COHERENT AND INCOHERENT REACTIONS AND DIFFRACTION

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The main object of these lectures will be to focus attention on some of the many uses of heavy atomic nuclei in high energy particle physics. As we shall see the physics of production and scattering of high energy particles on nuclei and on nucleons can be profitably investigated together and we shall do so.

A partial list of the information one can get from studying high energy reactions in nuclei includes

1. The determination of unstable particle cross-sections and scattering amplitudes.
2. Information on coupling constants *e. g.* the vector dominance coupling constants γ_V and possibly couplings between unstable particle *e. g.* $g_{\omega\varphi,\text{ Pomeron}}$.
3. Information on how asymptotia is approached in elastic scattering of K-mesons from nucleons (using coherent regeneration).
4. Information on hadron-hadron diffraction mechanisms *e. g.* does double pomeron exchange have meaning?
5. An answer to the question: Is vector dominance a good approximation over large ranges of "photon mass".
6. K^0 electric form factor through the interference of regeneration of K^0s on atomic electrons with coherent regeneration from nuclei.
7. The lifetimes of π^0, η, η', ... through Coulomb photoproduction in the field of a heavy nucleus.
8. Off diagonal couplings of hadrons with photons, *e. g.* $NN^*\gamma$, $KK^*\gamma$ through Coulomb production of the unstable hadrons.
9. Information on the dynamics of weak interactions, *e. g.* the deduction of neutrino-nucleon cross-sections from neutrino interaction with a block of iron.
10. A big nucleus may be useful for copious production of particles through cascade processes.

We will concentrate in these lectures on the calculation of some of these processes in nuclei and also to some extent on the phenomenology of diffraction on hadrons. The treatment will involve a rather physical approach with minimal emphasis on derivations.

Coherent and incoherent

We begin with a discussion of elastic and inelastic scattering of high energy hadrons by heavy nuclei treated in first Born approximation. We assume some effective spin and isospin independent interaction potential $v(\vec{r}-\vec{r}_i)$ between the incident particle and target

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particle i, whose position vectors are $\vec{r}$ and $\vec{r}_i$ respectively. Then the matrix element for scattering of the hadron at momentum transfer $\vec{q}$, the initial and final nuclear states being $|I\rangle$, $|F\rangle$ respectively, is given by

$$F_{FI}(\vec{q}) = \langle F|\int e^{i\vec{q}\cdot\vec{r}} \sum_i v(\vec{r}-\vec{r}_i)d^3r|I\rangle = v(\vec{q})\,\langle F|\sum_i e^{i\vec{q}\cdot\vec{r}_i}|I\rangle \tag{1}$$

where $v(\vec{q}) = \int e^{i\vec{q}\cdot\vec{r}}\, v(\vec{r})d^3r$ is the corresponding two body scattering amplitude.

We assume that we are at high enough energies ($\gtrsim 1$ GeV) so that the nuclear states of interest, $|F\rangle$, are effectively degenerate with the target ground state $|I\rangle$.

We now wish to sum over all the final nuclear states (assumed degenerate) including two body correlations in the nuclear wave function. We for the moment ignore the spin and iso-spin dependence of the correlations. We define the nucleon density $\varrho(\vec{r})$ and two body correlation function $g(\vec{r}_1, \vec{r}_2)$ through

$$\varrho(\vec{r}_1) \equiv \int d\vec{r}_2 d\vec{r}_3 \ldots d\vec{r}_A |u_I(\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A)|^2, \tag{2}$$

$$\varrho(\vec{r}_1)\varrho(\vec{r}_2)\,[1+g(\vec{r}_1, \vec{r}_2)] \equiv \int d\vec{r}_3 d\vec{r}_4 \ldots d\vec{r}_A |u_I(\vec{r}_1, \vec{r}_2 \ldots r_A)|^2. \tag{3}$$

We have than

$$\sum_F |F_{FI}|^2 = |v(\vec{q})|^2 \,\langle I|\sum_{i,j} e^{iq\cdot(r_i-r_j)}|I\rangle$$
$$= |v(\vec{q})|^2[A + A(A-1)\int \varrho(\vec{r}_1)\,\varrho(\vec{r}_2)\,[1+g(\vec{r}_1, \vec{r}_2)]e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}d^3r_1 d^3r_2]$$
$$= |v(\vec{q})|^2[A^2|F(\vec{q})|^2 + A(1-|F(\vec{q})|^2) + C(\vec{q})] \tag{4}$$

where $F(\vec{q}) = \int \varrho(\vec{r})e^{i\vec{q}\cdot\vec{r}}d^3r$ is the single particle nuclear form factor and the two body correlation form factor

$$C(\vec{q}) = A(A-1)\int \varrho(\vec{r}_1)\varrho(\vec{r}_2)g(\vec{r}_1, \vec{r}_2)\, e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}\, d^3r_1 d^3r_2. \tag{5}$$

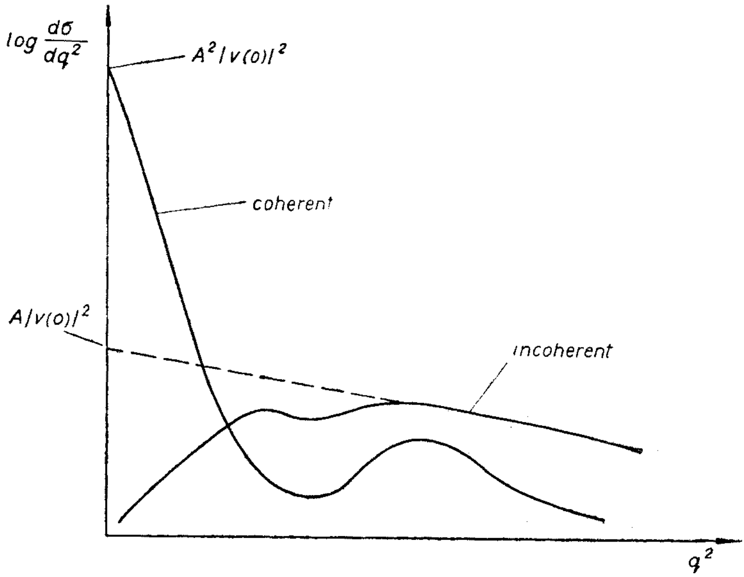
The differential cross-section given by (4) corresponds to elastic scattering and inelastic scattering to all excited target states including those that are particle unstable. The elastic scattering cross-section

$$\frac{d\sigma^{(E)}}{d\Omega} = A^2|v(\vec{q})|^2\,|F(\vec{q})|^2 \tag{6}$$

is the square of the coherent sum of scattering amplitudes off of each nucleon. It has an angular distribution characteristic of the size of the target nucleus falling off in a range $q \sim 1/R_{\text{nucleus}}$. The quantity

$$\frac{d\sigma^{(I)}}{d\Omega} = A|v(\vec{q})|^2[1-|F(\vec{q})|^2] \tag{7}$$

corresponds to inelastic scattering to all nuclear states. Once $|F(\vec{q})|^2$ falls off to near zero this expression corresponds to an incoherent sum of elastic scatterings from each target nucleon. The remaining term in (4) is a correction due to nuclear correlations and has a range in $q \sim 1/R_{\text{corr}}$, where the correlation length R_{corr} is of the order of 1 fermi. At zero momentum transfer we note that the inelastic scattering contributions to (4) vanish in first Born approximation due to orthogonality of the nucleon states. Sketched roughly the coherent and incoherent contributions to (4) look as follows



The coherent and incoherent features of high energy scattering (or production) of particles by nuclei are seen clearly already in first Born approximation. We now turn to the all important corrections due to the damping of the incident and outgoing particles by the target nucleons other than that on which the specific process of interest, whether scattering or production, occurs.

