LEE MODEL WITH ADDITIONAL FERMI INTERACTION AND THE COMPOSITE NATURE OF V-PARTICLE

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The generalized Lee model with additional nonlocal four-leg $N\Theta - N\Theta$ vertex is studied. The lowest sector is solved for arbitrary choice of the cut-off functions in both vertices. A complete discussion of the $Z_1 = Z_V = 0$ limit, defining the composite V-particle, is presented.

1. *Introduction*

One of the basic problems in quantum field theory is the understanding of notion of composite objects. In Lagrangian field theory we consider a particle as composite if its field[1] operator is not included in the original[2] Lagrangian. This procedure can be realized if we set the wave renormalization constant Z_3 for composite field operator equal to zero [1]. If we use the mixed interaction of composite-composite and composite-elementary type, it was conjectured by Salam [2] that the condition $Z_1 = 0$ removes the interaction of composite-elementary type, and we are left with a Lagrangian formulation[3] of a completely bootstraped world of particles. A complete classification of particles from the point of view of their compositeness properties has been proposed firstly by Ida [3]. Following Ida, in QFT one can introduce four categories of particles: superelementary ($Z_1 = 0$, $Z_3 \neq 0$), elementary ($Z_1 \neq 0$, $Z_3 \neq 0$), intermediate ($Z_1 \neq 0$, $Z_3 = 0$) and the composite one ($Z_1 = 0$, $Z_3 = 0$).

The compositeness conditions have been investigated intensively on the example of the Lee model [4]. Models were considered either with the conventional $VN\Theta$ vertex or

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[1] We call a field operator φ a field of the particle A with mass m (for simplicity we consider the spinless case), if the Kallen-Lehmann representation for the two-point function has in its spectral function the Dirac delta $\delta(\varkappa^2 - m^2)$, separated from other points of the mass spectrum.

[2] This sentence applies as well to the Hamiltonian formulation of QFT — it remains valid of we replace the word "Lagrangian" by "Hamiltonian".

[3] The condition $Z_1 = Z_3 = 0$ was also interpreted as the Reggeization condition for the elementary particle. See P. E. Kaus, Zachariasen, *Phys. Rev.*, **138**, B1304 (1965); M. Ida, *Progr. Theor. Phys.*, **34**, 990 (1965).

with the Fermi coupling $N\Theta N\Theta$, and their equivalence in the limit $g_0^2 \to \infty$, $\mu_0^2 \to \infty$, $\frac{g_0^2}{\mu_0^2}$ finite has been discussed [5]. It is easy to see, however, that in order to study the composite case one should consider the case of mixed Yukawa and Fermi coupling, because if only Yukawa coupling is present, we have $Z_1 = 1$, and in the Fermi case $Z_V = 1$. However the case of mixed coupling has been discussed by some authors [6]; the solution of the model with arbitrary cut-off functions is not known in the literature.

This paper has the following two aims:

a) To solve the lowest sector for the case when the nonlocality in both vertices are described by different cut-off functions $f(k)$ and $h(k)$,

b) to discuss the compositeness criterion $Z_1 = Z_3 = 0$.

In all our calculations the cut-off function is assumed such that all the integrals which occur in the $N\Theta - N\Theta$ scattering amplitude and in the formulae for the renormalization constants are finite.

The plan of our paper is the following. In Section 2 we define the model. In Section 3 we discuss the physical V-particle state. We calculate the formula for the mass renormalization and discuss the local limit $f(k) \to 1$, $h(k) \to 1$ for both cut-off functions. Besides we give the formula for the wave renormalization constant Z_3. In Section 4 we calculate the scattering amplitude for the case $f(k) = h(k)$. The case $f(k) \neq h(k)$ is considered in the Appendix. In Section 5 we discuss the charge renormalizations, and we express the $N\Theta - N\Theta$ scattering amplitude completely in terms of the renormalized parameters. In Section 6 we discuss the condition $Z_1 = Z_3 = 0$. It appears that in the case of mixed interaction the vanishing of Z_3 implies necessarity vanishing charge renormalization constant Z_1. One gets the result that the composite V particle is coupled in a definite way to the $N\Theta$ pair, but the self-interaction of elementary N and Θ particles remains undetermined, *i.e.* one parameter is left free. In particular one can choose as a free parameter the mass renormalization constant for the V-particle.

