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THE UNIVERSAL REGULATOR?

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By using the Goldstone's theorem, a Universal Regulator is discovered, *i.e.* the cosmic field being the Yang Mills field of the local scale transformations of space-time coordinates. Its physical role is as follows. Cosmon, the quantum of cosmic field, is a vanishing rest mass, spinless and neutral particle. It is a Universal particle in the sense that it participates in all the physical processes, carrying the interactions in the acausal region G_l and moreover, if its presence is taken into account, then interactions become microcausal and the theory is free of all the ultraviolet infinities.

At the present time, to consider whether a Universal Regulator actually exists constitutes a problem of principal value for the development of particle physics.

Recently, by a paper of De Witt [1] the following problem is arosen: Gravity, the Universal Regulator? Next, Salam and Strathdee have shown [2] that Gravity, in the simplest case of a scalar graviton, is a regulator in the perturbation approach.

Following Markov [3], it is possible that the so-called friedmon is a regulator. From our point of view, this idea is very interesting. However, it is necessary to clarify the role of friedmon in quantum theory of particles.

The purpose of this paper is to prove that it is possible that such a Universal Regulator exists.

1

The nonlocal theory of quantized fields and of elementary particles, which is free of the ultraviolet infinities without any regularization, has been constructed [4–11]. The two fundamental hypothesis of that theory are the following:

I. The macrocausality principle formulated by Efimov [12]

$$\frac{\delta}{\delta\varphi(x)}\left(\frac{\delta S}{\delta\varphi(y)}\, S^{-1}\right)=0 \tag{1.1}$$

outside the region G and G_l:

$$G: x^0 \geqslant y^0 \;\; (x-y)^2 > 0,$$

$$G_l: \; -l^2 \leqslant (x-y)^2 \leqslant l^2.$$

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II. The nonlocality is introduced by a following way.

The operators of free field are local, but in the interaction they are replaced by

$$\varphi(x) \rightarrow \Phi(x) = \int d^4y V(x-y)\varphi(y) = V(l^2\Box_x)\varphi(x)$$

where formfactors $V(l^2z)$ are generalized analytical functions of z.

Let us now study the groups of transformations leaving invariant the macrocausality (1.1) and the nonlocality (1.2).

First of all, it is easily seen that outside the Lorentz group, there is yet a group of transformations of space-time coordinates leaving invariant (1.1) and (1.2), *i.e.* the group of scale transformations:

$$x^\mu \rightarrow 'x^\mu = \eta x^\mu,$$
$$l \rightarrow 'l = \eta l,$$
$$m \rightarrow 'm = \eta^{-1} m \tag{1.3}$$

and then

$$\varphi(x) \rightarrow U\varphi(x)U^{-1} = \eta\varphi(x')$$

where η is an arbitrary positive number.

As it is known, this one-parameter group played a very interesting role in studying the vanishing rest mass particle, such as the neutrino [13], as well as the unified field theory of Heisenberg [14].

Therefore, in our theory, there two Universal groups in the sense that all the physical processes are invariant with respect to them.

In our view, in order to estimate the importance of the role of the group of scale transformations (1.3) it is necessary to base on the Goldstone's theorem [15] which pronounces that the degeneration of the fundamental state reduces to the existence of bosons with vanishing rest masses.

Thus, the asymmetry of the fundamental state with respect to the Lorent group reduces to the existence of gravitons and the asymmetry of fundamental state with respect to group (1.3) reduces to the existence of another vanishing rest mass boson. This boson is called cosmon.

Cosmons play the role similar to that of gravitons: they participate in all the physical processes since all these processes are invariant under the group of scale transformations (1.3) as well as under the Lorentz group.

Their role will be clarified by means of the following assumption:

In taking into account the existence of cosmons, the nonlocality of auto-interaction of scalar field $\varphi(x)$ is replaced by the locality of interaction of scalar field $\varphi(x)$ and the cosmic field, and then the theory becomes microcausal.

This fundamental position of our theory will be proved below.

2

It was known that the Yang-Mills field of local Lorentz transformations is the gravitational field [16, 17]. Let us now the Yang-Mills field of local scale transformations (1.3).

The transformations (1.3) can be rewritten in the following form:

$$x^\varrho \rightarrow 'x^\varrho = e^\eta x^\varrho \tag{2.1}$$

(2.1) is equivalent to the following transformations on the metric tensor

$$\overset{\circ}{g}_{\mu\nu} \rightarrow {}'\overset{\circ}{g}_{\mu\nu} = e^{2\eta}\, \overset{\circ}{g}_{\mu\nu}$$

where $\overset{\circ}{g}_{\mu\nu}$ is the metric tensor of Galileo. As it is known, (2.2) are called conformal transformations.

