),\mathscr{I}^\beta_0(sx')] = f^{\alpha\beta\gamma}\mathscr{I}^\gamma_0(sx)\,\delta^3(sx-sx'). \tag{3.26}$$

This means

$$s^{-2d}[\mathscr{I}^\alpha_0(x),\mathscr{I}^\beta_0(x')] = s^{-d-3}f^{\alpha\beta\gamma}\mathscr{I}^\gamma_0(x)\,\delta^3(x-x') \tag{3.27}$$

which implies $d = 3$. However other local operators are not expected to have integral dimension; they might well be functions of the coupling constant. This is supported by an explicit investigation [38] of the (soluble) twodimensional Thirring model. It is also supported by the aforementioned perturbation calculations [29, 30, 31]. Matrix elements are typically of the form $(g^2 \log x)^n x^{-2p}$; the logarithms are the same ones that give trouble to the $q_0 \to i\,\infty$ limit. These logarithms upon summation may well build up to a fractional power of x which is dependent upon coupling constants.

3. Wilson [35] assumes scale-invariance at small distances; *i. e.* as $x \to 0$ the operator--product expansion is invariant under scale-transformation. This implies that

$$f_{ijk}(sx) = s^{d_k - d_i - d_j} f_{ijk}(x). \tag{3.28}$$

Together with the requirements of Lorentz-covariance, this places a severe restriction on the structure of the functions $f_{ijk}(x)$; they essentially are just powers of x.

4. Ciccariello *et al.* [36] and Mack [37] assume that the only local operators with $d \leqslant 4$ are the currents $\mathcal{J}_\mu$ and the energy momentum tensor $\theta_{\mu\nu}$. Then to see how the procedure works, take the product $\mathcal{J}_\mu(x)\mathcal{J}_\nu(0)$ of dimension 6. It can be expanded in terms of $\mathcal{J}_\mu(x)$, of $\theta_{\mu\nu}(x)$ and of tensors of higher dimension. Leaving out $SU(3)$ indices,

$$\mathcal{J}_\mu(x)\mathcal{J}_\nu(0) = a\,\frac{g_{\mu\nu}x\cdot\mathcal{J}}{x^4} + b\,\frac{x_\mu\mathcal{J}_\nu}{x^4} + c\,\frac{x_\nu\mathcal{J}_\mu}{x^4} + d\,\frac{x_\mu x_\nu x\cdot\mathcal{J}}{x^6} +$$

$$+ e g_{\mu\nu}\frac{x^\alpha x^\beta}{x_4}\theta_{\alpha\beta} + f\,\frac{\theta_{\mu\nu}}{x_4} + g\,\frac{x_\mu x_\nu x^\alpha x^\beta}{x^6}\theta_{\alpha\beta} + \ldots \tag{3.29}$$

with dimensional analysis determining the power of x in the denominator. The neglected terms will have a coefficient function less singular than x^{-2} as $x \to 0$ as a consequence of assumption 4. Upon Fourier transformation the retarded product

$$T_{\mu\nu} = \int d^4x e^{iq\cdot x}\vartheta(t)\,[\mathcal{J}_\mu(x),\mathcal{J}_\nu(0)]$$

will again have an operator-product expansion, valid for $q_\mu = \lambda\hat{q}_\mu$, $\lambda \to \infty$; $\hat{q}_\mu$ a fixed vector. Then

$$T_{\mu\nu} = a' g_{\mu\nu}\frac{q\cdot\mathcal{J}}{q^2} + b'\,\frac{q_\mu\mathcal{J}_\nu}{q^2} + c'\,\frac{q_\nu\mathcal{J}_\mu}{q^2} + d'\,\frac{q_\mu q_\nu q\cdot\mathcal{J}}{q^4} +$$

$$+ e' g_{\mu\nu}\frac{q^\alpha q^\beta}{q_4}\theta_{\alpha\beta} + etc. + \ldots \tag{3.30}$$

The neglected terms go to zero faster than q^{-2}. Then taking $\hat{q}_\mu = (i, 0, 0, 0)$ we see that the coefficients of λ^{-1} and λ^{-2} can only involve $\mathcal{J}_\mu$ and $\theta_{\mu\nu}$. It then follows that the Callan-Gross integrals (3.12), (3.19), and (3.20) for neutrino-processes and electroproduction (in dimensionless form) are universal, *i. e.* depend only on matrix-elements of θ_{ij}. There is, of course, a strong assumption (number 4) necessary to get this result. Whether or not these specific assumptions are correct or not, it is certainly the case that method of operator-product expansions can do everything the use of equal-time commutators and asymptotic sum rules can. They can do much more because operators of fractional dimension can be dealt with and related to the data.

Ioffe [39], and others as well [40, 41], have provided further insight into the nature of the commutator in x-space. Given the data, which is the Fourier-transform of the commutator of currents according to (3.1), one may attempt to invert to obtain the commutator. With some apologies to rigor, this can be done. In the high energy limit ($\nu \gg 1$ GeV)

$$e^{iq\cdot x} = e^{i\nu t - i\sqrt{\nu^2+Q^2}\,z} \approx e^{i\nu(t-z) - \frac{iQ^2 z}{2\nu}}. \tag{3.31}$$

Ioffe observes that the important distances are those for which

$$(t-z) \lesssim \frac{1}{\nu} \text{(very small)} \tag{3.32}$$

$$\text{and } z \lesssim \left(\frac{2M\nu}{Q^2}\right)\frac{1}{M} = \frac{\omega}{M} \text{ (quite large for large } \omega). \tag{3.33}$$

From causality

$$x^2 = (t-z)\,(t+z) - x_\perp^2 \geqslant 0 \tag{3.34}$$

implying

$$x_\perp^2 \lesssim (t-z)\,(t+z) \lesssim \frac{1}{\nu}\frac{4\nu}{Q^2} = \frac{4}{Q^2}\,. \tag{3.35}$$