A classical treatment

One can get a good physical picture as well as results which stand up under more rigorous treatment with a completely classical picture, tracing rays through a medium. As an example [1] consider the incoherent production of particle *2* by a particle *1* in a one-step process. Let the density of the nucleus be $A\varrho(\vec{b}, z)$ where $(\vec{b}, z) = (b, \varphi, z)$ are cylindrical coordinates, let σ_1 and σ_2 be the total cross-section of the incident and outgoing particle and let $f(q^2)$ be the two body amplitude for the specific production process, assumed independent of spin and iso-spin for simplicity. Than the produced intensity of particle 2, is given by

$$N_{\text{eff}} |f(q^2)|^2 = \int d^2b \int_{-\infty}^{+\infty} dz\, e^{-\sigma_1 A \int_{-\infty}^{z} \varrho(b, z') dz'} |f(q^2)|^2 A \varrho(\vec{b}, z) e^{-\sigma_2 A \int_{z}^{\infty} \varrho(\vec{b}, z'') dz''} \tag{8}$$

Therefore

$$N_{\text{eff}} \equiv N(\sigma_1, \sigma_2) = \frac{1}{\sigma_2 - \sigma_1} \int d^2b [e^{-\sigma_1 T(\vec{b})} - e^{-\sigma_2 T(\vec{b})}] \tag{9}$$

$$T(\vec{b}) = A \int_{-\infty}^{+\infty} \varrho(\vec{b}, z) dz \tag{9a}$$

in the limit when $\sigma_1 \to \sigma_2 = \sigma$ we get

$$N(\sigma, \sigma) = \int T(\vec{b}) e^{-\sigma T(\vec{b})} d^2 b. \tag{10}$$

The quantity $T(\vec{b})$ as a function of b has roughly the same shape as $\varrho(\vec{r})$ as a function of r, assuming spherically symmetric distributions in which case for appreciable values of σ ($\geqslant 30$ mb) the integrand of (10) is surface peaked. Incoherent scattering or production is sensitive to the form of the nuclear surface density [2]. There are corrections to (9) due to elastic scattering before and after production [2, 3]. We will return to this question later.

Similarly by considering amplitudes rather than intensities one can produce expressions for coherent processes [3] which are equally valid.

Glauber theory: Eikonal methods

The Glauber theory [2] consists essentially of a generalization of (4), allowing for distortion of the ingoing and outgoing particle waves, this distortion or damping being expressed in terms of two body scattering amplitudes. Since you will have heard from others in this summer school concerning this treatment of high energy scattering by nuclei, we will content ourselves here with a description of the results we need. In elastic scattering from a heavy nucleus, neglecting correlations and spin and iso-spin dependence one gets

$$F_{II}(\vec{q}) = \frac{ik}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} [1 - e^{-\sigma/2(1-i\alpha) T(\vec{b})}] d^2 b \tag{11}$$

where α is the ratio of the real to the imaginary part of the forward scattering amplitude of the incident particle on a nucleon. Here $T(b)$ should be considered as the effective nuclear density thickness function obtained from convoluting the nuclear density with the range of the two body force.

The connection with the 1st Born approximation can be seen by expanding in powers of σ. One finds then

$$F_{II}(\vec{q}) = \frac{k\sigma}{4\pi} (\alpha + i) \int e^{i\vec{q}\cdot\vec{b}} T(\vec{b}) d^2 b + O(\sigma^2) \tag{12}$$

$$= f_{el}(0) A F(\vec{q}) + O(\sigma^2). \tag{12a}$$

This result can be obtained from an optical model [2] employing a Klein-Gordon type wave equation, solved in eikonal approximation. The optical model is

$$(\nabla^2 + k^2 - m^2)\psi = U\psi \tag{13}$$

where

$$U = -4\pi A f(0)\, \varrho(\vec{r}).$$

For reactions this can be generalized to include any number of channels coupled together, [4] as follows. We write the coupled wave equations

$$(\nabla^2 + k^2 - m_\alpha^2)\psi_\alpha = \sum_\beta U_{\alpha\beta}\psi_\beta \tag{14}$$

$$U_{\alpha\beta} = -4\pi A f_{\alpha\beta}(0) \varrho(\vec{r}), \quad k_\alpha^2 = k^2 - m_\alpha^2. \tag{14a}$$

Here ψ_α describes the scattering or produced wave corresponding to particle α, $f_{\alpha\beta}(0)$ is the amplitude for producing particle β on a nucleon with incident particle α at zero momentum transfer. We at this point assume spin and iso-spin independence again. These coupled equations automatically include longitudinal momentum transfer effects due to mass differences of the coupled channel particles, as well as multi-step processes to all orders.

The scattered or produced waves are strongly forward peaked. We therefore first solve the equations (14) as one dimensional wave equations in the incident direction. Neglecting the weak backward reflected wave in the integral representation of the one dimensional approximation to (14),

$$\psi_\alpha(\vec{b}, z) = \frac{e^{ik_\alpha z}}{2ik_\alpha} \int_{-\infty}^{z} e^{-ik_\alpha z'} \sum_\beta U_{\alpha\beta}\psi_\beta dz' + \frac{e^{-ik_\alpha z}}{2ik_\alpha} \int_{z}^{\infty} e^{ik_\alpha z'} \sum_\beta U_{\alpha\beta}\psi_\beta dz' \tag{15}$$

eads to a first order differential equation for $\varphi_\alpha(\vec{b}, z) \equiv e^{-ik_\alpha z}\psi_\alpha$

$$\frac{d}{dz}\varphi_\alpha(\vec{b}, z) = \frac{1}{2ik_\alpha}\sum_\beta U_{\alpha\beta}(\vec{b}, z)e^{i(k_\beta - k_\alpha)z}\varphi_\beta(\vec{b}, z) \tag{16}$$

and an elastic scattering or coherent production matrix element (the angular deflection is "provided" by the matrix element), the incident channel being *1*,

$$F_{\alpha 1}^{(c)} = \frac{1}{4\pi}\int e^{-i\vec{k}_\alpha\cdot\vec{r}} \sum_\beta U_{\alpha\beta}\psi_\beta d^3r \equiv \frac{k_\alpha}{2\pi i}\int e^{i\vec{q}\cdot\vec{b}}d^2b[\varphi_\alpha(\vec{b}, \infty) - \delta_{\alpha 1}]. \tag{17}$$

The coherent differential cross-sections are then given by

$$\frac{d\sigma_\alpha^{(c)}}{d\Omega} = |F_{\alpha 1}^{(c)}|^2. \tag{18}$$

Longitudinal momentum transfer effects are contained implicitly in $\varphi_\alpha(\vec{b}, \infty)$. One has included here any number of back and forth transitions among particles which can be connected coherently.