Next, η is considered to be an certain function of space-time coordinates; that is the transformations (2.1) become

$$x^{\varrho} \rightarrow {}'x^{\varrho} = a(s)x^{\varrho}$$

here $a(s)$ is a certain real differentiable function of $s = (t - r^2)^{\frac{1}{2}}$. These transformations are equivalent to

$$ds^2 \rightarrow e^{2\chi(s)}\, ds^2 \tag{2.3}$$

or

$$\overset{\circ}{g}_{\mu\nu} \rightarrow {}'\overset{\circ}{g}_{\mu\nu} = e^{2\chi(s)} \overset{\circ}{g}_{\mu\nu} \tag{2.4}$$

where

$$e^{2\chi(s)} = \left(\frac{\dot{a}}{a}\, s + a\right)^2 > 0.$$

In order to seek the Yang-Mills field corresponding to (2.3), let us, for simplicity, consider the vector field V^{ϱ}. It is easily seen that under the transformations (2.3), the ordinary derivative of vector V^{σ} is replaced by a covariant derivative, as follows:

$$V^{\sigma}_{,\mu} \rightarrow V^{\sigma}_{;\mu} = V^{\sigma}_{,\mu} + A^{\sigma}_{\mu\nu} V^{\nu}. \tag{2.5}$$

The transformation law of $A^{\mu}_{\nu\varrho}$ under (3.3) is as follows:

$$A^{\nu}_{\mu\sigma} \rightarrow {}'A^{\nu}_{\mu\varrho} = A^{\nu}_{\mu\varrho} - (-\delta^{\nu}_{\mu}\chi_{\varrho} - \delta^{\nu}_{\varrho}\chi_{\mu} + \overset{\circ}{g}_{\mu\varrho} g^{\nu\sigma}\chi_{\sigma}). \tag{2.6}$$

Here

$$\chi_{\sigma} = \frac{\partial}{\partial x^{\sigma}}\, \chi.$$

From (2.5) and (2.6) we deduce that $A^{\mu}_{\nu\varrho}$ transform as the Christoffel symbols of a certain Riemannian space:

$$ds^2 = G_{\mu\nu}(x) dx^{\mu} dx^{\nu} \tag{2.7}$$

under the conformal transformations

$$G_{\mu\nu}(x) \rightarrow {}'G_{\mu\nu}(x) = e^{+2\chi(s)} G_{\mu\nu}(x).$$

Moreover, the scale transformations (2.3) conserve the symmetry of space-time of special relativity theory. Therefore, the Riemannian space (2.7) needs to have that symmetry. This implies that its metric has the form:

$$ds^2 = H(x)(dt^2 - dr^2)$$

where $H(x)$ is a certain real scalar function of space-time coordinates.

Following Goldstone's theorem, the quanta of Yang-Mills field corresponding to (2.3) need to have a vanishing mass; therefore the transformation law of the potential $\varphi(x)$ of this field must have the form:

$$\varphi(x) \rightarrow {}'\varphi(x) = \varphi(x) + \frac{1}{l_1}\chi(s) \tag{2.8}$$

which is similar to the gauge transformations of the second kind if the potential is electromagnetic.

In order to do this, $H(x)$ must have the form:

$$H(x) = e^{-2l_1\varphi(x)} \tag{2.9}$$

here l_1 is a certain constant which can be identified with l and $\varphi(x)$ is potential of Yang--Mills field.

To summarize, the Yang-Mills field of local scale transformations is given by

$$ds^2 = e^{-2l\varphi(x)}(dt^2 - dr^2). \tag{2.10}$$

That is the cosmic field, the existence of which is characterized by a non-galilean metric of space-time.

Thus, at present time, it is possible to say that in nature there are two fields whose existences are characterized by the metric of space-time. One of them is the well-known Lorentz group and the other the cosmic field which plays the role, as will be seen below, of a Universal Regulator.

3

The wave equation of the "free" cosmic field is given

$$\Box\, \varphi(x) = 0 \tag{3.1}$$

in which $\varphi(x)$ transforms as

$$\varphi(x) \rightarrow {}'\varphi(x) = \varphi(x) + \frac{1}{l}\chi(s) \tag{3.2}$$

here

$$\Box\chi(s) = 0.$$

It is clear that the cosmon, the quantum of the cosmic field, is a vanishing rest mass, spinless and neutral particle.

Its propagator in the interaction representation is given as follows

$$D^c(x-y) = \underbrace{\varphi(x)\varphi(y)} = \frac{1}{(2\pi)^4}\int \frac{e^{ikx}}{k^2 - i\varepsilon}\, d^4k. \tag{3.3}$$

However, we shall need only the "effective" propagator[1] of this field, defined as

$$G(x-y) = \langle e^{-\bar{l}\varphi(x)}, e^{-\bar{l}\varphi(y)}\rangle_+ \quad (\bar{l} = 2l)$$

$$= \sum_{m,n} \frac{(-1)^m}{m!}\frac{(-1)^n}{n!}\bar{l}^{m+n}\langle\varphi^m(x), \varphi^n(y)\rangle_+. \tag{3.4}$$

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[1] Following Salam's terminology, it is called the super-propagator [18].

