ON THE RESIDUAL INTERACTIONS OF PAIRING AND QUADRUPOLE-PAIRING TYPE

By W. Stępień-Rudzka

Institute of Nuclear Research, Warsaw*

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The residual two-body interaction is investigated. A new term is added to the standard pairing force and its influence on the single particle excitation spectra in heavy nuclei (transuranic region) is examined with special attention given to the behaviour of the so-called "energy gap", a typical parameter in spectra of even-even nuclei.

The residual two-body interactions which cannot be incorporated into the average spherical nuclear potential can be divided into two parts. The long-range part which treated in the multipole expansion retains only a few low components, modifies our average field giving multipole-multipole interactions of various types. The short-range part is approximated by the so-called pairing force that couples two particles with total angular momentum equal zero. This kind of forces is now generally used in nuclear structure calculations in the form of so called BCS formalism or independent quasi-particle approximation (Bardeen, Cooper and Schrieffer, 1957). In connection with these short-range forces one can think of a procedure analogous to the including of higher terms in the multipole expansion, namely of taking into account interactions coupling two particles with non-zero total angular momentum. We shall call the part that couples particles with total angular momentum $J = 2$ the quadrupole-pairing interaction.

We assume the Hamiltonian of the problem to be:

$$H = H_0 + H_{\text{int}} \tag{1}$$

where

$$H_{\text{int}} = H_{\text{pair}} + H_{\text{qp}} \tag{2}$$

$$H_{\text{pair}} = -\frac{G_0}{4} \sum_{\nu_1,\nu_2} a_{\nu_1}^+ a_{-\nu_1}^+ a_{-\nu_2} a_{\nu_2} \tag{3}$$

$$H_{\text{qp}} = \sum_{\tilde{\nu}_1 \ldots \tilde{\nu}_4} \langle(\nu_1\nu_2)JM|v_{\text{qp}}|\nu_3\nu_4)JM\rangle \times [a_{\nu_1}^+ a_{-\nu_2}^+]_{JM} [a_{\nu_4} a_{-\nu_3}]_{JM} \tag{4}$$

* Address: Instytut Badań Jądrowych, Warszawa, Hoża 69, Poland.

We assume further that in our problem pairing forces play a dominant role, therefore we can apply selection rules found for them:

$$\nu_2 = -\nu_1, \quad \nu_4 = -\nu_3. \tag{6}$$

This leads us to the expression for H_{qp}:

$$H_{\text{qp}} = -\frac{G_2}{4} \sum_{\nu_1,\nu_2} \tilde{q}_{\nu_1\nu_1}\tilde{q}_{\nu_2\nu_2} a^+_{\nu_1} a^+_{-\nu_1} a_{-\nu_2} a_{\nu_2} \tag{7}$$

where

$$\tilde{q}_{\nu\nu} = (-1)^{j-m} q_{\nu\nu}, \quad q_{\nu_1\nu_2} = \left\langle \nu_1 \middle| 4\sqrt{\frac{\pi}{5}}\, r^2 Y_{20} \middle| \nu_2 \right\rangle. \tag{8}$$

Total Hamiltonian becomes:

$$H = H_0 - \tfrac{1}{4} \sum_{\nu_1,\nu_2} (G_0 + G_2 \tilde{q}_{\nu_1\nu_1}\tilde{q}_{\nu_2\nu_2})\, a^+_{\nu_1} a^+_{-\nu_1} a_{-\nu_2} a_{\nu_2}. \tag{9}$$

The BCS ground state wave function in the case of an even number of particles is:

$$|\Phi_{\text{BCS}}\rangle = \prod_{\nu} (u_\nu + v_\nu a^+_\nu a^+_{-\nu}) |\text{vacuum}\rangle \tag{10}$$

where u_ν, v_ν are given from the minimization condition for the expectation value of the Hamiltonian $H' = H - \lambda N$, with N-number of particles:

$$u^2_\nu = \frac{1}{2}\left[1 + \frac{\varepsilon_\nu - \lambda}{E_\nu}\right], \quad v^2_\nu = \frac{1}{2}\left[1 - \frac{\varepsilon_\nu - \lambda}{E_\nu}\right] \tag{11}$$

$$E_\nu = \sqrt{(\varepsilon_\nu - \lambda)^2 + \Delta^2_\nu}. \tag{12}$$

The parameters Δ_ν are found from the equation:

$$\Delta_\nu = \frac{1}{2} \sum_{\nu'} \frac{\langle \nu_1, -\nu | G | \nu', -\nu' \rangle}{E_{\nu'}} \Delta_{\nu'}. \tag{13}$$

When the interaction matrix element is explicitly written we get:

$$\Delta_\nu = \frac{G_0}{2} \sum_{\nu'} \frac{\Delta_{\nu'}}{E_{\nu'}} + \frac{G_2}{2} \tilde{q}_{\nu\nu} \sum_{\nu'} \frac{\Delta_{\nu'}}{E_{\nu'}} \tilde{q}_{\nu'\nu'}. \tag{14}$$