2. *The model*

We consider the following Hamiltonian

$$H = H_0 + H_{\text{int}} \tag{2.1}$$

where

$$H_0 = m_{V_0}\int d^3\vec{p}\, V^+(\vec{p})V(\vec{p}) + m_{N_0}\int d^3\vec{p}\, N^+(\vec{p})N(\vec{p}) + \int d^3\vec{k}\,\omega(k)\Theta^+(\vec{k})\Theta(\vec{k}) \tag{2.2}$$

$$H_{\text{int}} = \frac{g_0}{(2\pi)^{3/2}} \int \frac{d^3\vec{k} f(\vec{k})}{(2\omega(k))^{1/2}} \int d^3\vec{p}\,\{V^+(\vec{p})N(\vec{p}-\vec{k})\Theta(\vec{k}) + \text{H.C.}\} -$$

$$- \frac{\lambda_0}{(2\pi)^3} \iint \frac{d^3\vec{k}_1 d^3\vec{k}_2 h(\vec{k}_1)h(\vec{k}_2)}{(4\omega(k_1)\omega(k_2))^{1/2}} \int d^3\vec{p}\, N^+(\vec{p}-\vec{k}_1)\Theta^+(\vec{k}_1)N(\vec{p}-\vec{k}_2)\vec{\theta}(k_2) \tag{2.3}$$

an $[N(\vec{p}), N^+(\vec{p}')] = [V(\vec{p}), V^+(\vec{p})] = \delta(\vec{p}-\vec{p}')$

$$[\Theta(\vec{k}), \Theta^+(\vec{k}')] = \delta(\vec{k}-\vec{k}'). \tag{2.4}$$

We denote $k=|\vec{k}|$, $\omega(k)=(k^2+\mu^2)^{1/2}$ and assume that the form factors $f(k)$ and $h(k)$ are real. The local limit corresponds to the choices

$$f(k)\to 1 \tag{2.5a}$$

$$h(k)\to 1. \tag{2.5b}$$

It is easy to see that in the presence of additional $N\Theta-N\Theta$ interaction the following basic properties of the conventional Lee model are preserved:

a) The physical vacuum is defined by means of the relation

$$N(\vec{p})|0\rangle=\Theta(\vec{p})|0\rangle=V(\vec{p})|0\rangle=0. \tag{2.6}$$

b) The physical one-particle states are given by the following vectors:

$$N\text{-particle:}\quad |N(\vec{p})\rangle=N^+(\vec{p})|0\rangle \tag{2.7a}$$

$$\Theta\text{-particle:}\quad |\Theta(\vec{p})\rangle=\Theta^+(\vec{p})|0\rangle \tag{2.7b}$$

$$V\text{-particle:}\quad |V(\vec{p})\rangle$$

$$=Z_V^{1/2}\{V^+(\vec{p})|0\rangle+\int d^3\vec{k}\,\Phi(\vec{k})N^+(\vec{p}-\vec{k})\Theta^+(\vec{k})\}. \tag{2.7c}$$

c) The $N-\Theta$ scattering is elastic.

3. *The mass and wave function renormalization for V-particle*

The physical mass m_V of V-particle is determined by the equation

$$H|V(\vec{p})\rangle=m_V|V(\vec{p})\rangle. \tag{3.1}$$

Using (2.7c) and (2.3)–(2.4) one gets the following two equations

$$m_{V_0}+\frac{g_0}{(2\pi)^{1/2}}\int\frac{d^3\vec{k}f(k)}{(2\omega(k))^{1/2}}\,\Phi(k)=m_V \tag{3.2a}$$

$$[m_V-m_{N_0}-\omega(k)]\Phi(k)=\frac{g_0}{(2\pi)^{3/2}}\,\frac{f(k)}{(2\omega(k))^{1/2}}-$$

$$-\frac{\lambda_0}{(2\pi)^3}\,\frac{h(k)}{(2\omega(k))^{1/2}}\int\frac{d^3\vec{k}_1}{(2\omega(k_1))^{1/2}}\,h(k_1)\Phi(k_1). \tag{3.2b}$$