Thus as $Q^2 \to \infty$, only the commutator of currents on the light-cone contributes, and furthermore large separations $z \sim t \sim \dfrac{\omega}{M}$ are important. Then we can try to determine the light--cone singularity by inverting (3.1)

$$W_1(\nu, Q^2) \approx \frac{1}{M} F_1(\lambda) = \frac{1}{2M\lambda} F_2(\lambda)\langle\tau\rangle = \int_{\infty-}^{\infty} \frac{dtdz}{2\pi}\, e^{i\nu(t-z)-i\lambda Mz} \times C(t, z) \tag{3.36}$$

where

$$C(t, z) = \pi \int_0^\infty dx_\perp^2 \langle P|[J_x(x), J_x(0)]|P\rangle \tag{3.37}$$

and

$$\lambda = \omega^{-1} = Q^2/2M\nu.$$

Inverting we find

$$C(t, z) = \delta(t-z) \int_{-1}^{1} d\lambda e^{i\lambda Mz} F_1(\lambda). \tag{3.38}$$

With such a singularity, the commutator must have a singularity on the light cone proportional to $\delta'(x^2)$; we find that

$$\langle P|[J_x(x), J_x(0)]|P\rangle = \frac{4i}{\pi}\left\{\int_0^1 d\lambda[\sin \lambda Mt]F_1(\lambda)\right\}|t|\,\delta'(x^2). \tag{3.39}$$

Because according to (2.19), as $\lambda \to 0$, $F_1(\lambda) \approx (2\lambda)^{-1}F_2(0)$ we see that the singularity is just

$$\langle P|[J_x(x), J_x(0)]|P\rangle = \frac{2i}{\pi}\left\{\int_0^1 \frac{d\lambda}{\lambda}\,[\sin \lambda Mt]F_2(\lambda)\right\}|t|\,\delta'(x^2). \tag{3.40}$$

As $t \to \infty$, the integral in brackets is $\dfrac{\pi}{2} F_2(0)$, and asymptotically the light-cone singularity is $iF_2(0)|t|\,\delta'(x^2)$ (again consistent with dimensional analysis!).

4. *Dynamical models*

Two broad classes of models exist for interpreting the deep-inelastic data. The first, the parton model, considers the photon to be absorbed on the "real" current density associated with pointlike constituents within the nucleon. The calculations done with this model thus far are lamentably crude. The support for the idea comes not from details of the calculation but from two other sources: the first is the scale-invariance property of data, easily interpreted in general by the model, while the second is the existence of all the sum-rules such as written down in Section 3. These sum-rules can be interpreted in a simple-minded way [42] using the parton ideas and are also quite compatible with scale-invariance.

The second class of models may be called "diffractive" models, Pomeranchuk-exchange models, or vector-dominant models. The experimental case for this model lies in the shape of the $\nu W_2 = Q^2(\sigma_T + \sigma_S)/4\pi^2\alpha$ curve. As expected νW_2 is roughly constant with photon energy ν until ν is $\leqslant \frac{3Q^2}{2M}$, a value at which the longitudinal coherence-length $l \sim \frac{3}{M} \sim 0.6$ fermi. The theoretical case is nicely made by Ioffe's argument in Section 3 (above Eq. (3.36)). Ioffe furthermore argues that the hypothesis that small longitudinal distances (in the laboratory-frame) are important is inconsistent with experiment. This follows from (3.36). If the commutator tends to zero rapidly for $z > R_p$ (as might be anticipated if the picture is that the photon is absorbed on "real" charge in the nucleon), then the second exponential factor in (3.36) may be ignored for large ν/Q^2, with the result that W_1 is a function only of ν rather than of ν/Q^2. However, experimentally $W_1 \sim \nu/Q^2$ in that limit.

It is important that the diffraction-like models generally renounce the validity of the sum-rules written down in Section 3, or reduce them to triviality: $0 = 0$. All the sum rules discussed can be criticized, and no sacred principles (causality, locality, validity of the Gell-Mann algebra of charge-densities) need be abandoned if the sum rules fail. However, provided scale invariance holds, the Callan-Gross sum rules (3.12) and (3.20) will certainly have content; they imply that the $[\dot{J}, J]$ commutators are non-vanishing in the scale limit. The important question then becomes the $SU(3)$ structure (*e. g.* octet or singlet) of those commutators. The universal $[\dot{J}_i, J_j)$ commutator of Mack [37], Wilson [35], and Ciccariello *et al.* [36], would be compatible with the diffraction-like models.

Because detailed calculations or arguments regarding either class of model are far from convincing, and because for the present the primary importance of distinguishing between the two classes is to evaluate the role of the (reasonably respectable) sum rules, we shall not discuss any of the models in much detail, but summarize the results. We start with the parton-models:

1. Feynman [43] implements the parton idea as follows: When the proton at high momentum collides with the electron (as in the overall center-of-mass system) it may be considered during the interaction as a beam of temporarily non-interacting pointlike constituents (as in the lectures of Czyż) because the lifetime of fluctuations of the virtual states tends to ∞. The electron scatters incoherently from these pointlike constituents (partons). It follows, provided a calculation can be made successfully, that $\nu W_2 = F(\nu/Q^2)$ in this model because no scale of length other than given by the kinematical invariants has been introduced.

That is just a consequence of dimensional analysis. To make the calculation, notice that for a point particle, as $E_{\mathrm{CM}} \to \infty$

$$\frac{d\sigma}{dQ^2 d\nu} \to \frac{4\pi\alpha^2}{Q^4}\,\delta\left(\nu - \frac{Q^2}{2M}\right). \tag{4.1}$$

Suppose the parton of charge Q, has a fraction λ of the nucleon four-momentum P_μ (taken at high energy to be approximately null): Then

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\,\lambda Q^2\,\delta\left(\frac{\lambda q\cdot P}{M} - \frac{Q^2}{2M}\right). \tag{4.2}$$

The factor λ in front ensures that $d\sigma/dQ^2$ is given by the Rutherford formula. Then multiply this by $f_N(\lambda)$, the mean distribution of λ in a configuration N of partons, then sum over the partons in the configuration, multiply by the probability $\mathscr{P}_N$ of the configuration and sum over N. One gets [44]

$$\nu W_2 = \sum_N \lambda \mathscr{P}_N f_N(\lambda) \left\langle \sum_i Q_i^2 \right\rangle_N, \qquad \lambda = \frac{Q^2}{2M\nu}. \tag{4.3}$$