If we introduce

$$\Delta_0 = \frac{G_0}{2} \sum_{\nu'} \frac{\Delta_{\nu'}}{E_{\nu'}}, \quad \Delta_2 = \frac{G_2}{2} \sum_{\nu'} \frac{\Delta_{\nu'}}{E_{\nu'}} \tilde{q}_{\nu'\nu'} \tag{15}$$

we get

$$\Delta_\nu = \Delta_0 + \tilde{q}_{\nu\nu'} \Delta_2$$

or

$$\Delta_0 = \frac{G_0}{2}\left(\Delta_0 \sum_\nu \frac{1}{E_\nu} + \Delta_2 \sum_\nu \frac{\tilde{q}_{\nu\nu}}{E_\nu}\right)$$

$$\Delta_2 = \frac{G_2}{2}\left(\Delta_0 \sum_\nu \frac{\tilde{q}_{\nu\nu}}{E_\nu} + \Delta_2 \sum_\nu \frac{\tilde{q}^2_{\nu\nu}}{E_\nu}\right). \quad (16)$$

Numerical calculations were performed for the nuclei in the transuranic region. We wanted to verify if the addition of this new kind of interaction can account for the increase in the energy gap parameter Δ in fissioning nuclei as compared with the ground state values of Δ. This phenomenon discovered experimentally (Britt *et al.* 1963, 1965; Griffin 1963; Koneczny *et al.* 1967; Huizenga *et al.* 1968) cannot be interpreted in terms of pairing forces only even when one assumes (Stępień and Szymański 1968) that pairing forces are of surface character *i.e.* their strength increases with deformation (constant volume of the nucleus is assumed during this kind of calculation).

The calculation consisted of numerical solving of Eqs (16) together with the particle number equation:

$$n = \sum_\nu 2v_\nu^2 \equiv \sum_\nu \left(1 - \frac{\varepsilon_\nu - \lambda}{E_\nu}\right) \quad (16a)$$

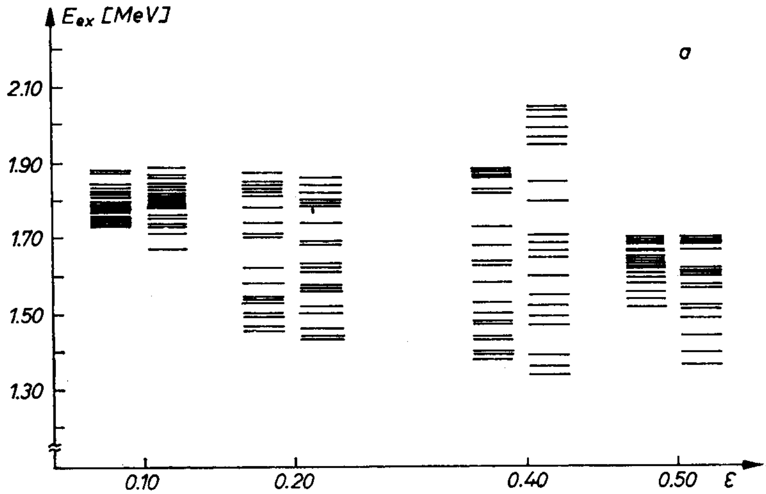
(n = number of particles, neutrons or protons) so as to get Δ_0, Δ_2 and λ. Single particle energies ε_ν were the energies of the new Nilsson model (Nilsson *et al.* 1966 and Nilsson 1967). This was done for neutrons and protons separately, for ^{236}U and ^{240}Pu nuclei. Quadrupole-pairing forces constant, G_2, was changed so as to be (0, 1/5, 1/2, 1, 1.5, 2, 2.5)% of G_0. Calculation covered the region of deformation described by the parameter ε: $0 \leqslant \varepsilon \leqslant 0.50$. The pairing forces constant, G_0, was taken as $G_0^n = 14$ MeV/A, $G_0^p = 19$ MeV/A for neutrons and protons respectively and the summation in (16) and (16a) included N (or Z) terms for neutrons (or protons).

Having found Δ_0, Δ_2 and λ we can calculate the lowest two-quasiparticle excitations:

$$E_i = E_{\nu_i} + E_{\mu_i} \quad \text{where} \quad E_{\nu_i} = \sqrt{(\varepsilon_{\nu_i} - \lambda)^2 + \Delta^2_{\nu_i}}. \quad (17)$$

The spectra of this kind of excitations are shown in Figs 1 and 2 and Table I. Fig. 1 shows twenty lowest levels of type (17) in ^{236}U for different deformations ε without (left-hand side of spectrum for each deformation) and with (right-hand side) quadrupole-pairing forces. Fig. 2 gives the same kind of excitations for different values of G_2 in ^{240}Pu (protons). These levels are also listed in Table I. One can see that quadrupole-pairing forces tend to decrease the energies of the lowest states relative to their positions in the case of pure pairing forces. Higher levels are somewhat shifted upwards but this change is too small to provide us with an increase in the "effective" energy gap.