Substituting in the equation (3.2b) the following Ansatz:

$$\Phi(k)=\frac{g_0}{(2\pi)^{3/2}}\,\frac{1}{(2\omega(k))^{1/2}(m_V-m_N-\omega(k))}\left[f(k)-\frac{\lambda_0}{(2\pi)^3}\,h(k)K\right] \tag{3.3}$$

we get after a straightforward calculation

$$K=\frac{\displaystyle\int d^3\vec{k}\,\frac{f(k)h(k)}{2\omega(k)\,(m_V-m_N-\omega(k))}}{\displaystyle 1+\frac{\lambda_0}{(2\pi)^3}\int d^3\vec{k}\,\frac{h^2(k)}{2\omega(k)\,(m_V-m_N-\omega(k))}} \tag{3.4}$$

and finally

$$\delta_{m_V} = m_V - m_{V_0} = \frac{g_0^2}{(2\pi)^3} \int d^3\vec{k}\, \frac{f^2(k)}{2\omega(k)\,(m_V - m_N - \omega(k))} -$$

$$- \frac{\lambda_0}{(2\pi)^3} \frac{g_0^2}{(2\pi)^3} \frac{\left[\int d^3\vec{k}\, \frac{f(k)h(k)}{2\omega(k)\,(m_V - m_N - \omega(k))}\right]^2}{1 + \frac{\lambda_0}{(2\pi)^3} \int d^3\vec{k}\, \frac{h^2(k)}{2\omega(k)\,(m_V - m_N - \omega(k))}}. \tag{3.5}$$

It is easy to see that if $\lambda_0 = 0$, we get the conventional formula for the mass renormalization in the usual Lee model

$$\delta m_V = \frac{g_0^2}{(2\pi)^3} \int d^3\vec{k}\, \frac{f^2(k)}{2\omega(k)\,(m_V - m_N - \omega(k))} \tag{3.6}$$

which is linearly divergent in the limit (2.5a). If we assume that $g_0 = 0$, one gets of course that $\delta m_V = 0$ because the V-particle is decoupled from $N\Theta$ pairs. We shall consider now the local limit (2.5) for the case when $\lambda_0 \neq 0$ and $g_0 \neq 0$. Let us assume that

$$f(k) = \Theta(\Lambda^2 - k^2)$$

$$h(k) = \Theta(\Lambda'^2 - k^2) \tag{3.7}$$

where

$$\lim_{\substack{\Lambda'^2 \to \infty \\ \Lambda'^2 \to \infty}} \frac{\Lambda^2}{\Lambda'^2} = a^2. \tag{3.8}$$

If $h(k)$ tends to the local limit faster than $f(k)$ *e.g.* $a^2 < 1$ we get the following expression for the mass renormalization:

$$\lim_{\Lambda'^2 > \Lambda^2 \to \infty} \delta m_V = -\frac{g_0^2}{(2\pi)^2} (1 - a^2)\Lambda^2. \tag{3.9}$$

In the opposite case $a^2 > 1$ it is easy to check that

$$\lim_{\Lambda^2 > \Lambda'^2 \to \infty} \delta m_V = -\frac{g_0^2}{(2\pi)^2} (a^2 - 1)\Lambda^2. \tag{3.10}$$

We see, therefore, that if $a^2 \neq 1$, δm_V is always linearly divergent. Only in the particular case $a^2 = 1$ δm_V achieves a finite value

$$\delta m_V = \frac{g_0^2}{\lambda_0} \tag{3.11}$$

and its sign depends on the sign of λ_0.

Thus we have shown that the mass renormalization constant depends on the way we perform the local limits (2.5).

It is interesting to compare the result (3.11) with the dispersion — theoretic calculations. It has been already recognized [7] that the local Lee model in the $N\Theta$ sector can be treated as a particular nonrelativistic limit of the Zachariasen model [8] with one coupling constant. This analogy remains valid if we introduce also the local Fermi vertex. In the Zachariasen model with two coupling constants g_0 and λ_0, the mass renormalization term is given by the formula (3.11). One can conclude, therefore, that the local limit, which is implicitly assumed if we write the dispersion relation, corresponds to the choice $a^2 = 1$ of the parameter (3.8).

The wave renormalization constant of the physical V-particle is obtained from the requirement:

$$\langle V(\vec{p}) | V(\vec{p}\,')\rangle = \delta(\vec{p}-\vec{p}\,').$$

From the formula (2.7) one gets

$$Z_V^{-1} = 1 + \int d^3\vec{k}\,\Phi^2(k). \tag{3.12}$$

Using the Eq. (3.3) and (3.7)in the case $f = h$ we obtain after a straightforward calculation, the following simple expression

$$Z_V = \frac{(1+\lambda_0 A)^2}{(1+\lambda_0 A)^2 + g_0^2 B} \tag{3.13}$$

where

$$A = \frac{1}{(2\pi)^3}\int d^3\vec{k}\,\frac{f^2(k)}{2\omega(k)\,(m_V - m_N - \omega(k))} \tag{3.14a}$$

$$B = \frac{1}{(2\pi)^3}\int d^3\vec{k}\,\frac{f^2(k)}{2\omega(k)\,(m_Y - m_N - \omega(k))^2}. \tag{3.14b}$$