Incoherent production, corresponding in multiple scattering theory to closure, that is to summing over all final nuclear states, can be treated by dealing with intensities of waves rather than amplitudes. The answer for the intensity $I_\alpha(\vec{b}, z)$ in any channel α is

$$I_\alpha(\vec{b}, z) = A\varrho(\vec{b}, z)\left|\sum_{\alpha',\alpha''}\psi_{\alpha'}^{(1)}(\vec{b}, z)f_{\alpha'\alpha''}(q^2)\psi_{\alpha''}^{(\alpha)}(\vec{b}, -z)\right|^2 \tag{19}$$

where $\psi_\alpha^{(\gamma)}(\vec{b}, z)$ is the wave amplitude in channel α for an incident beam in channel γ. This allows for any number of coherent steps (no nuclear excitation) but only one incoherent step and hence is not valid for $e^{-aq^2} \ll 1$ where a is the typical range of the two body amplitudes $f_{\alpha'\alpha''}$. Again we shall see that this expression must be corrected for correlations at small momentum transfers. The incoherent cross-section, bearing in mind the above restrictions, is then

$$\frac{d\sigma^{(I)}}{d\Omega} = \int I_\alpha(\vec{b}, z)d^2b dz. \tag{20}$$

The effect of nuclear correlations

In order to calculate the effect of nuclear correlations we go back to Glauber theory for elastic or inelastic scattering. The scattering amplitude is given by [2]

$$F_{FI}(\vec{q}) = \frac{ik}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} u_F^*(\vec{r}_1 \ldots \vec{r}_A) u_I(\vec{r}_1 \ldots \vec{r}_A) \left\{ 1 - \prod_{j=1}^{A} [1 - \Gamma(\vec{b} - \vec{s}_j)] \right\} d^2 b d\vec{r}_1 \ldots d\vec{r}_A \tag{21}$$

where the profile function

$$\Gamma(\vec{b}) = \int f(q^2)\, e^{-i\vec{q}\cdot\vec{b}} d^2 b \tag{22}$$

$f(q^2)$ being the amplitude for scattering of the incident hadron by a target nucleon. For $F = I$, *i. e.* elastic scattering, we get, assuming no correlations in the nuclear wave function,

$$(F_{II}(\vec{q}) = f(0) N(\vec{q}; 0, \tfrac{1}{2}\sigma') \tag{23}$$

$$N(\vec{q}; 0, \tfrac{1}{2}\sigma') = \frac{2}{\sigma'} \int e^{i\vec{q}\cdot\vec{b}} d^2 b [1 - e^{-\frac{1}{2}\sigma' T(\vec{b})}] \tag{24}$$

where $\sigma' = \sigma(1 - i\alpha)$. Using closure and assuming that only one inelastic step is important we get [5]

$$\frac{d\sigma}{d\Omega} = |f(0)|^2 \left\{ |N(\vec{q}; 0, \tfrac{1}{2}\sigma')|^2 + N_1(0; \sigma) - \frac{1}{A} |N_1(\vec{q}; \tfrac{1}{2}\sigma')|^2 + \right.$$
$$\left. + \frac{2}{A} \operatorname{Re} [N_2(\vec{q}; \tfrac{1}{2}\sigma') N^*(\vec{q}; 0, \tfrac{1}{2}\sigma)] \right\} \tag{25}$$

where

$$N_m(\vec{q}, \sigma) = \frac{1}{m!} \frac{1}{\sigma} \int e^{i\vec{q}\cdot\vec{b}} [\sigma T(\vec{b})]^m e^{-\sigma T(\vec{b})} d^2 b. \tag{26}$$

Letting $\sigma \to 0$ this reduces to (4) when $C(\vec{q}) = 0$. All three terms on the right side of (25) fall off in a range of $q \sim 1/R_{\text{nucleus}}$ except the familiar incoherent term $|f(0)|^2 N_1(0; \sigma)$.

Introducing a wave function with two body correlations leads, in a straightforward but tedious fashion to an elastic scattering cross-section [5]

$$\frac{d\sigma^{(c)}}{d\Omega} = |f(0)|^2 |M(\vec{q}; 0, \tfrac{1}{2}\sigma')|^2 \tag{27}$$

where $M(\vec{q}; 0, \frac{1}{2}\sigma')$ is obtained from $N(\vec{q}; 0, \frac{1}{2}\sigma')$ by the replacement of

$$T(\vec{b}) \to T_R(\vec{b}) = T(\vec{b}) - \xi Q(\vec{b}) \sigma \tag{28}$$

where

$$Q(\vec{b}) = A^2 \int_{-\infty}^{+\infty} \varrho^2(\vec{b}, z) dz \tag{29}$$

and the correlation length

$$\xi = \frac{1}{16\pi a} \int e^{-b^2/4a} g(\vec{b}, z) d^2 b dz. \tag{30}$$

To get (28) we have assumed a gaussian for the two body amplitude, $f(q^2) = f(0)\, e^{-aq^2}$ and have taken $g(\vec{r}_1, \vec{r}_2) \simeq g(\vec{r}_1 - \vec{r}_2)$ which cannot be exact but should be all right for the short range correlations. The $1/A$ corrections to the coherent cross-section have corrections similar in character but we do not discuss them further here as these are relatively small terms where they contribute at all.

In good approximation the result (27) is equivalent to using an uncorrelated wave function in which we write

$$F_{II}(\vec{q}) \equiv f(0) M(\vec{q}; 0, \tfrac{1}{2}\sigma') \equiv f^{(E)}(0) N(\vec{q}; 0, \tfrac{1}{2}\sigma^{(E)}) \tag{31}$$

where

$$f^{(E)}(0) = \frac{\sigma^{(E)}}{\sigma} f(0), \qquad \sigma^{(E)} = \sigma[1 - \xi\eta(\tfrac{1}{2}\sigma)\sigma], \tag{32}$$

$$\eta(\varkappa) = \int e^{-\varkappa T(\vec{b})} Q(\vec{b}) d^2b \Big/ \int e^{-\varkappa T(\vec{b})} T(\vec{b}) d^2b. \tag{33}$$

We can therefore use an effective optical model for diffractive processes with potentials $U_{\alpha\beta} = -4\pi f^{(E)}_{\alpha\beta}(0)\, A\varrho(\vec{b}, z)$. This is useful in production channels as well as for elastic scattering, *e. g.* photoproduction of vector mesons at finite energies. A reasonable value for ξ is -0.3 to -0.4 fermi. The biggest differences between $\sigma^{(E)}$ and σ are for the heaviest nuclei. In Pb we find the following values

σ(mb)	25.0	30.0	40.0
$\sigma^{(E)}$(mb)	27.3	33.0	42.4

Correlation effects then are not large and at the 10% level perhaps we have done well enough here.

The effect on incoherent cross-sections is as follows. We find [5]

$$\frac{d\sigma^{(I)}}{d\Omega} = |f(q^2)|^2 N_{\text{eff}}(\sigma, \xi)\, [1 + \eta(\sigma) G(\vec{q})] \tag{34}$$

where

$$N_{\text{eff}}(\sigma, \xi) = \int d^2b [T(\vec{b}) - 4\xi\sigma Q(\vec{b})]\, e^{-\sigma T_R(\vec{b})} \tag{35}$$

and

$$G(\vec{q}) = \int g(\vec{r})\, e^{i\vec{q}\cdot\vec{r}} d^3r. \tag{36}$$

Since $G(\vec{q})$ is generally negative at small q^2 because of nucleon-nucleon repulsion and the Pauli principle the incoherent cross-section generally decreases as we approach the forward direction. We note that unlike in 1st Born approximation the incoherent cross-section need not vanish in the forward direction, because of multiple scattering effects.

We return now to equation (32). The correlation length is generally negative because of short range repulsion between nucleons. It follows that $\sigma^{(E)} > \sigma$. Physically this can be understood as follows. The repulsion decreases the shadow cast by each nucleon, as it keeps the nucleons further apart than if they were uncorrelated. Hence the effective cross-section of each nucleon is larger.

Production in the infinite energy limit

In the infinite energy limit the longitudinal momentum transfer $q_l \simeq (m_2^2 - m_1^2)/2p_{\text{inc}}$ vanishes. Here m_1 and m_2 are the masses of the incident and produced particles respectively. For simplicity we consider several processes in this limit. We will then come back to the corresponding processes at finite energy.