2. Paschos and I [44] try out the above formula assuming that the only partons are quarks. We find the calculated νW_2 somewhat larger ($\lesssim 50\%$) than the data. We furthermore find $\sigma_S/\sigma_T = 0$ (were the partons spin zero we would get $\sigma_T/\sigma_S = 0$). It is difficult to make νW_2 smaller without introducing some neutral partons (perhaps the neutral vector boson of the "gluon" model). Using the same calculation as leading to (4.3), the process $\gamma + N \to \gamma + \textit{hadrons}$ can be estimated. (To see what the proton is made of, a good way is to look!) Under kinematical conditions identical to those for electron-scattering (just replace e by γ) we find

$$\left(\frac{d\sigma}{d\Omega dE'}\right)_\gamma \bigg/ \left(\frac{d\sigma}{d\Omega dE'}\right)_e = \frac{\nu}{EE'}\,\frac{\langle \sum_i Q_i^4\rangle}{\langle \sum_i Q_i^2\rangle}. \tag{4.4}$$

The ratio of signal to background (from π^0 decay γ's) is marginal at present SLAC energies; however even at $E_\gamma = 25$ GeV the experiment appears feasible.

The implications of this model for neutrino-processes has also been studied [5], [20], [45]. The situation is similar to that for electroproduction. However, σ_L tends to be greater than σ_R, leading to $\sigma_{\nu N} > \sigma_{\bar\nu N}$ as well as $\sigma_{\nu n} > \sigma_{\nu p}$.

3. Drell, Levy, and Yan [46], [47], [48] start with a relativistic γ_5 theory of pions and nucleons, mutilated with a transverse-momentum cutoff. In this model, partons are "bare nucleons" and "bare pions". They are able to derive the scaling property of νW_2 and estimate νW_2 as $\nu/Q^2 \to \infty$. In this limit νW_2 behaves as a power of ν/Q^2. The exponent depends upon the pion-nucleon coupling constant and the transverse-momentum cutoff. With a resonable cutoff the power can be taken to be near zero. Among the interesting consequences of the model are the following:

a) The distribution of secondary nucleons is expected to be peaked along the direction of $\boldsymbol{q}$; in the laboratory frame

$$\frac{dN}{dE_p} \approx \frac{1}{\nu} f\left(\frac{\nu}{Q^2}, \frac{E_p}{\nu}\right) \tag{4.5}$$

where E_p is the energy of the recoil nucleon. This leads to a relatively large number of energetic nucleons with $\langle E_p \rangle = \nu \int_0^1 d\varepsilon \varepsilon f\left(\frac{\nu}{Q^2}, \varepsilon\right)$. Tihs result occurs because the momentum of the virtual photon is often absorbed by the "bare nucleon", and a finite fraction ends up in the emergent physical nucleon. The nucleon and secondary pions have low $p_\perp \lesssim 500$ MeV relative to the direction of $\boldsymbol{q}$.

b) The threshold behaviour of νW_2 near $\omega=1$ is related to the asymptotic behaviour of the elastic form factor. If $G_M(q^2) \xrightarrow[q^2\to\infty]{} q^{-4}$, then $\nu W_2 \xrightarrow[\omega\to 1]{} (\omega-1)^3$.

c) The crossed reaction $e^+ + e^- \to \bar{p} + hadrons$ is expected to exhibit scaling; in particular

$$\frac{d\sigma}{dE_p} = \frac{\alpha^2}{Q^2} \frac{1}{\sqrt{Q^2}} \Phi \left(\frac{E_p}{\sqrt{Q^2}}\right) \tag{4.6}$$

leading to a large total cross-section

$$\langle n_{\bar{p}} \rangle \sigma_{\text{tot}} \approx \sigma_{\text{tot}} = \frac{\alpha^2}{Q^2} \int_0^1 d\varepsilon \Phi(\varepsilon) \tag{4.7}$$

which is consistent with dimensional analysis. Again many $\bar{p}$ should be produced, and $\langle E_{\bar{p}} \rangle \sim \sqrt{Q^2}$. The accompanying pions should have low $p_T \lesssim 500$ MeV relative to the direction of the $p\bar{p}$ pair. These results for the annihilation channel are specific to this model and are nota general consequence of crossing + scaling for electroproduction.

4. Chang and Fishbane [49] take the Drell, Lewy, Yan model and remove the transverse-momentum cutoff. This removes the scaling property of νW_2 as well, and $\nu W_2 \approx f\left(\frac{\nu}{Q^2}\right) \times \times (Q^2)^p$, with p dependent on the pion coupling. The diagrams they sum are multiperipheral in character, and they find the pion secondaries lie in two jets, along the directions of initial and final nucleons, somewhat as would occur in a bremsstrahlung model, or in the Drell, Levy, Yan model. They also find the pion multiplicity $\langle n_S \rangle \sim \log Q^2$. We now turn to the diffractive or Pomeranchuk-exchange models:

1. The most detailed hypothesis is that of Sakurai [50], which proposes ϱ-dominance is dynamical mechanism, even at high Q^2. This means, for large ν

$$\sigma_T(Q^2, \nu) \cong \sigma_\gamma(m_p^2/(Q^2+m_p^2))^2 \tag{4.8}$$

whereas the data implies (at least for $\nu \lesssim 10$ GeV) $\sigma_T \sim \frac{1}{Q^2}$. The discrepancy at high Q^2 is a factor 2 to 3 and the model in this unadorned from fails. However, it has been suggested that it should only have validity in the pure diffractive region (Sakurai [51] proposes $\nu/Q^2 > 5\ \text{GeV}^{-1}$) where σ_S/σ_T is not yet measured. In this region the prediction is

$$\sigma_S/\sigma_T \approx Q^2/m_\varrho^2. \tag{4.9}$$

It is $\sigma_S \sim \frac{1}{Q^2}$ which provides the scaling property.

