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ANALYSIS OF THE ELASTIC SCATTERING OF ALPHA PARTICLES

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An analysis was carried out of the elastic scattering of 25 MeV alpha particles measured for 24 elements. Experimental angular distributions were reproduced by the Frahn-Venter models, Springer-Harvey models, and the optical model. The models were compared and the angular range applicable for each model and the reproducibility of the σ_R values were examined. For the Springer-Harvey models ambiguity was found, and sets of parameters without physical significance were noted. The geometrical parameters of nuclei were calculated, and the energy dependence of parameters was observed.

1. *Introduction*

For the scattering of alpha particles, the experimental values of angular distribution (differential cross-section *versus* scattering angles) $\sigma(\Theta)$ and the total reaction cross-section σ_R were analysed on the basis of several elastic scattering models reported further.

The differential scattering cross-section is determined by the scattering amplitude $F(\Theta)$. For scattering of charged particles

$$F(\Theta) = f_c(\Theta) + f(\Theta), \tag{1}$$

where

$$f(\Theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) e^{2i\sigma_l}(\eta_l - 1) P_l(\cos \Theta). \tag{2}$$

In these formulae $f_c(\Theta)$ is the Coulomb scattering amplitude and σ_l are the Coulomb phase shifts.

Nuclear reflection coefficients η_l are parametrized in the strong absorption models or computed from an optical potential. With their values obtained from the angular distri-

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bution by a fitting procedure, the total reaction cross-section was calculated from the following formula:

$$\sigma_R = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1-|\eta_l|^2). \tag{3}$$

The ratio $\sigma(\Theta)$ to the Rutherford cross-section $K(\Theta)$ is given in the figures showing angular distributions.

2. *Strong absorption models*

Different strong absorption models assume different parametrizations in order to describe η_l. All of them take into consideration terms of sums over l (formulae (2), (3)) for $l \leqslant l_{max}$ only.

In the Springer-Harvey five parameter model [22], [23]:

$$\eta_l = f(l) + A\Delta \frac{df}{dl} + l\left[B\Delta \frac{df}{dl} + \Delta^2 C \frac{d^2 f}{dl^2}\right], \tag{4}$$

where

$$f(l) = \left(1 + \exp \frac{L-l}{\Delta}\right)^{-1}. \tag{5}$$

Parameters A and C are equal zero in the Springer-Harvey three parameter model. The values of parameters L, Δ, B or L, Δ, B, A, C are obtained from fitting the calculated angular distribution to the experimental one.

In the Frahn-Venter [13] five parameter model:

$$\begin{aligned} \mathrm{Re}\ \eta_l &= g(t) + \varepsilon_1[1-g(t)], \\ \mathrm{Im}\ \eta_l &= \mu g(t) + \varepsilon_2[1-g(t)], \end{aligned} \tag{6}$$

where

$$t = l+1/2, \quad T = L+1/2, \quad g(t) = \left(1 + \exp \frac{T-t}{\Delta}\right)^{-1}.$$

Parameters ε_1 and ε_2 are equal zero in the Frahn-Venter three parameter model. For both parametrizations, with three or five parameters, Frahn and Venter obtained closed-form expressions for $\sigma(\Theta)/K(\Theta)$ and σ_R by means of analytical treatment in which values of L and Δ are related to geometrical parameters: interaction radius r_F and width of the surface zone d. The parameters to be fitted are: r_F, d, μ or r_F, d, μ, ε_1, ε_2.

3. *The optical model*

According to the assumption of the model, the interaction of the alpha particle with the target nucleus is described by the following potential

$$V = (U + i \cdot W) f(r), \tag{7}$$

where

$$f(r) = \left(1 + \exp\frac{r - r_0 A_T^{1/3}}{a}\right)^{-1}. \tag{8}$$

The optical potential is added to the Coulomb potential which is taken as the potential of a uniformly charged sphere with radius $r_c = 1.34$ fm.

The values of η_l needed for the calculation of angular distributions are obtained by solving the Schrödinger equation. The parameters to be fitted are: U, W, r_0, a.