We are prevented from using much higher strength for quadrupole-pairing forces because of the fact that the pure pairing forces are a good approximation to the delta-type forces. The latter, when compared with experimental evidence, are believed to give a good descrip-



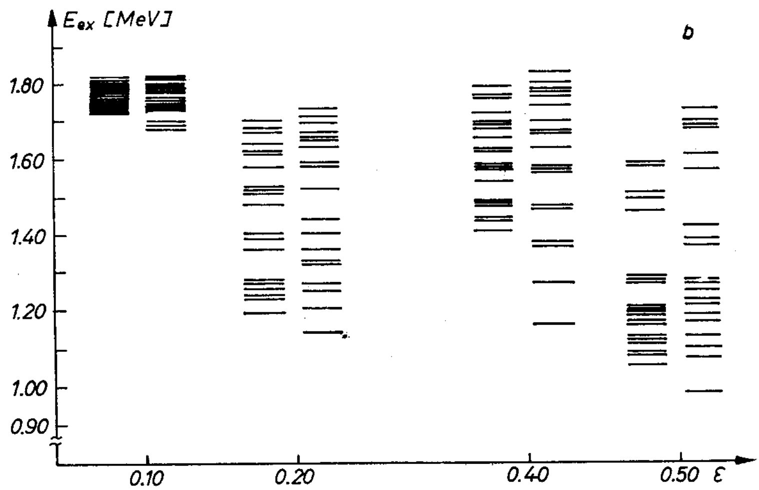


Fig. 1. Twenty lowest single-particle excitations in ^{236}U for 4 different deformations. For each ε left-hand side of spectrum is the pure pairing force case ($G_2 = 0$) whereas the right-hand side is for $G_2 = G_0/100$. *a*. protons, *b*. neutrons

tion of the short-range part of residual interaction. In that way our system has to be described mainly by pairing forces with all other terms treated as small perturbations.

In order to test the influence of the number of levels used in sums (16) on the value and behaviour of the gap parameter, the value of the expression:

$$D = \frac{\partial \Delta^2/\Delta^2}{\partial G/G} = \frac{4\sum_\nu \frac{1}{E_\nu^3}}{G\left[\left(\sum_\nu \frac{\varepsilon_\nu - \lambda}{E_\nu}\right)^2 + \Delta^2\left(\sum_\nu \frac{1}{E_\nu^3}\right)^2\right]} = \frac{4B}{G(A^2 + \Delta^2 B^2)} \tag{18}$$

was calculated in the case $G_0 = G$, $G_2 = 0$.

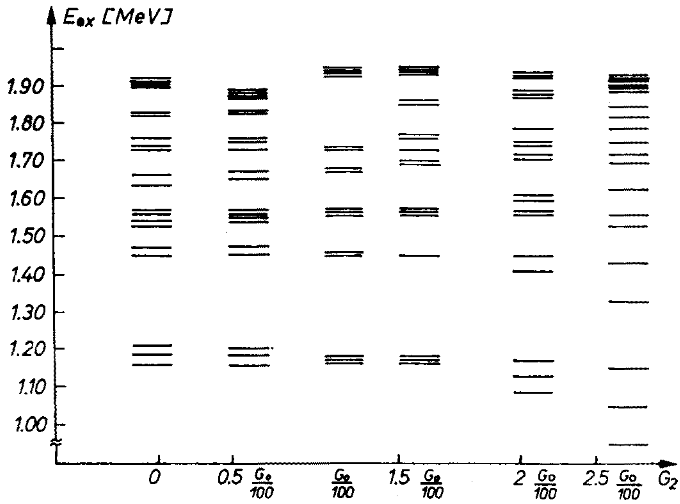


Fig. 2. Comparison of lowest excitations in ^{240}Pu (protons) for different values of quadrupole-pairing forces strength G_2

TABLE I

Twenty lowest two-quasiparticle excitations (MeV) in ^{240}Pu (protons) for different values of G_2