The role of ϱ dominance in electroproduction can be further tested by studying ϱ-electroproduction (the analogous process for neutrino reactions is $\varrho^\pm$ and $A1^\pm$ production for ν/Q^2 large; all the coherent phenomena encounted with photons should be encountered in that region also).

Fraas and Schildknecht [52], and also Dieterle [53], have given a thorough kinematical analysis of this process. One of the interesting questions, for example is the ratio

$$\sigma^i(Q^2, \nu)_{ep\to ep\varrho}/\sigma^i(Q^2, \nu)_{ep\to e\ hadrons}$$

ϱ-dominance predicts it to be similar to that for real photons ($\sim$ 15%) independent of Q^2 and ν.

2. A generalization of this idea has been given by Gribov [54] and by Brodsky and Pumplin [55] in their discussion of electroproduction on nuclei. They generalize ϱ-dominance to dominance of all vector states coupled to the γ. Gribov concludes that σ_S/σ_T on nuclei should increase with Q^2 (for large Q^2) independently of the value of σ_S/σ_T on the nucleon, provided ν/Q^2 is so large that the coherence length exceeds the hadron mean-free path in nuclear matter. This will be an interesting point for future very high-energy muon experiments.

3. Abarbanel, Goldberger and Treiman [56] argued that Pomeranchuk-exchange should dominate the virtual Compton amplitude when Q^2 gets large because $\cos\theta_t \gg 1$ throughout the deep inelastic region. Harari [57] gives a nice argument based on finite-energy sum rules. He argues that the coupling of virtual photon to non-vacuum trajectories (such as A2) should have the same Q^2-dependence as the form-factors of the resonances, because the finite-energy sum rules connect them. Because the resonance-form-factors vanish rapidly with Q^2, so also will all trajectories except the Pomeranchuk trajectory, which is not connected to the resonances.

The experimental distinction between parton and diffraction models is very simple; in diffraction models everything is approximately the same:

$$W_{2p}(\nu, Q^2) \approx W_{2n}(\nu, Q^2) \qquad \sigma_L \approx \sigma_R$$
$$(\sigma_S/\sigma_T)_p \equiv R_p(Q^2, \nu) = R_n(Q^2, \nu). \tag{4.10}$$

For neutrinos

$$W_i(\nu, Q^2) = \overline{W}_i(\nu, Q^2) \qquad (i = 1, 2)$$
$$\sigma_L(Q^2, \nu) = \sigma_R(Q^2, \nu);\ \sigma_{tot}^{\nu p} \approx \sigma_{tot}^{\nu n} \approx \sigma_{tot}^{\bar\nu p} \approx \sigma_{tot}^{\bar\nu n}. \tag{4.11}$$

In parton-models one expects everything different, sufficiently different that sum-rules such as (3.13) can be satisfied. The differences are typically 20% or larger. The next round of experiments with e, μ, and ν beams should be able to distinguish between the two classes of models.

5. *Quantum electrodynamics of infinite momentum*

High-energy quantum electrodynamic scattering processes have been recently shown [58], [59] to possess many simple features, as discussed by Professor Czyż. We shall here reproduce them by an infinite-momentum formalism developed by John Kogut and Davison Soper at Stanford [60], [61]. Much of this — and probably much more — we believe also exists in Feynman's notebooks.

The basic idea is to construct a limiting formalism for high energy systems ($P_z \to \infty$) which is analogous to the formalism for the nonrelativistic electron coupled to the radiation field. A key element in doing this is to recognize that just as there is a preferred axis in space-time, the t-axis, along which non-relativistic systems propagate, so also in there a preferred direction in space-time, that line $t = z$, $x = y = 0$, along which these extreme-relativistic systems propagate. So the first step lies in a change of variables [62], [63] from $\hat{x}^\mu = (t, x, y, z)$ to

$$x^\mu = (\tau, x, y, \zeta) \tag{5.1}$$

with

$$\tau = \frac{t+z}{\sqrt{2}}, \qquad \zeta = \frac{t-z}{\sqrt{2}}. \tag{5.2}$$

We introduce the contravariant four-vector

$$x_\mu = (\zeta, -x, -y, \tau) = g_{\mu\nu}x^\nu \tag{5.3}$$

so that

$$x_\mu x^\mu = \hat{x}_\mu \hat{x}^\mu = t^2 - z^2 - x^2 - y^2, \; g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \tag{5.4}$$

Noteworthy is the effect of Lorentz transformations in the z-direction; they are merely scale-transformations:

$$\tau \to e^{-\omega}\tau, \; \zeta \to e^{\omega}\zeta. \tag{5.5}$$

Given this change of variables, Kogut and Soper develop a Hamiltonian formalism in the new variable τ. Because of the simple scaling property (5.5), this leads to a formalism which is especially convenient in the high-energy limit. The procedure of Kogut and Soper is twofold. First they take the ordinary Feynman diagrams for quantum electrodynamics, change variables as above, τ-order the amplitudes and obtain rules for "old-fashioned" perturbation-diagrams *a la* Heitler. Then they take the equations of motion in the new variables and construct a canonical field theory which formally reproduces the old-fashioned rules. Finally an external field is introduced, and the high-energy limit of the S-matrix is considered. It turns out to be very simple:

$$S = \exp\,[i \int d^2x_\perp \varrho(\boldsymbol{x}_\perp)\chi(\boldsymbol{x}_\perp) \tag{5.6}$$

where $\varrho(\boldsymbol{x}_\perp)$ is a transverse-charge-density operator and $\chi(\boldsymbol{x}_\perp)$ is the eikonal phase: the integral of the potential over τ.

It is hard to believe the formalism which emerges from all this is really equivalent to ordinary quantum electrodynamics. It looks very different. It may in fact be different. Several logarithmic pitfalls have been noticed and thus far ignored. We shall here only sketch the formalism, leaving out many of the details and proofs of equivalence with conventional quantum electrodynamics.

We start with a very brief description of the free-field theory.