We consider first photoproduction of vector mesons (*e.g.* the ϱ^0-meson) as a coherent one-step process. Tracing the eikonal ray for any impact vector b leads to a photoproduced amplitude

$$\int_{-\infty}^{\infty} f_{\gamma\varrho}(0) A\varrho(\vec{b}, z) dz e^{-\sigma/2A \int_z^{\infty} \varrho(\vec{b}, z') dz'} . \tag{37}$$

Here σ is the vector meson nucleon total cross-section. To get the angular distribution we integrate the Fourier distribution over all impact parameters, obtaining a production amplitude

$$F_{\gamma\varrho}^{(c)}(\vec{q}) = f_{\gamma\varrho}(0) \int d^2b e^{i\vec{q}\cdot\vec{b}} \int_{-\infty}^{\infty} dz A\varrho(\vec{b}, z)\, e^{-\sigma/2A \int_z^{\infty} \varrho(\vec{b}, z') dz'} \tag{38}$$

$$= f_{\gamma\varrho}(0) \frac{2}{\sigma} \int e^{i\vec{q}\cdot\vec{b}} [1 - e^{-\sigma/2 T(\vec{b})}] d^2b. \tag{39}$$

We have assumed a ϱ^0-nucleon scattering amplitude which is purely imaginary here. One notes that the amplitude for coherent photoproduction of a particle is proportional to the amplitude for elastic scattering of this particle [3]. This is an example of the optical effect known as Babinet's principle [6]. Babinet's principle says that diffraction by an aperture A in an opaque screen S is the same as diffraction when A is opaque and S transparent. In photoproduction the mesons emanate from the nucleus which plays the role of the aperture A. In elastic ϱ-scattering the nucleus acts as the opaque obstacle. Using similar arguments one finds, again neglecting longitudinal momentum transfer effects, the coherent production amplitude for producing particle *2*, with a hadron *1* incident in a one step process

$$F_{12}^{(c)}(\vec{q}) = \frac{2}{\sigma_2 - \sigma_1} f_{12}(0) \int [e^{-\frac{1}{2}\sigma_1 T(\vec{b})} - e^{-\frac{1}{2}\sigma_2 T(\vec{b})}] e^{i\vec{q}\cdot\vec{b}} d^2b \tag{40}$$

where σ_1, and σ_2 are the total cross-sections for particle *1* and *2* respectively on a nucleon, scattering amplitudes assumed to be purely imaginary. It is of course true that (39) and (40) follow from the coupled channel equations (14) with the assumptions above.

In the case when $\sigma_1 = \sigma_2 = \sigma$ equation (40) reduces to

$$F_{12}^{(c)}(\vec{q}) = f_{12}(0) \int T(\vec{b}) e^{-\frac{1}{2}\sigma T(\vec{b})} e^{i\vec{q}\cdot\vec{b}} d^2b. \tag{41}$$

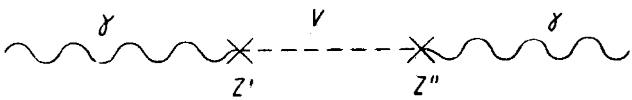
Again as in incoherent production the integral is surface peaked (take $\vec{q} = 0$) as a function of b, but less so, because of the factor 1/2 multiplying σ in the exponent. Combined studies of coherent and incoherent production are capable of producing information on the details of radial nuclear distributions.

We next consider two step processes at infinite energy, or to be more precise, particles which are produced as a result of strong contributions from both one and two step processes. Examples are (*i*) elastic scattering of photons [6, 7], (*ii*) elastic scattering of neutrinos [8] and (*iii*) possibly many totally hadronic processes [4]. We consider in detail here elastic scattering of photons. Through the optical theorem one may obtain total photon cross-sections from the imaginary part of the forward scattering amplitude.

The one step process in forward photon scattering is simply A times the forward amplitude on a nucleon

$$F^{(1)}_{\gamma\gamma}(0) = Af_{\gamma\gamma}(0). \tag{42}$$

The two step process, assuming dominance by one intermediate vector meson V, consists of photoproducing the vector meson, V, on one nucleon followed by radiative capture of V on a second nucleon as below



We have than

$$F^{(2)}_{\gamma\gamma}(0) = \frac{2\pi i}{k}\int d^2b \int_{-\infty}^{\infty} A\varrho(\vec{b}, z')f_{\gamma V}(0)dz' \int_{z'}^{\infty} e^{-\frac{1}{2}\sigma A\int_{z'}^{z''}\varrho(\vec{b},z)dz} A\varrho(b, \vec{z}'')f_{V\gamma}(0)dz'' \tag{43}$$

$$= \frac{4\pi i}{k\sigma} f_{\gamma V}(0)f_{V\gamma}(0)\,\{A-N(0, \tfrac{1}{2}\sigma)\} \tag{44}$$

where

$$N(0, \tfrac{1}{2}\sigma) = \int d^2b[1-e^{-\frac{1}{2}\sigma T(\vec{b})}). \tag{44a}$$

In the above σ is the V-nucleon total cross-section and we have assumed that the V-nucleon forward scattering amplitude is purely imaginary.

If we now assume vector dominance (by a single meson V) on a nucleon then

$$f_{\gamma V}(0)f_{V\gamma}(0) = f_{\gamma\gamma}(0)f_{VV}(0) \tag{45}$$

and since $f_{VV}(0) = \dfrac{k\sigma}{4\pi}\, i$ we have

$$F^{(2)}_{\gamma\gamma}(0) = -f_{\gamma\gamma}(0)[A-N(0, \tfrac{1}{2}\sigma] \tag{46}$$

so that

$$F_{\gamma\gamma}(0) = F^{(1)}_{\gamma\gamma}(0)+F^{(2)}_{\gamma\gamma}(0) = f_{\gamma\gamma}(0)\int d^2b[1-e^{-\frac{1}{2}\sigma T(\vec{b})}] \tag{47}$$

which is proportional to the amplitude for elastic scattering of the vector meson V from a nucleus. Vector dominance on a nucleon implies vector dominance on a nucleus.

On the other hand, at low energies ($\sim$1 GeV), longitudinal momentum transfer effects supress the two step coherence and the photon "behaves" as a photon might be expected to. The scattering amplitude is given by (42).

As an example of hadronic induced single and two step processes we consider, for reasons which will become clearer below, coherent production of particles A_1 (1.06 GeV) and A_3 (1.65 GeV) assuming a coupled π, A_1, A_3 system and neglecting all other couplings [4]. Assuming that these three particles have the same cross-section on a nucleon, σ, (which follows from a simple quark model) we find by methods similar to the above that the production amplitude for A_1 mesons is

$$F^{(c)}_{\pi A_1}(0) = f_{\pi A_1}(0) \left[N_1\left(0; \frac{\sigma}{2}\right) - \frac{f_{\pi A_3}(0) f_{A_3 A_1}(0)}{f_{\pi A_1}(0) f_{A_3 A_3}(0)} N_2\left(0; \frac{\sigma}{2}\right)\right] \tag{48}$$

$$N_m\left(0, \frac{\sigma}{2}\right) = \frac{1}{m!}\left(\frac{2}{\sigma}\right) \int \left[\frac{\sigma}{2} T(\vec{b})\right]^m e^{-\frac{1}{2}\sigma T(\vec{b})} d^2 b. \tag{49}$$

We have ignored higher order back and forth transitions between mesons here.