4. *Analysis of scattering of 25 MeV alpha particles*

An analysis of the large experimental material consisting of the results of measurements of $\sigma(\Theta)$ [1],[2],[7],[8],[10],[11],[24] and σ_R [9],[24] obtained for different elements using the 25 MeV alpha particle beam from the 120 cm cyclotron of the Institute of Nuclear Physics in Cracow was carried out. The best fits of the predictions of each model to the experimental angular distribution were obtained by varying the parameters of the model. The quality of the fits was estimated either visually in the Frahn-Venter model or with the χ^2/N test in the Springer-Harvey and optical models. These models give the absolute value of the differential cross-section. Whenever the difference between the absolute values of the differential cross-section predicted by a model and the experimental values was found to be greater than six per cent, the experimental values were normalized by the correction factor AH [15]:

$$AH = \left\{\sum_{i=1}^{N}\left[\frac{\sigma^{\text{calc}}(\Theta_i)}{\Delta\sigma(\Theta_i)}\right]^2\right\}\left\{\sum_{i=1}^{N}\frac{\sigma^{\text{calc}}(\Theta_i)\cdot\sigma^{\text{exp}}(\Theta_i)}{[\Delta\sigma(\Theta_i)]^2}\right\}^{-1}, \tag{9}$$

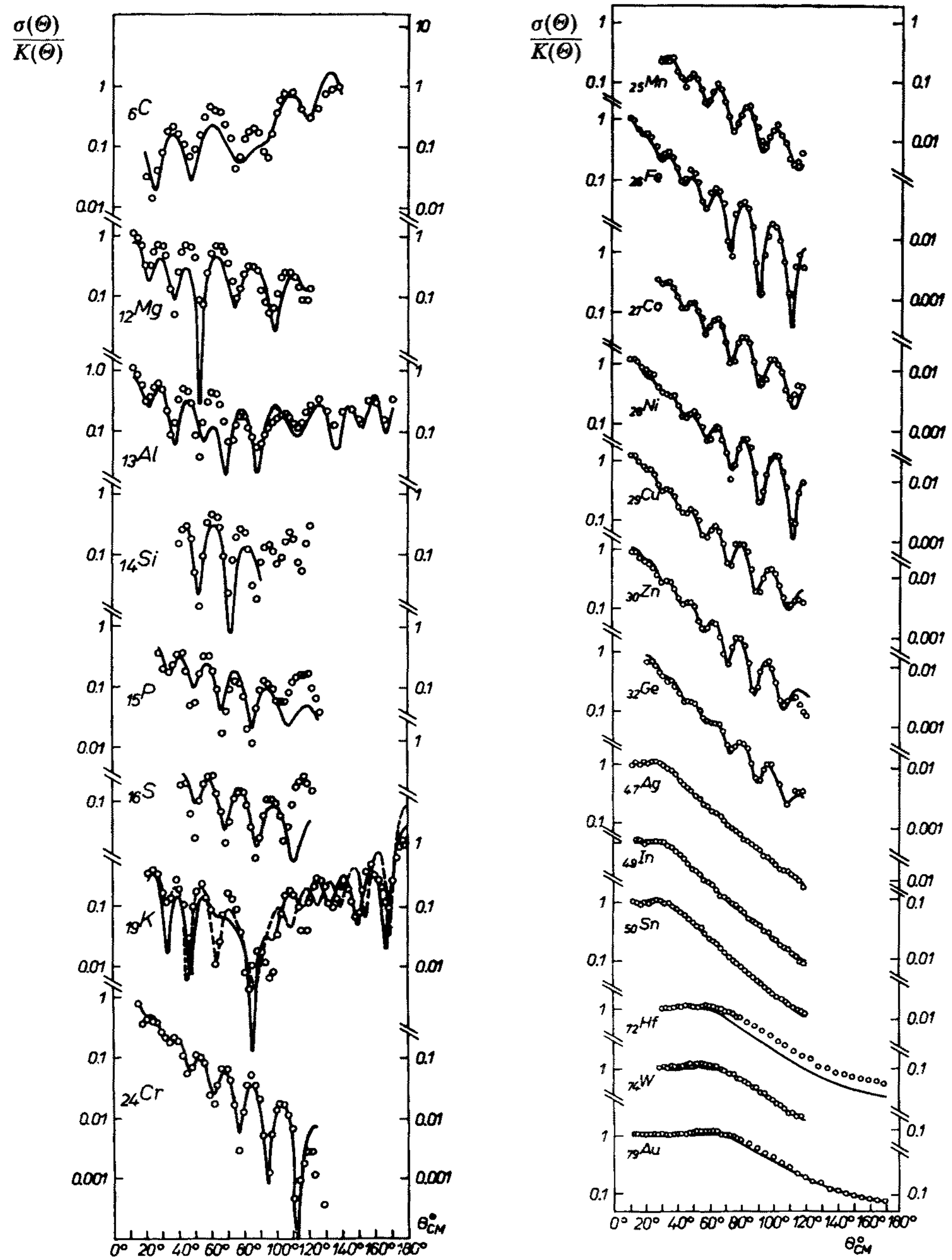
where $\Delta\sigma(\Theta_i)$ indicates the error of the experimental value at the angle Θ_i.