4. $N-\Theta$ *scattering*

We shall now calculate the S-matrix element which describes the elastic scattering of the $N-\Theta$ pair. This element has the form

$$\langle N(\vec{q}\,'), \Theta(\vec{k}') | S | N(\vec{q}), \Theta(\vec{k})\rangle = {}_-\langle N(\vec{q}\,'), \Theta(\vec{k}') | N(\vec{q}), \Theta(\vec{k})\rangle_+ \tag{4.1}$$

where $|N(\vec{q}), \Theta(\vec{k})\rangle_\pm$ are the eigenstates of the total Hamiltonian

$$H|N(\vec{q}), \Theta(\vec{k})\rangle_\pm = (\omega(k) + m_N)|N(\vec{q}), \Theta(\vec{k})\rangle_\pm. \tag{4.2}$$

The index "+" indicates the outgoing wave, and "−" means the incident one.

Performing standard simple calculations (see *e.g.* [9]) we have

$$|N(\vec{q}), \Theta(\vec{k})\rangle_\pm = |N(\vec{q}), \Theta(\vec{k})\rangle + \frac{g_0}{(2\pi)^{3/2}}\frac{f(k)}{(2\omega(k))^{1/2}}\frac{1}{m_N+\omega(k)-H\pm i\varepsilon}V^+(\vec{q}+\vec{k})|0\rangle -$$

$$-\frac{\lambda_0}{(2\pi)^3}\frac{h(k)}{(2\omega(k))^{1/2}}\frac{1}{m_N+\omega(k)-H\pm i\varepsilon}\int d^3\vec{k}_1\,\frac{h(k_1)}{(2\omega(k_1))^{1/2}}N^+(\vec{k}+\vec{q}-\vec{k}_1)\,\Theta^+(\vec{k}_1)|0\rangle \tag{4.3}$$

and one obtains

$$
{}_{-}\langle N(\vec{q}\,'), \Theta(\vec{k}\,')|N(\vec{q}), \Theta(\vec{k})\rangle_{+} = \delta(\vec{q}-\vec{q}\,')\delta(\vec{k}-\vec{k}\,') + \\
+2\pi i\delta(\omega(k)-\omega(k'))\langle N(\vec{q}\,'), \Theta(\vec{k}\,')|R|N(\vec{q}), \Theta(\vec{k})\rangle \tag{4.4}
$$

where

$$
\begin{aligned}
\langle N(\vec{q}\,'), \Theta(\vec{k}\,')|R|N(\vec{q}), \Theta(\vec{k})\rangle &= \frac{\lambda_0}{(2\pi)^3}\frac{h(k)h(k')}{2\omega(k)}\,\delta(\vec{k}\,'+\vec{q}\,'-\vec{k}-\vec{q}) - \\
&- \frac{\lambda_0^2}{(2\pi)^6}\frac{h(k)h(k')}{2\omega(k)}\iint d^3\vec{k}_1 d^3\vec{k}_2\,\frac{h(k_1)h(k_2)}{(4\omega(k_1)\,\omega(k_2))^{1/2}}\times \\
&\times \langle 0|N(\vec{k}\,'+\vec{q}\,'-\vec{k}_2)\,\Theta(\vec{k}_2)\,\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,N^{+}(\vec{k}+\vec{q}-\vec{k}_1)\,\Theta^{+}(\vec{k}_1)|0\rangle + \\
&+ \frac{2g_0}{(2\pi)^{3/2}}\frac{\lambda_0}{(2\pi)^3}\frac{h(k)f(k)}{2\omega(k)}\int d^3\vec{k}_1\,\frac{h(k_1)}{(2\omega(k_1))^{1/2}}\times \\
&\times\langle 0|N(\vec{k}\,'+\vec{q}\,'-\vec{k}_1)\,\Theta(\vec{k}_1)\frac{1}{m_N+\omega(k_1)-H+i\varepsilon}\,V^{+}(\vec{k}+\vec{q})|0\rangle - \\
&- \frac{g_0^2}{(2\pi)^3}\frac{f^2(k)}{2\omega(k)}\,\langle 0|V(\vec{k}\,'+\vec{q}\,')\,\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,V^{+}(\vec{k}+\vec{q})|0\rangle.
\end{aligned} \tag{4.5}
$$