Experiments of Morrison *et al.* [9] yield $f_{\pi A_3}(0)/f_{\pi A_1}(0) = 0.35$, $f_{\pi A_1}(0)/f_{\pi\pi}(0) = 0.27$ at 8GeV/c. Since these processes are diffractive, we can expect little energy dependence of these ratios. In order to calculate A_1 production in this model we also need

$$f_{A_3 A_1}(0)/f_{A_3 A_3}(0).$$

The coefficient of N_2 in (48) which we call R_1 is then a parameter in the calculation. One can expect that R_1 is not bigger than 0.1. Therefore A_1 production is expected to go predominantly as a one step process in this model. It is to be noted with respect to our conclusions for A_1 production that at finite energies the two step pròcess for A_1 production through

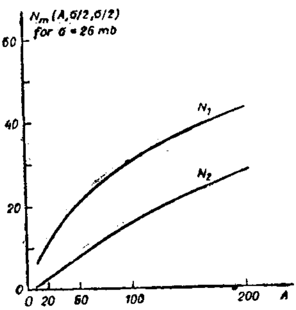


Fig. 1. $N_m(A\frac{1}{2}\sigma)$ $m = 1, 2$ as a function of A for $\sigma = 26$ mb. The nuclear density is taken to be $\varrho(r) = \varrho_0/(1+\exp[(r-c)/a])$ with $a = 0.545$ fm and $c = 1.14\, A^{1/3}$ fm

the A_3 is further inhibited by longitudinal momentum transfer effects due to the higher mass of the A_3. The same could be true for possible couplings to other higher mass bosons. It is unlikely that there is any important coherent coupling to mass states lower than the A_1.

In A_3 production we are in a more complicated situation. The labels A_1 and A_3 must be interchanged in (48) and this makes the coefficient of N_2 which we call $R_3 \sim$unity. The fact that one and two step processes are comparable (note in Fig. 1 that N_1 and N_2 are not too different in value) means that in detailed calculations we must know or be able to determine R_3 as well as $\sigma^{\text{tot}}_{A_1 n}$ and $\sigma^{\text{tot}}_{A_3 n}$. In principle we should be able to determine all of these since we have the whole periodic table to work with as targets. There is however the real possibility that other bosons contribute as intermediate states. Clearly the sign of R_3 drastically affects the production rate. This sign could be negative according to some models of diffractive production on nucleons [10]. There is then, interesting dynamical information possibly available from diffractive production in nuclei.

Coherent and incoherent production at finite energy

We will discuss results of measurements and calculations of the following processes in nuclei.

Coherent photoproduction of vector mesons.

Incoherent photoproduction of vector mesons.

Total photon cross-sections.

Photoproduction of charged pions.

Coherent photoproduction of the ϱ_0 meson

Because of the weak strength of the electromagnetic coupling constant and since the vector mesons φ and ω cannot be connected to the ϱ^0 by pomeron exchange, coherent photoproduction of ϱ_0 mesons may reasonably be expected to be describable as a one step process.

The amplitude at finite energies is

$$F_{\gamma\varrho}(0) = f_{\gamma\varrho}(0) A \int e^{i\vec{q}\cdot\vec{b}} e^{iq_l z} \varrho(\vec{b}, z) e^{-\frac{1}{2}\sigma(1-i\alpha)A \int_z^\infty \varrho(\vec{b}, z')dz'} d^2b\,dz \tag{50}$$

neglecting nuclear correlations for the moment.

Careful measurement of diffractive production of ϱ^0-mesons is a crucial experiment for vector dominance. Using the results of measurements across the periodic table one can determine

1. The nuclear radius from the diffractive slope.
2. The cross-section $\sigma^{\text{tot}}_{\varrho n}$ from the relative A dependence in medium to heavy nuclei.
3. The coupling constant γ_ϱ from the absolute photoproduction cross-section using

$$\frac{d\sigma_{\gamma\varrho}(0)}{dq^2} = -\frac{1}{16}\frac{\alpha}{4\pi}\left(\frac{\gamma_\varrho^2}{4\pi}\right)^{-1} \sigma^2_{\varrho n}(1+\alpha^2) \tag{51}$$

which is the result of vector dominance.

The most comprehensive series of measurements are those of Ting *et al.* [11] at DESY. They lead to nuclear radius $R = 1.12\,A^{1/3}$ (see Fig. 2) assuming a Woods Saxon form with surface thickness $C = 0.545$ fm; $\gamma_\varrho^2/4\pi = 0.57 \pm 0.10$ and $\sigma^{\text{tot}}_{\varrho n} = 26.7 \pm 2$ mb. The ratio of forward real to imaginary scattering amplitude is taken to be $\alpha = -0.2$ and correlation effects are included using $\xi = -0.3$ fm. The effective photon energy was 6.2 GeV. It is to be noted that the broad width of the rho meson presents some problems which you will have heard about in other lectures here.

The calculation of the total cross-section of photons is shown in Fig. 3 along with measured values. One sees here the increasing importance of two step process as energy

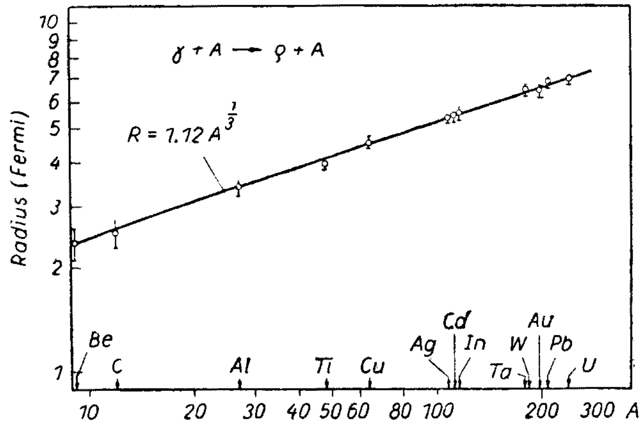


Fig. 2. Nuclear radii determined from photoproduction of rho mesons

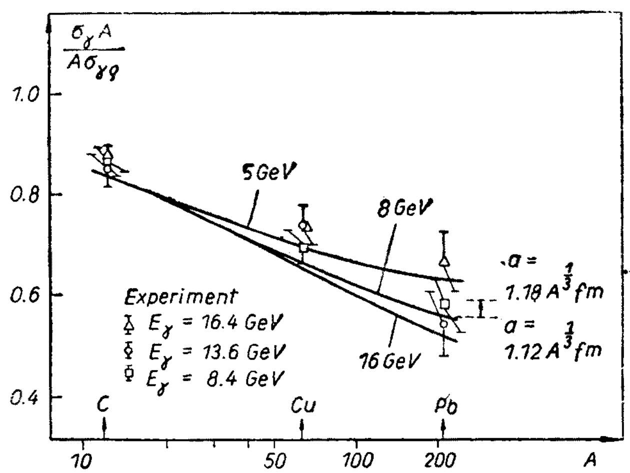


Fig. 3. Total photon cross-section as a function of A divided by A times the total photon cross-section on hydrogen. a is the nuclear radius

increases. It is not clear that there is complete agreement between theory and experiment. We have taken [12] the vector mesons ϱ^0, ω and φ into account with coupling constants from storage ring experiments in which the vector mesons are on the mass shell. In photon scattering, of course, it is the photon that is on the mass shell. Further experiments especially at energies in the range 2–5 GeV would be of interest. The value of other parameters of the

TABLE I

The parameters for the calculation as functions of the incident photon energy

E_γ (GeV)	$[d\sigma(\gamma p \to \varrho^0 p)/dt]_{t=0}$ (fit of exp. data) (μb/GeV^2)	$\beta = \mathrm{Re}\, f/\mathrm{Im}\, f$	$\sigma^{\mathrm{tot}}_{\pi+}$ (mb)	$\sigma^{\mathrm{tot}}_{\varrho p}$ from VMD[a] with $\gamma^2/4\pi = 0.5$ (mb)
3	152	−0.26	...	27.6
5	124.2	−0.22	...	25.2
8	113.1	−0.185	25	24.2
16	106.2	−0.135	24	23.65

[a] Vector-meson dominance.

calculation are given in Table I. They have been calculated using two body photoproduction data and formula (51).