Let

$$H = \frac{E-p_z}{\sqrt{2}}, \qquad \eta = \frac{E+p_z}{\sqrt{2}} \tag{5.7}$$

and

$$P_\mu = (H, p_x, p_y, \eta) = i\,\frac{\partial}{\partial x^\mu}. \tag{5.8}$$

Then for a free particle

$$H = \frac{p_\perp^2+m^2}{2\eta} \tag{5.9}$$

and already the theory attains a non-relativistic flavor. Dirac theory turns out to be equivalent to the wave equation

$$i\,\frac{\partial\psi(x)}{d\tau} = \left(\frac{p_\perp^2+m^2}{2\eta}\right)\psi(x) \tag{5.10}$$

where

$$\frac{1}{\eta}\;\psi(\tau, \boldsymbol{x}_\perp, \zeta) = -\frac{i}{2}\int_{-\infty}^{\infty} d\zeta'\varepsilon(\zeta-\zeta')\psi(\tau, \boldsymbol{x}_\perp, \zeta') \tag{5.11}$$

with

$$\varepsilon(\zeta) = \begin{cases} +1 & \zeta > 0 \\ -1 & \zeta < 0 \end{cases}. \tag{5.12}$$

The field $\psi(x)$ is two-component: only two of the 4 original components are independent variables. A "spin up" plane-wave solution corresponds to that spin state which, if boosted to $P_z = +\infty$, has helicity $+1/2$. Solutions with negative η (and therefore negative H) are identified with antiparticles as in the usual theory. The particle interpretation of the field theory follows from postulated anticommutators:

$$\delta(\tau-\tau')\{\psi_\alpha(x), \psi_\beta^+(x')\} = \delta_{\alpha\beta}\delta^4(x-x'). \tag{5.13}$$

It is interesting to see how the 4-component theory reduces to 2 components. After rotation of coordinates we still have the equation

$$i\gamma^\mu\,\frac{\partial\psi}{\partial x^\mu} = m\psi \tag{5.14}$$

but the γ-matrices are new and have crazy properties:

$$\gamma^0\gamma^0 = g^{00} = 0 = \gamma^3\gamma^3 \tag{5.15}$$

$$\gamma^0\gamma^3+\gamma^3\gamma^0 = 2g^{30} = 2 \tag{5.16}$$

$$\gamma_\mu^+ = \gamma^\mu. \tag{5.17}$$

It is not hard to see that

$$P^+ = \frac{1}{2}\gamma^3\gamma^0, \qquad P^- = \frac{1}{2}\gamma^0\gamma^3 \tag{5.18}$$

are projection operators, and that

$$\psi^\pm = P^\pm\psi^\pm \tag{5.19}$$

have two independent degrees of freedom apiece. But ψ^- does not satisfy an equation of motion because $\gamma^0\psi^- = 0$:

$$0 = -i\gamma^0\frac{\partial\psi^-}{\partial\tau} = i\gamma^3\frac{\partial\psi^-}{\partial\zeta} + i\gamma^i\frac{\partial\psi}{\partial x^i} - m\psi. \tag{5.20}$$

Thus at any τ, ψ^- can be expressed in terms of ψ^+, which in turn is easily shown to satisfy (5.10), with an appropriate representation for the γ^μ.

The free electromagnetic field satisfies

$$i\frac{\partial A_i(x)}{d\tau} = \frac{p_\perp^2}{2\eta}A_i(x). \tag{5.21}$$

The Hamiltonian turns out to be

$$H = -\frac{1}{2}\sum_{i=1}^{2}\int d^3x A_i(x)\nabla^2 A_i(x) \tag{5.22}$$

while the commutation-relations are exotic:

$$\delta(\tau-\tau')[A_i(x), A_j(x')] = \frac{1}{2\eta}\delta^4(x-x')$$

$$= -\frac{i}{4}\delta(\tau-\tau')\delta^2(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)\varepsilon(\zeta-\zeta'). \tag{5.23}$$

The commutator leads to the correct equation of motion

$$[H, A_i(x)] = \frac{p_\perp^2}{2\eta}A_i(x). \tag{5.24}$$

Normalized plane wave solutions are, for positive helicity,

$$\psi(x) = \begin{pmatrix}1\\0\end{pmatrix}e^{-ip\cdot x} = \begin{pmatrix}1\\0\end{pmatrix}e^{-iH\tau-i\eta\zeta+i\boldsymbol{p}_\perp\cdot\boldsymbol{x}_\perp}$$

$$\boldsymbol{A}_\perp(x) = \frac{1}{\sqrt{2\eta}}\frac{1}{\sqrt{2}}(1, i)e^{-ip\cdot x}. \tag{5.25}$$

The reader may understandably be unconvinced tby all this, but these results were actually constructed from Lagrangian formalism, in the new coordinates, by standard techniques.

Continuing on to the interacting case we return to the Dirac-equation, writing it as (*cf.* Eq. (5.20))

$$[H, \psi(x)] = (m - i\boldsymbol{\sigma} \cdot \boldsymbol{p}_\perp) \frac{1}{2\eta} (m + i\boldsymbol{\sigma} \cdot \boldsymbol{p}_\perp)\psi(x) \tag{5.26}$$

and then introducing the gauge-invariant substitution

$$P_\mu \to P_\mu - eA_\mu(x),$$

$$[H, \psi(x)] = eA_0(x)\psi(x) + [m - i\boldsymbol{\sigma}(\boldsymbol{p}_\perp - e\boldsymbol{A}_\perp)] \frac{1}{2\eta} [m + i\boldsymbol{\sigma}(\boldsymbol{p}_\perp - e\boldsymbol{A}_\perp)]\psi(x). \tag{5.27}$$

In writting (5.27) we have chosen the gauge

$$A^0 = A_3 = 0 \tag{5.28}$$

which clearly leads to an advantage in keeping A_3 out of the denominator $(2\eta)^{-1}$. A_0 also turns out, as in conventional electrodynamics, to be a dependent variable. We eliminate it using Maxwell's equations

$$\frac{\partial}{\partial x^\mu} \left(\frac{\partial A^\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right) = j_\nu . \tag{5.29}$$

Choosing $\nu = 3$, we get (recalling $A^0 = A_3 = 0$)

$$\frac{\partial}{\partial \zeta} \frac{\partial A^3}{\partial \zeta} + \sum_{i=1}^{2} \frac{\partial}{\partial x^i} \frac{\partial A^i}{\partial \zeta} = j_3 = j^0 = e\psi^+\psi \tag{5.30}$$