If we take into consideration Eq. (2.7c) and use the fact that $|V(\vec{k})\rangle$ is an eigenstate of the Hamiltonian (Eq. (3.1)) it is easy to see that we have:

$$
\begin{aligned}
\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,V^{+}(\vec{k}+\vec{q})|0\rangle &= -\frac{1}{m_N+\omega(k)-H+i\varepsilon}\int d^3\vec{k}_1\Phi(\vec{k}_1)|N(\vec{k}+\vec{q}-\vec{k}_1), \Theta(\vec{k}_1)\rangle + \\
&+ \frac{1}{m_N+\omega(k)-m_V+i\varepsilon}\int d^3\vec{k}_1\Phi(\vec{k}_1)|N(\vec{k}+\vec{q}-\vec{k}_1), \Theta(\vec{k}_1)\rangle + \\
&+ \frac{1}{m_N+\omega(k)-m_V+i\varepsilon}\,V^{+}(\vec{k}+\vec{q})|0\rangle.
\end{aligned} \tag{4.6a}
$$

Now, from the identity (see [13])

$$
\frac{1}{x-H+i\varepsilon}\,\Theta^{+}(\vec{k}) = \Theta^{+}(\vec{k})\,\frac{1}{x-H-\omega(k)+i\varepsilon} + \frac{1}{x-H+i\varepsilon}\,[H_I, \Theta^{+}(\vec{k})]\,\frac{1}{x-H-\omega(k)+i\varepsilon}
$$

we obtain:

$$
\begin{aligned}
\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}_1), \Theta(\vec{k}_1)\rangle &= \frac{1}{\omega(k)-\omega(k_1)+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}_1), \Theta(\vec{k}_1)\rangle + \\
&+ \frac{1}{\omega(k)-\omega(k_1)}\,\frac{1}{m_N+\omega(k)-H+i\varepsilon}\left\{\frac{g_0}{(2\pi)^{3/2}}\,\frac{f(k_1)}{(2\omega(k_1))^{1/2}}\,V^{+}(\vec{k}+\vec{q})|0\rangle - \right. \\
&\left. - \frac{\lambda_0}{(2\pi)^3}\,\frac{h(k_1)}{(2\omega(k_1))^{1/2}}\int d^3\vec{k}\,'\,\frac{h(k')}{(2\omega(k'))^{1/2}}\,|N(\vec{k}+\vec{q}-\vec{k}\,'), \Theta(\vec{k}\,')\rangle\right\}.
\end{aligned} \tag{4.6b}
$$

This is a pair of integral equations, which one can solve after simple but tedious manipulations (see Appendix). Here we shall discuss only the case $f(k) = h(k)$. Then we obtain for the scattering matrix a relatively simple expression, namely

$$\langle N(\vec{q}'),\, \Theta(\vec{k}')|R|N(\vec{q}),\, \Theta(\vec{k})\rangle = -\delta(\vec{q}'+\vec{k}'-\vec{q}-\vec{k})\,\langle N(\vec{q}'),\, \Theta(\vec{k}')|M|N(\vec{q}),\, \Theta(\vec{k})\rangle$$

$$= -\frac{\delta(\vec{q}'+\vec{k}'-\vec{q}-\vec{k})}{2\omega(k)G(k)} f^2(k) \left\{ \frac{g_0^2}{(2\pi)^3}\,\frac{1}{m_N+\omega(k)-m_V} - \frac{\lambda_0}{(2\pi)^3}\,(1+\lambda_0 A)\right\} \tag{4.7}$$

where

$$G(k) = (1+\lambda_0 A)\left\{1+\frac{\lambda_0}{(2\pi)^3}\int d^3\vec{k}_1\,\frac{f^2(k_1)}{2\omega(k_1)\,(\omega(k)-\omega(k_1)+i\varepsilon)}\right\} +$$

$$+\frac{g_0^2}{(2\pi)^3}\int d^3\vec{k}_1\,\frac{f^2(k_1)}{2\omega(k_1)\,(m_V-m_N-\omega(k_1))\,(\omega(k)-\omega(k_1)+i\varepsilon)}. \tag{4.8}$$

5. *The renormalization procedure*

In this Section we shall replace the non-renormalized quantities g_0, λ_0 by finite renormalized values g, λ. We introduce the renormalized three-leg coupling g^2 by means of the following requirement:

$$\operatorname*{Res}_{\omega(k)=m_V-m_N}\,[2\omega(k)\,\langle N(\vec{k}'),\, \Theta(\vec{q}')|M|N(\vec{k}),\, \Theta(\vec{q})\rangle] = f^2(m_V-m_N)\,\frac{g^2}{(2\pi)^3}. \tag{5.1}$$

The renormalized coupling constant λ can be introduced by means of the $\frac{N}{D}$ decomposition of the scattering amplitude. Normalizing the denominator function at the point $\omega(k) = m_V-m_N$ to unity, one defines λ as the constant determining the asymptotic behaviour $k \to \infty$ of $N(k)$ in the local limit $f(k) \to 1$.