Following [2],

$$\langle \varphi^m(x), \varphi^n(y)\rangle_+ = m!\delta^{mn}(\langle \varphi(x), \varphi(y)\rangle_+)^m$$

we obtain

$$G(x-y) = e^{\bar{l}^2 D^c(x-y)}. \tag{3.5}$$

Let us notice that $G(x)$ is not singular in the light cone. In effect, by using the explicit form of

$$D^c(x) = \frac{1}{4\pi^2}\left\{\pi\delta(x^2) - P\frac{1}{x^2}\right\}$$

we obtain

$$G(x) = \exp\frac{\bar{l}^2}{4\pi^2}\left\{\pi\delta(x^2) - P\frac{1}{x^2}\right\}. \tag{3.6}$$

(3.6) shows that $G(x)$ tends to zero very quickly as $x^2 \to 0$.

Its Fourier transform is given

$$G(x) = \int e^{ipx} G(p) d^4p \tag{3.7}$$

and therefore

$$G(p) = \int e^{ipx} G(x) d^4x$$

or

$$G(p) = \int e^{ipx} \exp\frac{\bar{l}^2}{4\pi^2}\left\{\pi\delta(x^2) - \frac{P}{x^2}\right\} d^4x. \tag{3.8}$$

Let us note that $\delta(x^2)$ has no remarkable contribution to the integral, it can thus be neglected and $G(p)$ becomes

$$G(p) = \int e^{ipx} \exp\left\{-\frac{\bar{l}^2}{4\pi^2 x^2}\right\} d^4x. \tag{3.9}$$

From (3.7) and (3.9) one deduces that for an arbitrary positive number k we obtain

$$\lim (p^2)^k G(p) = 0 \quad \text{as } p^2 \to \infty. \tag{3.10}$$

A detailed study of the propagator $G(p)$ will be presented in the next paper.

4

Similarly to the gravitational field, the cosmic field interacts with all the other fields following an unique way.

For instance, in the presence of the cosmic field, the interaction Lagrangian of two fields $Q(x)$ and $R(x)$ had the form:

$$\mathcal{L}_I = \sqrt{-g}\, L_I(Q, R) = e^{-\bar{l}\varphi(x)} L_I(Q, R) \tag{4.1}$$

where $L_I(Q, R)$ is the interaction Lagrangian of Q and R fields in the absence of cosmic field.

By [19, 20], this is a non-polynomial interaction.

By (4.1) it is seen that in reality the participation of cosmic field into the interactions is characterized always not by $\varphi(x)$ but by the "effective" potential $\exp \bar{l}\varphi(x)$.

In order to establish the physical meaning of the effective potential, let us field the interacting radius of the cosmic field.

To do that, we consider the cosmic field to have the "static" potential. Then we obtain

$$\Delta\varphi(\boldsymbol{r}) = 0$$

from which one deduces that the spherically symmetrical potential created by a point--source has the form

$$\varphi(x) = +\frac{1}{r}$$

and consequently, the static effective potential takes the form

$$\Phi(r) = \exp\left(-\frac{\bar{l}}{r}\right). \tag{4.2}$$

It is seen that the interacting radius of this potential is equal to $\bar{l}$!

Hence, although the cosmon is a vanishing rest mass particlè, its acting radius is finite (equal to $\bar{l}$) when it interacts with other fields.

This fact illustrates the above mentioned assumption that the cosmon is an object carrying the interaction in the acausal region G_l and if its presence is taken into account, thè interaction becomes microcausal.

Next, let us consider the role of cosmons in the scattering processes. By using the interaction Lagrangian (4.1) we obtain

$$S = T \exp\{\textstyle\int e^{-\bar{l}\varphi} L_I(Q, R) d^4x\}. \tag{4.3}$$

By means of the Feynman diagrams we obtain

$Q(x)$ denoted by ————

$R(x)$ denoted by — — — —

$\Phi(x)$ denoted by 

Then, in principle, following the convergence of the diagrams, we classify them into two type of diagrams, diagrams to have at least an external cosmic line and diagrams without any external cosmic line:

I *II*

However, basing on the obtained results of *2*, the diagrams of type *I* are eliminated, since the free cosmons have vanishing rest masses, and they participate in the interaction as particles with rest masses equal $\bar{l}^{-1}$. In this sense, it is possible to say that they play the role of the background of all the physical processes.