Therefore

$$A^3 = A_0 = -\frac{1}{\eta^2} (e\psi^+\psi) + \frac{i}{\eta} \frac{\partial A^i}{\partial x^i}. \tag{5.31}$$

This yields the full Hamiltonian density $\mathscr{H}$

$$\mathscr{H} = \frac{e^2}{2} (\psi^+\psi) \frac{1}{\eta^2} (\psi^+\psi) + e(\psi^+\psi) \frac{1}{\eta} (\boldsymbol{p}_\perp \cdot \boldsymbol{A}_\perp) +$$

$$+ \psi^+[m - i\boldsymbol{\sigma} \cdot \boldsymbol{p}_\perp + ie\boldsymbol{\sigma} \cdot \boldsymbol{A}_\perp] \frac{1}{2\eta} [m + i\boldsymbol{\sigma} \cdot \boldsymbol{p}_\perp - ie\boldsymbol{\sigma} \cdot \boldsymbol{A}_\perp]\psi + \frac{1}{2} \sum_{i=1}^{2} A_i p_\perp^2 A_i \tag{5.32}$$

where

$$\frac{1}{\eta^2} f(\zeta) = \frac{1}{2} \int_{-\infty}^{\infty} d\zeta' |\zeta - \zeta'| f(\zeta'). \tag{5.33}$$

From this Hamiltonian, the veteran field-theorist will be able to construct rules for old--fashioned perturbation diagrams by whatever method pleases him. The vertices are especially simple if the helicities of the particles are specified. Denoting positive helicity by R (right-handed) and negative helicity by L, we get the table below, corresponding to the diagrams in Fig. 5

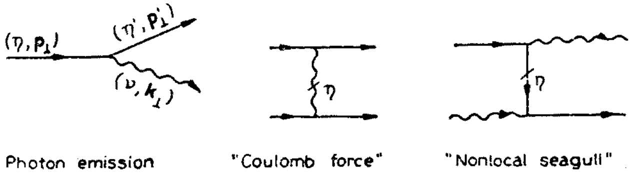

Fig. 5

photon-emission
$(e \to e+\gamma)$

$$R \to R+R:\ H' = \frac{e}{\sqrt{2\nu}}\left(\frac{\boldsymbol{k}_\perp \cdot \boldsymbol{\varepsilon}^*}{\nu} - \frac{\boldsymbol{p}_\perp \cdot \boldsymbol{\varepsilon}^*}{\eta}\right) \tag{5.34}$$

$$R \to R+L:\ H' = \frac{e}{\sqrt{2\nu}}\left(\frac{\boldsymbol{k}_\perp \cdot \boldsymbol{\varepsilon}^*}{\nu} - \frac{\boldsymbol{p}'_\perp \cdot \boldsymbol{\varepsilon}^*}{\eta'}\right) \tag{5.35}$$

$$R \to L+L:\ H' = \frac{e}{\sqrt{2\nu}}\frac{im}{\sqrt{2}}\left(\frac{1}{\eta} - \frac{1}{\eta'}\right) \tag{5.36}$$

$$R \to L+L:\ H' = 0 \tag{5.37}$$

"Coulomb force"

$$H' = \frac{e^2}{\eta^2} \tag{5.38}$$

$(e+e \to e+e)$ (no helicity flip at either vertex)
"Nonlocal seagull"

$$H' = \frac{1}{2\sqrt{\nu\nu'}}\frac{e^2}{2\eta} \tag{5.39}$$

(no helicity flip for either electron or photon).
With these rules go quite conventional rules for δ-functions and phase-space factors:

a) $\dfrac{d^2p_\perp d\eta}{(2\pi)^3}\dfrac{d^2k_\perp d\nu}{(2\pi)^3}$ for final-particle phase space

b) $\dfrac{1}{H_f - H_n + i\varepsilon}$ for each intermediate state

c) $(2\pi)^3\delta^3(\Sigma p_i)$ for momentum conservation at vertices

d) Minus signs from exchange, *etc.*, as in conventional theory.

With these rules, Feynman diagrams can be reconstructed. Subtle things happen in closed loops, *e.g.* self-energies or wave-function renormalization. The renormalization program in this formalism — as in the non-relativistic case — is a mess and this kind of fundamental problem appears no better, and perhaps worse, in this way of looking at electrodynamics.

We now turn to the scattering problem. Returning to (5.27) we include also an external field $\mathscr{A}_\mu(x)$ and find the Hamiltonian

$$\mathscr{H}+\mathscr{V} = e\,\mathscr{A}_0(\psi^+\psi) + \frac{e}{2}(\psi^+\psi)\frac{1}{\eta^2}(\psi^+\psi)+e(\psi^+\psi)\frac{1}{\eta}[\boldsymbol{p}_\perp\cdot(\boldsymbol{A}_\perp+\mathscr{A}_\perp)] +$$
$$+\psi^+[m-i\boldsymbol{\sigma}(\boldsymbol{p}_\perp-e\,\boldsymbol{A}_\perp-e\,\mathscr{A}_\perp]\frac{1}{2(\eta-e\,\mathscr{A}_3)}[m+i\boldsymbol{\sigma}\cdot(\boldsymbol{p}_\perp-e\boldsymbol{A}_\perp-e\,\mathscr{A}_\perp)] +$$
$$+\frac{1}{2}\sum_{i=1}^{2}A_i\boldsymbol{p}_\perp^2A_i. \tag{5.40}$$

To continue, we examine the scattering-operator

$$S=\mathscr{T}e^{i\int d^4x\mathscr{V}(x)} \tag{5.41}$$

where $\mathscr{T}$ refers to τ-ordering and the operators in $\mathscr{V}$ are interaction-picture ($\mathscr{A}_\mu=0$) operators. We now consider S between given states $\langle f_0|$ and $|i_0\rangle$ and study the effect of boosting those states to infinite momentum in the z-direction; this is accomplished by the Loretz-boost operator $e^{i\omega K_3}$. Kogut and Soper construct K_3 from the Noether theorem; the result is simple:

$$K_3=\int d^2xd\zeta\left\{\psi^+\frac{i}{2}\frac{\overleftrightarrow{\partial}}{\partial\zeta}\psi+\sum_{i=1}^{2}\left(\frac{\partial A^i}{\partial\zeta}\right)^2\right\}_{\tau=0}. \tag{5.42}$$