Let us consider firstly g^2. One gets, using (4.7)–(4.8) and (5.1), after simple calculations

$$g^2 = \frac{g_0^2}{(1+\lambda_0 A)^2+g_0^2 B} \tag{5.2}$$

or equivalently

$$g_0^2 = \frac{g^2(1+\lambda_0 A)^2}{1-g^2 B}. \tag{5.3}$$

Because in the $V-N\Theta$ vertex only the V-line is renormalized by interaction, one defines the charge renormalization constant Z_1 as follows:

$$g_0 = Z_1\cdot Z_V^{-1/2} g. \tag{5.4}$$

Using (3.13) and (4.4) one gets

$$Z_1 = 1+\lambda_0 A. \tag{5.5}$$

The renormalized coupling constant can be defined by means of the formula:

$$\lambda = \frac{\lambda_0(1+\lambda_0 A)}{(1+\lambda_0 A)^2+g_0^2 B} \tag{5.6}$$

or

$$\lambda_0 = \frac{\lambda}{1-\lambda A-g^2 B}. \tag{5.7}$$

The charge renormalization of the four-leg vertex can be expressed in terms of Z_V and Z_1 as follows:

$$\lambda_1 = \frac{Z_V}{Z_1}\lambda_0. \tag{5.8}$$

The physical scattering amplitude can be expressed in the $\frac{N}{D}$ form as follows:

$$\langle N(\vec{q}'), \Theta(\vec{k}')|R|N(\vec{q}), \Theta(\vec{k})\rangle = -\frac{\delta(\vec{k}'+\vec{q}'-\vec{k}-\vec{q})}{2\omega(k)}\frac{N(k)}{D(k)} \tag{5.9}$$

where

$$N(k) = \frac{f^2(k)}{(2\pi)^3}\left[\frac{g^2}{m_N+\omega(k)-m_V}-\lambda\right] \tag{5.10}$$

and

$$D(k) = 1-g^2B-\lambda A+\int d^3\vec{k}_1\frac{N(k_1)}{2\omega(k_1)(\omega(k)-\omega(k_1)+i\varepsilon)}. \tag{5.11}$$

We can easily check up that

$$D(k)|_{\omega(k)=m_V-m_N} = 1 \tag{5.12}$$

in consistency with (5.1).

Finally, one can express the mass renormalization term δm_V and wave renormalization constant Z_V in terms of the physical parameters. One gets (see (3.5))

$$\delta m_V = \frac{g^2 A}{1-g^2B-\lambda A} \tag{5.13}$$

$$Z_V = 1-g^2B. \tag{5.14}$$

Besides, one can write

$$Z_1 = \frac{1-g^2B}{1-\lambda A-g^2B} \tag{5.15}$$

and also

$$\delta m_V = \frac{g^2}{\lambda}(1-Z_1). \tag{5.16}$$

6. *The compositeness criteria for the V-particle*

Let us consider firstly the case when $|A| < \infty$, $|B| < \infty$, particularly if

$$\int f^2(k)dk < \infty. \tag{6.1}$$

In this case all the formulae in Section 4 are well defined. We shall assume that $f(k)$ is fixed and satisfies the condition (6.1).

We see from the formulae (3.13) and (5.6) that if $Z_V \to 0$, then necessarily $Z_1 \to 0$. Let us consider such a case. We obtain from (5.5)

$$\lambda_0 = -\frac{1}{A} \tag{6.2}$$

and from (5.14) it follows that

$$g^2 = \frac{1}{B}. \tag{6.3}$$

The parameter g_0^2 remains undetermined. The limiting properties of the ratio $\frac{Z_1^2}{Z_V}$ are known. We have

$$\frac{Z_1^2}{Z_V} = (1+\lambda_0 A)^2 + g_0^2 B \to g_0^2 B \tag{6.4}$$

and the relation (5.4) becomes an identity for any choice of g_0^2. We have, therefore, one free parameter in the theory. One may also chose λ or δm_V as such a free parameter. Assuming in particular that λ is non-vanishing, we obtain from (5.8) that

$$\frac{Z_1}{Z_V} = \frac{\lambda_0}{\lambda} = -\frac{1}{\lambda A} \tag{6.5}$$

and

$$g_0^2 \xrightarrow[Z_1 \to 0]{} -\frac{Z_1}{\lambda AB} \to 0. \tag{6.6}$$

The case $\lambda = 0$ corresponds to the choice $g_0^2 \to \infty$.