$G_2 = 0$	$0.5\,\frac{G_0}{100}$	$\frac{G_0}{100}$	$1.5\,\frac{G_0}{100}$	$2\,\frac{G_0}{100}$	$2.5\,\frac{G_0}{100}$
1.1642	1.1654	1.1682	1.1508	1.0910	0.9542
1.1888	1.1827	1.1748	1.1608	1.1309	1.0538
1.2134	1.2000	1.1814	1.1708	1.1708	1.1534
1.4486	1.4492	1.4499	1.4302	1.4066	1.3283
1.4732	1.4665	1.4565	1.4502	1.4465	1.4289
1.5336	1.5422	1.5562	1.5641	1.5562	1.5343
1.5449	1.5508	1.5602	1.5668	1.5694	1.5642
1.5582	1.5595	1.5628	1.5741	1.5961	1.6339
1.5695	1.5681	1.5668	1.5768	1.6093	1.7024
1.6353	1.6505	1.6692	1.6904	1.7130	1.7230
1.6599	1.6678	1.6764	1.7004	1.7222	1.7535
1.7330	1.7330	1.7316	1.7296	1.7522	1.7921
1.7403	1.7468	1.7562	1.7608	1.7529	1.8226
1.7649	1.7641	1.7628	1.7708	1.7921	1.8531
1.8180	1.8260	1.8379	1.8535	1.8717	1.8917
1.8293	1.8346	1.8419	1.8562	1.8850	1.9024
1.9030	1.9190	1.9330	1.9369	1.8957	1.9084
1.9110	1.9229	1.9396	1.9462	1.9296	1.9170
1.9143	1.9276	1.9436	1.9469	1.9356	1.9250
1.9164	1.9322	1.9442	1.9495	1.9446	1.9385

Equation (18) describes how does Δ^2 change when G is changed. The calculation was performed for several numbers of levels used in sums (18) but for each case the value of G was taken such as to reproduce the values of Δ.and λ for N (or Z) levels when inserted into (16). The results for ^{236}U are given in Table II. One can see that this renormalization of G is not enough to recompensate the influence of cutting-off some of the single particle levels.

TABLE II

D-values for different numbers of levels used in sums (18)

Neutrons: Number of levels	Neutrons: D	Protons: Number of levels	Protons: D
24	5.23177	24	4.88737
40	6.36788	40	5.90425
52	6.98909	52	6.44821
60	7.32730	60	6.72614
80	7.99029	80	7.26799
100	8.50045	92	7.44469
120	8.90625	—	—
144	9.21207	—	—

In addition the total ground state energy of ^{236}U was calculated with and without quadrupole-pairing forces:

$$E = E^n_{BCS} + E^p_{BCS} + E_{Coul}$$

$$E_{BCS} = \langle \Phi_{BCS} | H' | \Phi_{BCS} \rangle$$

$$E_{Coul} = 0.6\, Ze^2 \frac{g(\varepsilon)}{R_0}$$

$$g(\varepsilon) = (1 - \tfrac{2}{3}\varepsilon)^{\frac{2}{3}} (1 + \tfrac{1}{3}\varepsilon)^{\frac{1}{3}} \times \begin{cases} \ln \left(\dfrac{1 - \frac{2}{3}\varepsilon}{1 + \frac{1}{3}\varepsilon - \sqrt{2\varepsilon - \frac{1}{3}\varepsilon^2}} \right) \dfrac{1}{\sqrt{2\varepsilon - \frac{1}{3}\varepsilon^2}} & \varepsilon > 0 \\ \operatorname{arctg} \left(\dfrac{\sqrt{\frac{1}{3}\varepsilon^2 - 2\varepsilon}}{1 + \frac{1}{3}\varepsilon} \right) \dfrac{1}{\sqrt{\frac{1}{3}\varepsilon^2 - 2\varepsilon}} & \varepsilon < 0 \end{cases} \qquad (19)$$

TABLE III

Ground state energy in ^{236}U

ε	Ground state energy (MeV)	
	$G_2 = 0$	$G_2 = G_0/100$
0	9389.47	9389.26
0.10	9388.41	9388.33
0.20	9375.39	9375.26
0.25	9399.65	9399.50
0.35	9400.63	9400.59
0.40	9406.43	9406.62
0.45	9412.67	9412.39
0.50	9418.78	9420.80

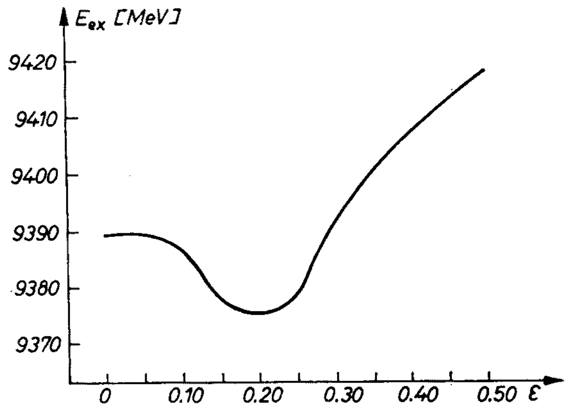


Fig. 3. Total ground state energy of ^{236}U as a function of deformation

The results, in the cases of $G_2 = 0$ and $G_2 = G_0/100$, are identical (see Table III and Fig. 3).

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