Now, let us prove that all the matrix elements of the diagrams of type *II* are convergent. Indeed, let us write the so-called index of diagrams $\omega(G)$ of Bogolyubov [21]:

$$\omega(G) = \sum_i \omega_i + 4 - \tfrac{1}{2} \sum_{L_{\text{ext}}} (r_{L_{\text{ext}}} + 2), \tag{4.4}$$

where, in the first term the summation is over all the vertexes and in the second term, over all the external lines. Quantities ω_i are called the indexes of i-th vertex, defined as follows

$$\omega_i = \tfrac{1}{2} \sum_j (r_j + 2) - 4 \tag{4.5}$$

the summation is taken over all the lines entering the j-th vertex and r_j is related to the rate of increase of the propagator for a given particle:

$$(p^2)^{\frac{r+2}{2}} \, \Pi(p) = 0(1) \quad \text{for} \quad p^2 \to \infty \tag{4.6}$$

where $\Pi(p)$ is certain propagator.

From (3.10), (4.4), (4.5) and (4.6) we deduce that for all diagrams of type *II* we obtain $\omega(G) < 0$; this implies that all these diagrams converge.

To summarize, if the presence of cosmic field is taken into account, then the theory is free from the ultraviolet infinities.

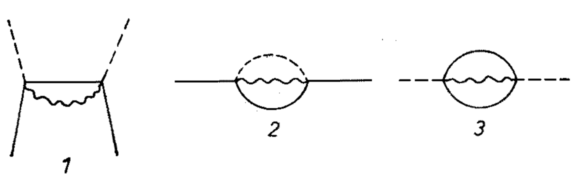
As an exemple, let us study quantum electrodynamics. Its interaction Lagrangian is given by

$$\mathscr{L}_{em} = ie\bar{\psi}\hat{A}\psi e^{-\bar{l}\varphi}$$

and therefore, S-matrix is defined as

$$S = T \exp \{ie \int \bar{\psi}\hat{A}\psi e^{-\bar{l}\varphi} d^4x\}.$$

The diagrams of type *II* in the second approximation of perturbation theory are as follows



1 — represents Compton scattering,

2 — represents the so-called self-energy of electron,

3 — represents the so-called self-energy of photon.

As it is known, in "ordinary" quantum electrodynamics two diagrams *1* and *2* give divergent matrix elements. Now, let us examine one of them, for instance, diagram *2*. We have

$$S_2 = -e^2 \int \gamma^\sigma S^c(x) \gamma^\sigma D^c(x) G(x) d^4x. \tag{4.7}$$

In the neighbourhood of the light cone

$$D^c_{\mu\nu}(x) \approx -\overset{\circ}{g}_{\mu\nu} \frac{1}{x^2}, \tag{4.8}$$

$$S^c(x) = \frac{\gamma_\alpha x^\alpha}{(x^2)^2} + \frac{m}{x^2} + \text{less singular terms} \tag{4.9}$$

and

$$G(x) \approx \exp\left\{-\frac{\bar{l}^2}{4\pi^2 x^2}\right\}. \tag{4.10}$$

Substituting (4.8), (4,9) and (4.10) into (4.7) we obtain

$$\frac{\delta m}{m} \approx e^2 \int \frac{1}{(x^2)^2} \exp\left\{-\frac{\bar{l}^2}{4\pi^2 x^2}\right\} d^4x \tag{4.11}$$

which is free from the ultraviolet infinity.

For the self-energy of photon, an analoguous result is also obtained.

5

As was already mentioned, the presence of the cosmic and gravitational fields is characterized by the non-Galilean metrics of space-time:

$$ds^2 = e^{-2l\varphi(x)}(dt^2 - dr^2), \tag{5.1}$$

and

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \quad (x^0 = t). \tag{5.2}$$

In the nature, these two fields exist "parallelly" and participate into all the physical processes. But, in principle, they have entirely different properties:

1. The cosmon is spinless and the graviton is a particle of spin 2.
2. The cosmon plays an important role at very small distances whereas the graviton plays the role at very large distances.
3. The cosmic field cannot be eliminated in any way as opposed to the gravitational field, possible to eliminate in the neighbourhood of a given point by using an adequate transformation of coordinates.

It would be very interesting to study the connection between the cosmic field and the gravitational field.

6

In order to conclude, let us discuss some points about the obtained results.

Firstly, it is seen that if the existence of the cosmic field is a reality, then it is not necessary to build the nonlocal theory, since, in fact, the nonlocality of interactions in the absence of cosmic field is equivalent to the locality of interactions in the presence of this field.

Next, the connection between nonlocality of interactions and cosmons seems to have a certain analogy to that between the negativeness of energy and the positron in Dirac's theory. The holes in the energy background are replaced by the holes in space-time background of interaction.

Finally, it is possible to say that the cosmons of our theory are probably connected in some way with friedmons of the theory of Markov.

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