The mass renormalization δm_V in the limit $Z_1 \to 0$ is given by the formula

$$\delta_{mV} = \frac{g^2}{\lambda}. \tag{6.7}$$

We see that in the composite particle limit the mass renormalization is expressed in the same way by the renormalized coupling constants as by the bare parameters in the local limit $f(k) \to 1$ (see (3.11)), and that it cannot be fixed by the compositeness conditions.

Let us now consider the particular cases $\lambda_0 = 0$ and $g_0 = 0$. If $\lambda_0 = 0$ one gets necessarily $Z_1 = 1$. From the formula for Z_V one obtains

$$Z_V|_{\lambda_0=0} = \frac{1}{1+g_0^2 B}. \tag{6.8}$$

One gets $Z_V|_{\lambda_0=0} \to 0$ if $g_0^2 \to \infty$. In such a case again

$$g^2|_{\lambda_0=0} \to \frac{1}{B} \tag{6.9}$$

and $\delta m_V \to \infty$. According to the classification proposed in the *Introduction*, the V-particle is an intermediate one.

If $g_0 = 0$ one obtains always $Z_V = 1$. One gets $Z_1 = 0$ if λ_0 is given by the formula (6.2). We obtain in such a case $\lambda \to \infty$, which corresponds to the unrealistic case of an infinite physical coupling constant. We should assume, therefore, that $Z_1 \neq 0$ and then one finds that the V-particle is elementary.

Finally, one can study the local limit $f(k) \to 1$. Because, however, in this case A is linearly and B logarithmically divergent, one gets $Z_V \to -\infty$ and we have to deal with the problem of ghost states. This last difficulty lead to the introduction of the cut-off function $f(k)$, satisfying the condition (5.1).

APPENDIX

$N\Theta$-scattering amplitude if $f \neq h$

Let us consider the pair of the integral equations (4.6). We perform a reduction of this system to the one equation substituting Eq. (4.6a) into Eq. (4.6b). Then one obtains:

$$\begin{aligned} &\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}'),\,\Theta(\vec{k}')\rangle \\ &= A(\vec{k},\vec{q};\vec{k}')+g(k;k')\int d^3\vec{k}_1\Phi(k_1)\,\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}_1),\Theta(\vec{k}_1)\rangle+ \\ &+l(k;k')\int d^3\vec{k}_1 s(k_1)\,\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}_1,\,\Theta(\vec{k}_1)\rangle \end{aligned} \tag{A.1}$$

where we denoted:

$$\begin{aligned} A(\vec{k},\vec{q};\vec{k}') &= \frac{1}{\omega(k)-\omega(k')+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}',\,\Theta(\vec{k}')\rangle+ \\ &+\frac{g_0}{(2\pi)^{3/2}}\,\frac{f(k')}{(2\omega(k'))^{1/2}}\,\frac{1}{(\omega(k)-\omega(k')+i\varepsilon)\,(m_N+\omega(k)-m_V+i\varepsilon)}\,\{V^+(\vec{k}+\vec{q})|0\rangle+ \\ &+\textstyle\int d^3k_1\Phi(k_1)|N(\vec{k}+\vec{q}-\vec{k}_1),\,\Theta(\vec{k}_1)\rangle\} \end{aligned}$$

$$g(k;k') = -\frac{g_0}{(2\pi)^{3/2}}\,\frac{f(k')}{(2\omega(k'))^{1/2}}\,\frac{1}{\omega(k)-\omega(k')+i\varepsilon}$$

$$l(k;k') = -\frac{\lambda_0}{(2\pi)^3}\,\frac{h(k')}{(2\omega(k'))^{1/2}}\,\frac{1}{\omega(k)-\omega(k')+i\varepsilon}$$

$$s(k) = \frac{h(k)}{(2\omega(k))^{1/2}}.$$

We substitute

$$\frac{1}{m_N+\omega(k)-H+i\varepsilon}\,|N(\vec{k}+\vec{q}-\vec{k}'),\,\Theta(\vec{k}')\rangle = A(\vec{k},\vec{q};\vec{k}')+g(k,k')K_1(\vec{k},\vec{q})+l(k;k')K_2(\vec{k},\vec{q}) \tag{A.2}$$