We have also calculated [12] incoherent production of ϱ^0 and π^+ mesons from nuclei assuming vector dominance. The appropriate diagrams are shown in Fig. 4. The results are shown in Figs 5 and 6. While the agreement between experiment and theory is not bad at all it is not clear that the relatively modest energy dependence (due to longitudinal momentum transfer effects, but modified by the energy variation of σ and α) of the calculations is reproduced by the measurements. Again further experiments could be of value.

We will not go into the many possible modifications of vector dominance that could be invoked to modify the calculated results at this time.

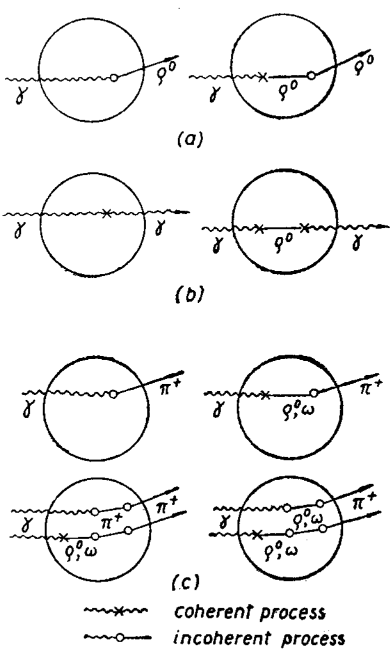


Fig. 4. Diagrams for one-step and multi-step photoreaction processes. (*a*) Incoherent ϱ^0 production. (*b*) Photon forward elastic scattering. (*c*) Incoherent π^+ photoproduction

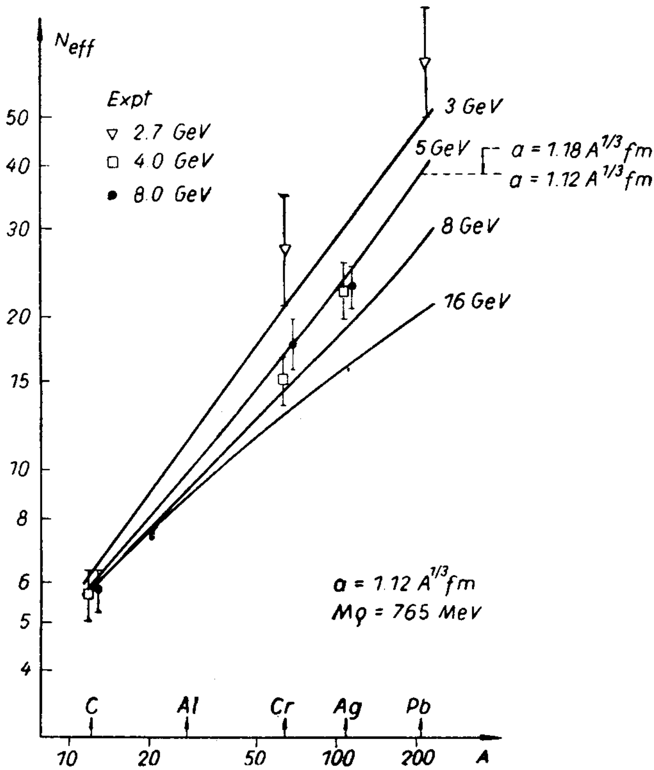


Fig. 5. ϱ^0 photoproduction: Experimental and theoretical values of N_{eff} for several energies as a function of A. The effect of a change in the nuclear radius from $a = 1.12\, A^{1/3}$ fm to $a = 1.18\, A^{1/3}$ fm is shown for $A = 208$ at $E_\gamma = 5$ GeV

Diffractive processes on protons

Elastic scattering

We will now study models of elastic scattering of hadrons (and photons using vector dominance) from protons. At high energies we may profitably use the eikonal approximation [10], [13], [2] to a partial wave expansion of an elastic scattering amplitude. Ignoring spin considerations we can write

$$A(s, t) = ik\sqrt{s} \int \frac{d^2b}{2\pi} \left[1 - e^{2i\delta(\vec{b})}\right] e^{i\vec{b}\cdot\vec{q}} \tag{52}$$

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2 s} |A(s, t)|^2; \qquad t = \vec{q}^{\,2}. \tag{53}$$

Here k is the C.M. momentum and $\vec{q}$ the three momentum transfer.

The physics of the situation determines the eikonal phases $\delta(\vec{b})$. It is of interest of compare (52) with the Glauber expression for elastic scattering by a heavy nucleus. Here

$$F(s, \vec{q}) = \frac{ik}{2\pi} \int d^2b \left[1 - e^{-\frac{1}{2}\sigma(1-i\alpha)T(\vec{b})}\right] e^{i\vec{b}\cdot\vec{q}}. \tag{54}$$

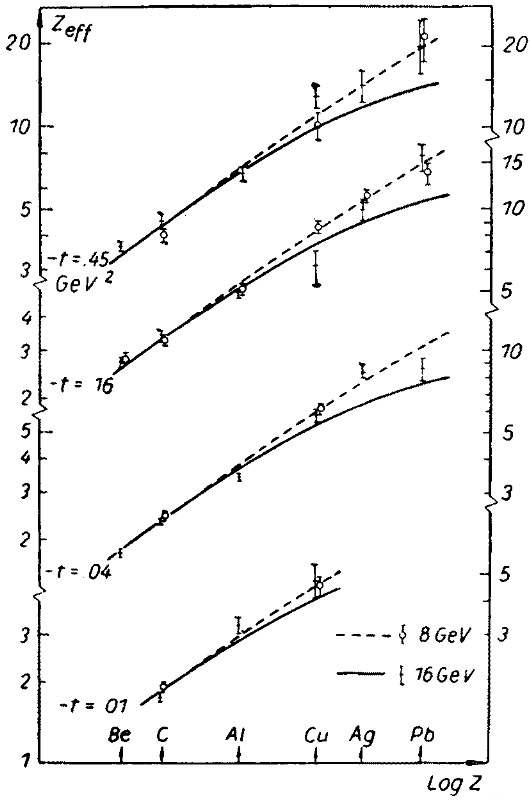


Fig. 6. The A dependence of $Z_{eff} = \dfrac{d\sigma(\gamma A \to \pi^+ A')/dt}{d\sigma(\gamma \varrho \to \pi^+ n)/dt}$ for four different momentum transfers

There is an intimate connection between the eikonal phase $\delta(\vec{b})$ and the density of nuclear matter

$$2i\delta(\vec{b}) = -\frac{1}{2}\,\sigma(1-i\alpha)T(\vec{b}) \propto T(\vec{b}). \tag{55}$$

The question arises, if we are to make an analogy between nuclear and nucleon scattering, what shall we use for the density of hadronic matter.

We will describe three classes of models which have been proposed.

(*i*) Yang [4]

$$2i\delta(\vec{b}) \propto \int_{-\infty}^{\infty} \varrho(\vec{b}, z)\,dz \tag{56}$$

where $\varrho(\vec{b}, z)$ is the convoluted density of the two interacting hadrons. These densities are taken to be the Fourier transforms of electromagnetic form factors.