In the analysis GIER, URAL-II, ODRA-1003, and UMC-1 computers were used. Tables of the sets of optimum parameter values and further details of analysis are given in Ref. [18] and Ref. [19]. Fits to experimental angular distributions are shown in Fig. 1. (optical model), Fig. 2 (three and five parameter Springer-Harvey models), and Fig. 3 (three parameter Frahn-Venter model). The experimental values of the differential cross-section are marked with dots or circles in the figures presented in this paper.

With values of η_l obtained in the analysis of angular distributions the values of σ_R were calculated. Values of σ_R both given by different models and measured are shown in Fig. 4 and presented in the tables of Ref. [18] and Ref. [19]. Values of σ_R are very close, although the experimental ones were not used in the fitting procedure.

5. *Discussion*

Variations of the reflection coefficients with the angular momentum (*i.e.* shapes of reflections coefficients) of the strong absorption models and the optical model are very close (Fig. 5) when obtained by fitting to the same experimental data.



a b

Fig. 1*a*, *b*. Optical model fits to experimental angular distributions; sets of optimum parameter values with U ≈ 200 MeV were chosen. Fits to the K(α, α) K data were performed up to 75°LAB (broken curve) and up to 177.5°LAB (solid curve). For Cu, data of Ref. [10] were multiplied by 0.80