where K_1 and K_2 are some unknown functions. Because generally $g(k;k')$ and $l(k;k')$ are linearly independent functions, we obtain two algebraic equations for K_1 and K_2 which have the solution in a form:

$$K_1(\vec{k},\vec{q}) = -\frac{1}{D(k)}\int d^3\vec{k}_1[R_{\Phi l}(k)s(k_1)+R_{sl}(k)\Phi(k_1)]A(\vec{k},\vec{q};\vec{k}_1)$$

$$K_2(\vec{k},\vec{q}) = -\frac{1}{D(k)}\,d^3\vec{k}_1[R_{sg}(k)\Phi(k_1)+R_{\Phi g}(k)s(k_1)]A(\vec{k},\vec{q};\vec{k}_1) \tag{A.3}$$

where

$$\begin{aligned}
R_{\Phi l}(k) &= \int d^3\vec{k}_1\Phi(k_1)l(k;k_1)\\
R_{sg}(k) &= \int d^3\vec{k}_1 s(k_1)g(k;k_1)\\
R_{sl}(k) &= 1-\int d^3\vec{k}_1 s(k_1)l(k;k_1)\\
R_{\Phi g}(k) &= 1-\int d^3\vec{k}_1\Phi(k_1)g(k;k_1)\\
D(k) &= R_{\Phi l}(k)R_{sg}(k)-R_{\Phi g}(k)R_{sl}(k).
\end{aligned}$$

After some tedious calculations we obtain the S-matrix for the elastic $N\Theta-N\Theta$ scattering in such a form:

$$\begin{aligned}
{}_-\langle N(\vec{q}'),\,\Theta(\vec{k}')|N(\vec{q}),\,\Theta(\vec{k})\rangle_+ &= \delta(\vec{q}-\vec{q}')\,\delta(\vec{k}-\vec{k}')+\\
+2\pi i\delta(\omega(k)-\omega(k'))\,\delta(\vec{k}'+\vec{q}'-\vec{k}-\vec{q})\,\frac{1}{D(k)}&\left\{\frac{g_0^2}{(2\pi)^3}\frac{f^2(k)}{2\omega(k)}\frac{R_{sl}(k)}{m_N+\omega(k)-m_V+i\varepsilon}+\right.\\
+\frac{2g_0}{(2\pi)^{3/2}}\frac{\lambda_0}{(2\pi)^3}\frac{f(k)h(k)}{2\omega(k)}\frac{R_{sg}(k)}{m_N+\omega(k)-m_V+i\varepsilon}&-\frac{\lambda_0}{(2\pi)^3}\frac{h^2(k)}{2\omega(k)}R_{\Phi g}(k)-\\
&\left.-\frac{\lambda_0^2}{(2\pi)^6}\frac{g_0}{(2\pi)^{3/2}}\frac{h^2(k)}{2\omega(k)}\frac{R_{sg}(k)}{m_N+\omega(k)-m_V+i\varepsilon}K\right\}
\end{aligned} \tag{A.4}$$

K in the last term this is the constant which had appeared in the formula for $\Phi(k)$ (see (3.4)).

Finally one gets for $D(k)$ the following result:

$$\begin{aligned}
D(k) = &-\frac{g_0^2}{(2\pi)^3}\frac{\lambda_0}{(2\pi)^3}\frac{1}{m_N+\omega(k)-m_V+i\varepsilon}\left[\int d^3\vec{k}_1\frac{h(k_1)f(k_1)}{2\omega(k_1)\,(\omega(k)-\omega(k_1)+i\varepsilon)}\right]^2-\\
&-\left\{1+\frac{\lambda_0}{(2\pi)^3}\int d_3\vec{k}_1\frac{h^2(k_1)}{2\omega(k_1)\,(\omega(k)-\omega(k_1)+i\varepsilon)}\right\}\left\{1+\right.\\
&+\frac{g_0^2}{(2\pi)^3}\int d^3\vec{k}_1\frac{f^2(k_1)}{2\omega(k_1)\,(m_V-m_N-\omega(k_1))\,(\omega(k)-\omega(k_1)+i\varepsilon)}-\\
&\left.-\frac{\lambda_0}{(2\pi)^3}\frac{g_0^2}{(2\pi)^3}\frac{K}{m_N-m_V+\omega(k)+i\varepsilon}\int d^3\vec{k}_1\frac{f(k_1)h(k_1)}{2\omega(k_1)\,(m_V-m_N-\omega(k_1))}\right\}.
\end{aligned} \tag{A.5}$$

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