(*ii*) Chiu and Finkelstein [15]

In this model one adds to the eikonal phase of Yang, eikonal phases corresponding to the exchange of the Regge pole trajectories on which lie the ϱ^0, A_2, ω, and f_0. Specifically the Regge phase for a given trajectory is taken as

$$2i\delta_R(\vec{b}) = \int \frac{d^2q}{2\pi} \left[\frac{A_{\text{Regge pole}}(s,-q^2)}{-ik\sqrt{s}}\right] e^{-i\vec{b}\cdot\vec{q}}. \tag{57}$$

(*iii*) Frautschi and Margolis [10], Barger and Phillips [16], Ter Martirosyan (Gribov [17], Jacob and Pokorski [10])

Here one assumes that we are at high enough energies that the dominant trajectory, *i.e.* the pomeron trajectory, gives the main features of the scattering. The eikonal phase then is given by an expression of the form of (57) with

$$A_{\text{Regge pole}}(s, -q^2) = A_{\text{Pomeron}}(s, -q^2).$$

The model of Yang corresponds to a fixed pole pomeron. It has no energy dependence and no shrinkage. The angular distribution features diffractive dips which should be seen if one has an accelerator of high enough energy. The model is meant to represent the infinite energy limit.

Model (*ii*) produces energy dependence, shrinkage or anti-shrinkage at finite energies, depending on the nature of the projectile and again one has a flat pomeron. Total cross-sections in the fits of these authors are monotonic decreasing. At high enough energies the shrinkage or anti-shrinkage will cease.

The recent measurements of total cross-sections and elastic scattering of protons at small t, at Serpukhov [18], [19] as well as photoproduction of φ mesons at SLAC and earlier K^+p elastic scattering measurements tend to favour models of type (*iii*) and we now explore these in more detail. Specifically we will study the simple model of Frautschi *et al.* [10], [20].

Our assumptions in this model in its simplest form are

(*i*) We consider only non-spin flip scattering, *i.e.* we neglect all spin dependence.

(*ii*) We assume a straight line trajectory

$$\alpha_P(t) = 1+\alpha' t. \tag{58}$$

(*iii*) We take a constant residue and have no signature zeros so that

$$A_{\text{pole}}(s, t) = c\left[\frac{s}{s_0}\, e^{-i\pi/2}\right]^{\alpha_P(t)}. \tag{59}$$

Note that signature zeros at $\alpha_P = -1, -3, -5, \dots$ occur for $|t| \geqslant 2\ (\text{GeV})^2$. These are not important, as we shall see, since multiple scattering takes over for $|t| \leqslant 1\ (\text{GeV})^2$.

Then it follows from (57) that

$$2i\delta(\vec{b}) = -\frac{\xi}{\mu}\, e^{-b^2/4\alpha'\mu} \tag{60}$$

$$\mu = \ln\frac{s}{s_0} - i\,\frac{\pi}{2}\,,\ \xi = -\frac{c\sqrt{s}}{2k}\,\alpha' s_0. \tag{61}$$

Using (52) one finds then

$$A = 2ik\sqrt{s}\,\alpha'\xi \sum_{n=1}^{\infty} \frac{1}{nn!}\left(-\frac{\xi}{\mu}\right)^{n-1} e^{\frac{t\alpha'\mu}{n}}. \tag{62}$$

The terms with $n < 1$ correspond to multiple scattering (multiple pomeron exchange) and exhibit an inverse power of $\ln(s/s_0)$ behaviour, characteristic of Regge cuts. This can be seen by writing the contour integral around the cut as

$$\int_{-\infty}^{\alpha_{\max}} d\alpha(\alpha-\alpha_{\max})^{n-2}\left(\frac{s}{s_0}\,e^{-i\pi/2}\right)^{\alpha} = \frac{(n-2)!}{\mu^{n-1}}\left(\frac{s}{s_0}\,e^{-i\pi/2}\right)^{\alpha_{\max}} \tag{63}$$

where

$$\alpha_{\max} = 1 + t\alpha'/n.$$

This should be compared with the n^{th} term of (62) which is proportional to

$$\frac{s}{s_0}\,\frac{1}{\mu^{n-1}}\exp\left(\frac{t\alpha'\mu}{n}\right) = \frac{1}{\mu^{n-1}}\left(\frac{s}{s_0}\right)^{1+t\alpha'/n}(e^{-i\pi/2})^{t\alpha'/n}. \tag{64}$$

The main features of the multiple scattering amplitude are the following:

1. From the optical theorem

$$\sigma^{\text{tot}} = \frac{4\pi}{k\sqrt{s}}\,\text{Im}\,A(s,0)$$

$$= 8\pi\alpha'\xi\sum_{1}^{\infty}\frac{1}{nn!}\,\text{Re}\left(-\frac{\xi}{\mu}\right)^{n-1}$$

$$= 8\pi\alpha'\xi\left[1 - \frac{\xi\ln\dfrac{s}{s_0}}{4\ln^2\dfrac{s}{s_0} + \pi^2} + 0(\xi^2)\right]. \tag{65}$$

It follows from this total cross-sections will invariably approach their infinite energy limit from below. As an example in the case of p—p scattering we estimate, using values of ξ and α' determined from elastic differential cross-sections ($\xi \sim 7$, $\alpha' \sim 0.8\,(\text{GeV})^{-2}$, $s_0 = 1\,(\text{GeV})^2$), that $\sigma^{\text{tot}}(\infty) \simeq 50$ mb. It is to be noted that the effect of lower trajectories will have to fall away (as inverse powers of s/s_0) before the features presented by this model show themselves clearly. One should then see the slow logarithmic rise to the assymptotic value $\sigma_{\text{tot}}(\infty)$.

2. $d\sigma/dt$ will continue shrinking until $\sigma_{\text{el}} = 0$ at infinite energy, (see Fig. 7).

3. The ratio $\alpha = \text{Re}\,A(0)/\text{Im}\,A(0)$ for the forward amplitude, which is generally negative at present energies, will cross over and become positive at higher energies. It is to be expected however that by the time this happens α will be quite small (< 0.1).

4. Elastic scattering amplitudes exhibit a cross-over effect. Given a universal pomeron slope the total cross-section determines the residue strength ξ. The larger σ^{tot} the larger ξ and hence the greater the strength of the double scattering which interferes destructively with single scattering according to (62). This destructive interference sharpens the slope. The larger is σ^{tot} (and hence $d\sigma/dt|_{t=0}$) the sharper the slope. Hence we have a cross--over effect in angular distributions of processes with different total cross-sections.

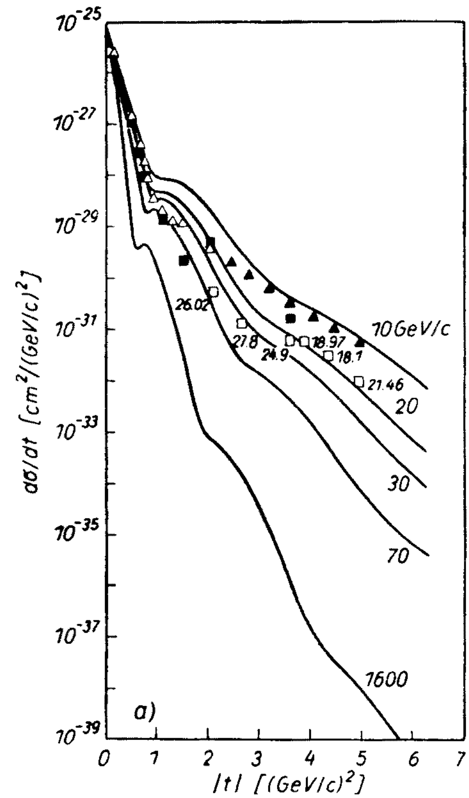


Fig. 7. Proton-proton scattering

5. The square of the multiple scattering series can be summed approximately converting a sum to an integral and using the method of steepest descent [20]. Roughly speaking, the net result is the following at all but the smallest t values ($\geqslant 2\ (\mathrm{GeV})^2$)

$$|A| \,\widetilde{\propto}\, \frac{s}{s_0} \exp\left[-2\sqrt{t\alpha' \ln\frac{s}{s_0} \ln\left|\frac{\xi}{\bar{n}\mu}\right|}\right] \tag{66}$$

where $\bar{n}$ is the dominant exchange at any t value given by

$$\bar{n} \simeq \sqrt{\frac{-t\alpha' \ln s/s_0}{\ln|\bar{n}\mu/\xi|}}. \tag{67}$$

In detail great care must be taken in evaluating (62) approximately because of the alternate signs in the series.