The (interaction-picture) fields and Hamiltonian transform simply under boosts:

$$e^{i\omega K_3}He^{-i\omega K_3}=e^{-\omega}H$$
$$e^{i\omega K_3}\boldsymbol{A}_\perp(\tau,\boldsymbol{x}_\perp,\zeta)e^{-i\omega K_3}=\boldsymbol{A}_\perp(e^{-\omega}\tau,\boldsymbol{x}_\perp,e^{\omega}\zeta)$$
$$e^{i\omega K_3}\psi(\tau,\boldsymbol{x}_\perp,\zeta)e^{-i\omega K_3}=e^{\omega/2}\psi(e^{-\omega}\tau,\boldsymbol{x}_\perp,e^{\omega}\zeta). \tag{5.43}$$

Thus letting

$$|f\rangle=e^{-i\omega K_3}|f_0\rangle$$
$$|i\rangle=e^{-i\omega K_3}|i_0\rangle \tag{5.44}$$

we study the limit of S as $\omega\to\infty$. It is

$$S=\mathscr{T}\exp i\int d^4xe^{i\omega K_3}\mathscr{V}(x)e^{-i\omega K_3}. \tag{5.45}$$

Changing variables to $\zeta'=e^{\omega}\zeta$, implying $\eta=e^{\omega}\eta'$, we find that the term that dominates all others in (5.40), by a power of e^{ω}, is the $\mathscr{A}_0(\psi^+\psi)$ term; therefore

$$\langle f|S|i\rangle=\langle f_0|\mathscr{T}\exp ie^{-\omega}\int d\tau d\zeta'd^2x_\perp\{e\mathscr{A}_0(\tau,\boldsymbol{x}_\perp,e^{-\omega}\zeta')\times$$
$$\times e^{\omega}\varrho(e^{-\omega}\tau,\boldsymbol{x}_\perp,\zeta')+\mathscr{O}(1)\}|i_0\rangle. \tag{5.46}$$

In the limit $\omega \to \infty$, all field operators ϱ are evaluated at $\tau = 0$, so that the τ-ordering is irrelevant. This can be checked by expanding out the exponential in a power series and examining each term. Thus

$$\langle f|S|i\rangle \underset{\omega\to\infty}{\to} \langle f_0| \exp i \int d^2x_\perp \varrho_\perp(\boldsymbol{x}_\perp)\chi(\boldsymbol{x}_\perp)|i_0\rangle \tag{5.47}$$

where

$$\varrho_\perp(\boldsymbol{x}_\perp) = \int_{-\infty}^{\infty} d\zeta\varrho(0, \boldsymbol{x}_\perp, \zeta) \ (\varrho=\psi^+\psi) \tag{5.48}$$

and

$$\chi(\boldsymbol{x}_\perp) = e \int_{-\infty}^{\infty} d\tau\mathscr{A}_0(\tau, \boldsymbol{x}_\perp, 0) \tag{5.49}$$

is the eikonal phase.

The form (5.47) has a direct interpretation in terms of partons. Suppose $|i_0\rangle$ is expanded in a series of terms composed of a normal product of operators ψ, ψ^+, $\boldsymbol{A}^i$, evaluated at $\tau = 0$, acting on the vacuum state. Those operators are the parton creation and destruction operators: the partons are the bare quanta; the eigenstates of H_0. The coefficient functions for the various terms are the wave functions for that particular configuration of partons. The action of S on a configuration is simply to

1) Leave the number and kind of constituent partons unchanged.

2) Leave the longitudinal momentum of each parton unchanged.

3) Multiply the wave function by a factor $e^{i\sum_i(\pm)\chi(\boldsymbol{x}_{i\perp})}$, where $\boldsymbol{x}_{i\perp}$ is the coordinate of the i^{th} charged electron, the (+) is for particle and (−) for antiparticle.

This is just the model discussed by Czyż [65]. Here the profile-function can be computed, order by order in perturbation-theory, using the rules (5.34)–(5.39) we have constructed. Only the initial and final parton wave-functions are needed; the rest of the dynamics is simple and given by (5.47).

We believe the method can be generalized to 2-body scattering, with the answer

$$S \to \exp ie^2 \int d^2x_\perp d^2x'_\perp \varrho_1(\boldsymbol{x}_\perp)D(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)\varrho_2(\boldsymbol{x}'_\perp) \tag{5.50}$$

where $\varrho_1(\boldsymbol{x}_\perp)$ is the charge-density operator for the "right movers"

$$\varrho_1(\boldsymbol{x}_\perp) = \int_{-\infty}^{\infty} d\zeta\varrho(0, \boldsymbol{x}_\perp, \zeta) \tag{5.51}$$

and

$$\varrho_2(\boldsymbol{x}_\perp) = \int_{-\infty}^{\infty} d\tau\varrho(\tau, \boldsymbol{x}_\perp, 0) \tag{5.52}$$

is the charge-density operator for "left-movers", with ϱ_1 and ϱ_2 acting only on the hopefully separate Hilbert spaces of partons within the right and left-movers respectively. D is then the "transverse" Coulomb-field

$$D(\boldsymbol{x}_\perp) = \int \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp}}{q_\perp^2} \tag{5.53}$$

complete with its infrared divergence. However, although the functional method of Abarbanel and Itzykson [66] applies well to this case, we have not yet convinced ourselves that (5.50) properly reproduces the Feynman diagrams.