We see that the multiple scattering corrections convert the Gaussian momentum transfer of the pole term at small t into an exponential dependence $A \propto e^{-b\sqrt{-t}}$. What we have is an amplitude which is of the character of Orear's Law [21] but which also shrinks. One can write an effective form for the multiple scattering trajectory

$$\alpha_{\text{eff}} = 1-2[t\alpha' \ln^{-1}(s/s_0) \ln|\xi/\bar{n}\mu|]^{1/2}. \tag{68}$$

Diffractive production

Going back to Glauber theory we can write amplitudes for one step and two step diffractive production, neglecting longitudinal momentum transfers which are much less important than on nuclei because of the small size hadrons, in the form

$$A^{ij}_{\text{one-step}} = s \int \frac{d^2b}{2\pi}\, \delta_{ij}(b) e^{2i\delta(b)} e^{i\vec{b}\cdot\vec{q}} \tag{69}$$

$$A^{ij}_{\text{two-step}} = is \int \frac{d^2b}{2\pi}\, (\sum_m \delta_{im}(b)\, \delta_{mj}(b)) e^{2i\delta(b)} e^{i\vec{b}\cdot\vec{q}} \tag{70}$$

where, as before, $2i\delta(b) = -\xi/\mu \exp[-b^2/4\alpha'\mu]$ and

$$\delta_{ij}(b) = -\frac{\xi_{ij}}{2i\mu} \exp[-b^2/4\alpha'\mu].$$

The fact that $\delta_{ij}(b)$ has the same b dependence as $\delta(b)$ follows from the fact that we are considering diffractive processes having a single non-spin flip amplitude coming from pomeron exchange. The ξ_{ij} are the residue strengths for each pomeron exchange. i and j represent the initial and final particles *e.g.* p and $N^*(1470)$, K and $K^*{}_{1/2}$, π and A_1. In the two step process we can go through a number of intermediate states which we label by m.

If a diffractive process occurs dominantly as a one step process the diffractive slope at small t is, in the pole approximation, the same as that elastic scattering, being determined in this model by the slope of the pomeron. Actually the slope is slightly larger than in elastic scattering as the double scattering absorptive correction is twice as strong.

In the two step process the slope at small t is half as big at small t in the pole approximation.

Thus in this model A_1 production, which has a slope $a \sim 10\ (\text{GeV})^{-2}$ at 16 GeV would be taken to be predominantly one step. On the other hand A_3 production has a slope $a \sim 6\ (\text{GeV})^{-2}$ and would therefore be predominantly two step. Further, equations (69) and (70) imply a phase difference of near 180° at $t = 0$ for one and two step processes.

At large t values the multiple scattering series look similar and one again expects expressions like that given by equation (66). A universal slope for large t diffractive production is found in $N^*_{1/2}$ production experiments.

K-regeneration and elastic K-scattering

Consider a beam of K^0_L incident on a nuclear target for the purpose of regenerating K^0_S. The amplitude for this process is given by

$$\tfrac{1}{2}\langle K^0+\overline{K}{}^0|A|K^0-\overline{K}{}^0\rangle = \tfrac{1}{2}\langle K^0|A|K^0\rangle - \tfrac{1}{2}\langle \overline{K}{}^0|A|\overline{K}{}^0\rangle \equiv \tfrac{1}{2}[F(\vec{q})-\overline{F}(\vec{q})] \tag{71}$$

by conservation of strangeness. Hence using an extension of results obtained in a preceding section, the regeneration amplitude is given by

$$F(\vec{q})-\overline{F}(\vec{q}) = \frac{ik}{2\pi}\int [e^{i\overline{P}(\vec{b})} - e^{iP(\vec{b})}] e^{i\vec{q}\cdot\vec{b}} d^2b, \tag{72}$$

$$P(\vec{b}) = \frac{2\pi}{k}\left(\frac{Z}{A}f_p + \frac{N}{A}f_n\right) T(\vec{b}). \tag{72a}$$

$$\bar{P}(\vec{b}) = \frac{2\pi}{k}\left(\frac{Z}{A}\bar{f}_p + \frac{N}{A}\bar{f}_n\right) T(\vec{b}). \tag{72b}$$

We have assumed equal neutron and proton densities. Here

$$f_p \equiv f_{K^0p}(0) = f_{K^+n}(0);\ \bar{f}_p \equiv f_{\bar{K}^0p}(0) = f_{K^-n}(0);$$

$$f_n \equiv f_{K^0n}(0) = f_{K^+p}(0);\ \bar{f}_n \equiv f_{\bar{K}^0n}(0) = f_{K^-p}(0). \tag{73}$$

All quantities are forward scattering amplitudes. We have assumed charge independence of two body amplitudes to relate charged to neutral K-meson scattering amplitudes in (73).

Coherent regeneration of K^0_s mesons using a beam of K^0_L mesons incident on a nuclear target and measuring the time dependence of the $\pi^+\pi^-$ decay mode yields the phase [22]

$$\varphi = \arg\,[F(0)-\bar{F}(0)] + \tfrac{1}{2}\pi - \varphi_{\pm} \tag{74}$$

where

$$\varphi_{\pm} = \arg\,\eta_{\pm}\,, \qquad \eta_{\pm} = \frac{A(K^0_L \to \pi^+\pi^-)}{A(K^0_s \to \pi^+\pi^-)}.$$

At $k \sim 5$ GeV/c an experiment done with a lead target [22] yields $\varphi = -79.4° \pm 8°$.

We have calculated arg $[F(0)-\bar{F}(0)]$ assuming the following for the two body amplitudes $f_p, f_n, \bar{f}_p, \bar{f}_n$: the imaginary parts of the amplitudes are calculated from the optical theorem using measured total cross-sections for $K^{\pm}$ mesons or protons and neutrons. The real parts of $f_{K^{\pm}p}$ have been taken from a dispersion relation calculation of Horn and Yahil [23] which assumes Regge amplitudes satisfying the Pomeranchuk theorem fitted to existing data. The trajectories P, ϱ, ω, A_2 and f_0 (all but P being exchange degenerate) are included. For $f_{K^+n}(0)$, we have taken $\arg f_{K^+n}\,(0) = \arg f_{K^+p}(0)$ since $\sigma^{\text{tot}}_{K^+n} \simeq \sigma^{\text{tot}}_{K^+p}$. The real part of $f_{K^-n}(0)$ is obtained from the relation

$$\text{Re}\,[f_{K^-n}(0) - f_{K^+n}(0)] = \text{Im}\,[f_{K^-n}(0) - f_{K^+n}(0)] \tag{75}$$

which follows from the that only the $C = -1$ trajectories ω and ϱ contribute, if $\alpha_\omega(0) = \alpha_\varrho(0) = 1/2$. We obtain, using [24] $\varphi\pm = 40° \pm 6°$, $\varphi = -77° \pm 6°$ in good agreement with experiment.

On the other hand, Horn and Yahil produce another set of amplitudes which satisfy existing $K^+ - p$ total cross-section data but violate the Pomeranchuk theorem by about 20%. I will not go into detail on the nature of the assumptions but refer you to reference [23]. The corresponding phase φ in this case we find to be $\varphi \simeq -100° \pm 6°$.

Similar information can be obtained using regeneration on Hydrogen. However, the experiments have yet to be done. They are more difficult.

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