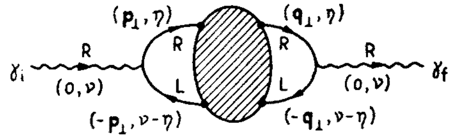

Fig. 6

As an example of how the formalism works, we consider forward Delbruck scattering [67] of a virtual photon as shown in Fig. 6. The structure of the amplitude is

$$\sum_{\substack{(e^+e^-)\\(e^+e^-)'}} \langle\gamma_f|H' \frac{|e^+e^-\rangle\langle e^+e^-|}{\Delta H}\left\{\exp i\int d^2x_\perp \varrho(x_\perp)\chi(x_\perp)-1\right\}\frac{|(e^+e^-)'\rangle\langle(e^+e^-)'|}{\Delta H'} H'|\gamma_i\rangle. \tag{5.54}$$

The matrix element H' for this configuration of helicities is obtained from (5.34). The energy denominators are of the form

$$\Delta H = \frac{q_\perp^2+m^2}{2\eta}+\frac{q_\perp^2+m^2}{2(\nu-\eta)}-\frac{(-Q^2)}{2\nu} = \frac{(q_\perp^2+m^2)\nu}{2\eta(\nu-\eta)}+\frac{Q^2}{2\nu}. \tag{5.55}$$

Unity is subtracted from the eikonal factor in order to eliminate a disconnected contribution. After all δ-functions are removed one finds for T the expression

$$T = \frac{ie^2}{2\nu}\int \frac{d^2q_\perp d\eta}{(2\pi)^3}\frac{\varepsilon^*\cdot \boldsymbol{q}_\perp}{\eta}\left[\frac{(q_\perp^2+m^2)\nu}{2\eta(\nu-\eta)}+\frac{Q^2}{2\nu}\right]^{-1}\times$$

$$\times\int\frac{d^2p_\perp}{(2\pi)^2}\int d^2x_{1\perp}d^2x_{2\perp}\exp i(\boldsymbol{q}_\perp-\boldsymbol{p}_\perp)\cdot(\boldsymbol{x}_{1\perp}-\boldsymbol{x}_{2\perp})\,\{\exp i[\chi(x_{1\perp})-\chi(x_{2\perp})]-1\}\times$$

$$\times\left[\frac{(p_\perp^2+m^2)\nu}{2\eta(\nu-\eta)}+\frac{Q^2}{2\nu}\right]^{-1}\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{p}_\perp}{\eta}. \tag{5.56}$$

It is most transparent to go into coordinate-space by performing the $\boldsymbol{q}_\perp$ and $\boldsymbol{p}_\perp$ integrations first. Introducing the longitudinal fraction y of the positron by the equation

$$\eta = \nu(1-y) \tag{5.57}$$

one easily finds

$$T = \frac{i\alpha}{\pi^2}\int_0^1 dy\,y^2\int d^2x_1d^2x_2|\boldsymbol{\epsilon}\cdot\boldsymbol{\nabla}_1K_0(\sqrt{m^2+Q^2y(1-y)}|\boldsymbol{x}_1-\boldsymbol{x}_2|)|^2\times$$

$$\times\{\exp i[\chi(x_1)-\chi(x_2)]-1\}. \tag{5.58}$$

The wave function of the pair in the photon is given by the factor $\boldsymbol{\epsilon}\cdot\nabla K_0$. Notice that because $K_0(z)$ falls off exponentially for large z, the important transverse separations of the pair are, for large Q^2, given by

$$|\boldsymbol{x}_1-\boldsymbol{x}_2| \leqslant \frac{1}{\sqrt{Q^2y(1-y)}}. \tag{5.59}$$

Only for y near zero or unity is the transverse separation of order m^{-1}. If we presume that these values of y are not too important, we can make an additional simplification by writing

$$\mathrm{Re}\,\{\exp i[\chi(\boldsymbol{x}_1)-\chi(\boldsymbol{x}_2)]-1\}\approx-\frac{1}{2}[(\boldsymbol{x}_1-\boldsymbol{x}_2)\cdot\nabla\chi(\boldsymbol{x}_1)]^2\rightarrow-\frac{1}{4}|\boldsymbol{x}_1-\boldsymbol{x}_2|^2|\nabla\chi(x)|^2 \tag{5.60}$$

an approximation sufficient for computing Im T at high Q^2. Then we get

$$-\mathrm{Im}\,T=\frac{\alpha}{4\pi^2}\int\limits_0^1 dyy^2\int d^2x_\perp x_\perp^2|\boldsymbol{\epsilon}\cdot\nabla_\perp K_0(\sqrt{m^2+Q^2y(1-y)}\,|\boldsymbol{x}_\perp|)|^2\int d^2b|\nabla_b\chi(b)|^2. \tag{5.61}$$

With our normalization, Im T is proportional to $\lim\limits_{\nu\to\infty}\sigma_\gamma(\nu)$ for real photons, and to $\lim\limits_{\nu\to\infty}\sigma_T(Q^2,\nu)$ for virtual photons. If for large Q^2 one can neglect m relative to $\sqrt{Q^2\,y(1-y)}$, then one obtains (by dimensional analysis) $\sigma_T\sim\frac{1}{Q^2}$, consistent with the scaling found in hadron electroproduction. This however does not apply, because for small z, $K_0(z)\sim\log z$. Thus $\nabla K_0\sim\sim[y(1-y)]^{-1/2}$ and the integral in (5.61) diverges logarithmically at $y=1$. Consequently as [68] $Q^2\to\infty$

$$\sigma_T\sim\frac{1}{Q^2}\log\frac{Q^2}{m^2}. \tag{5.62}$$

However, when one computes σ_S (which comes from the "Coulomb" photons) the wave function is K_0, not $\boldsymbol{\epsilon}\cdot\nabla K_0$ and the endpoints of the y-integration are protected. In that case

$$\sigma_S\sim\frac{1}{Q^2} \tag{5.63}$$

consistent with the scaling property. The result (5.59) suggests that virtual photons are "small", of order $(Q^2)^{-1/2}$ wide. This is compatible with Ioffe's arguments [39], and may have important implications for the properties of secondary hadrons produced by virtual--photon or virtual-W collisions (for which the same considerations hold). It may also mean that the diffraction-pattern for ϱ^0 electroproduction may broaden [68] as Q^2 increases as a consequence of the smaller net impact parameter necessary to initiate the process.

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