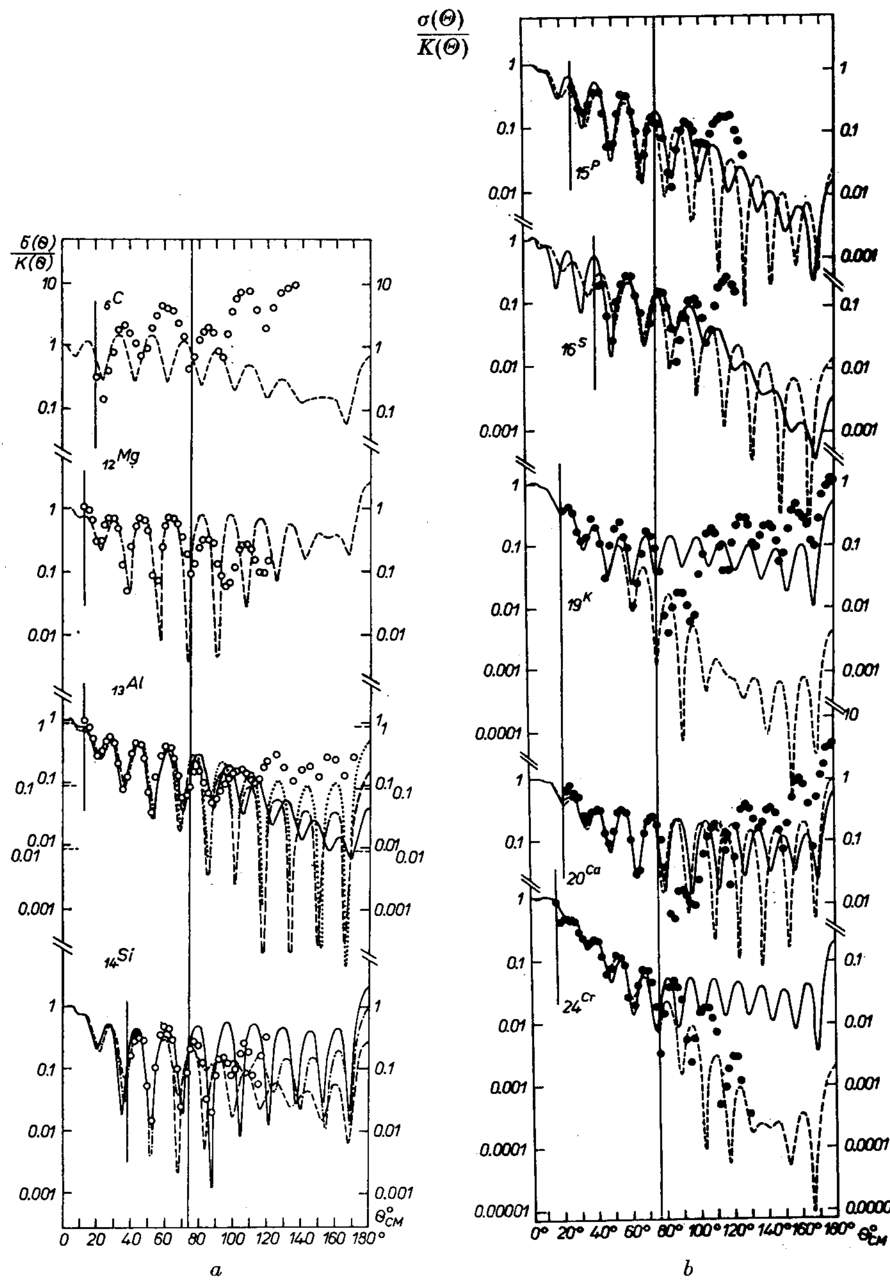


Fig. 2*a*, *b*. Springer-Harvey models fits to experimental angular distributions. All broken curves present the three parameter model fits. For Al the dotted curve presents three parameter model fit to non-normalised data; other fits were carried out for data of Ref. [10] multiplied by 0.883. For Si ambiguity of the model was found; fit with greater value of *B* parameter (Table I) is presented as dot-dash curve. Solid curves present Springer-Harvey five parameter model fits. Vertical lines show angular fitting ranges

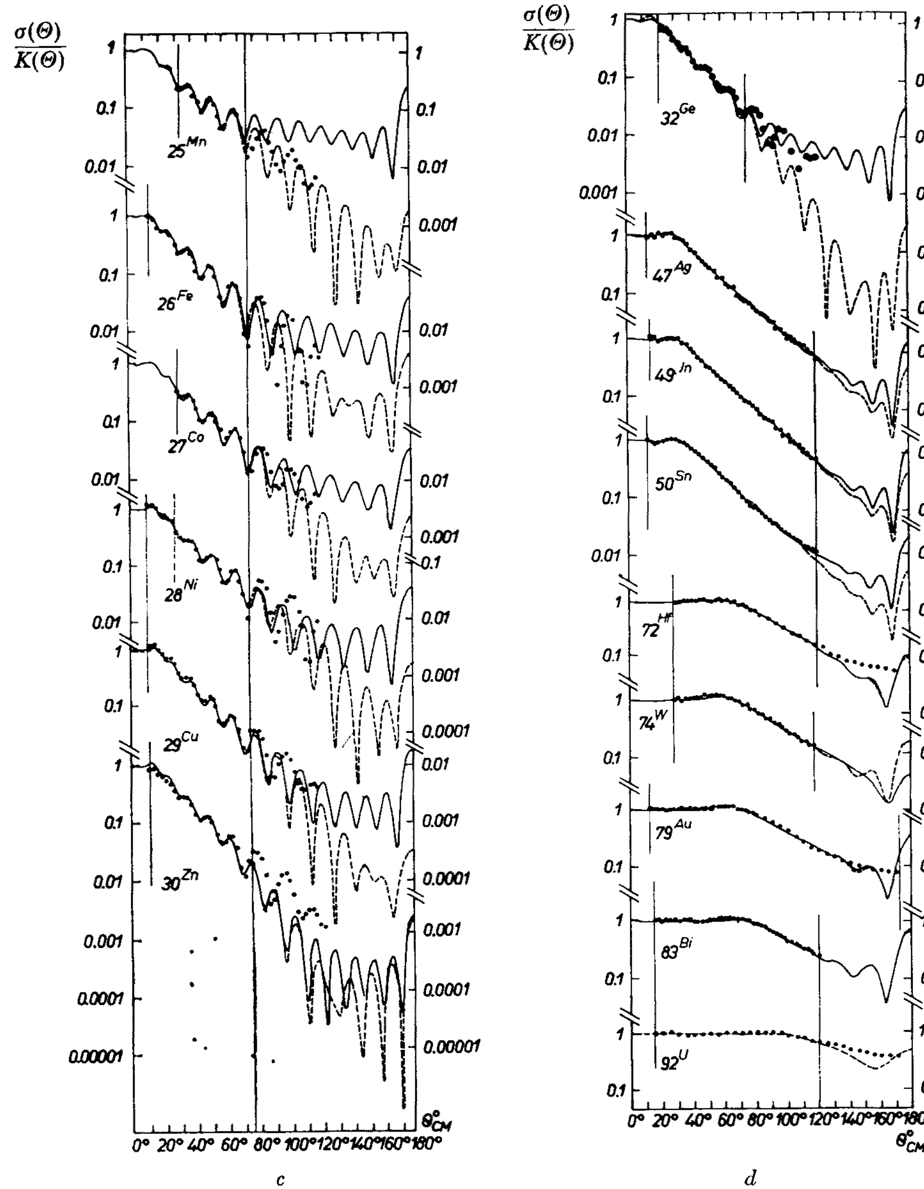


Fig. 2c, d. Springer-Harvey three (broken curves) and five (solid curves) parameter model fits to experimental angular distributions. For Cu, data of Ref. [10] were multiplied by 0.820. Vertical lines show angular fitting ranges

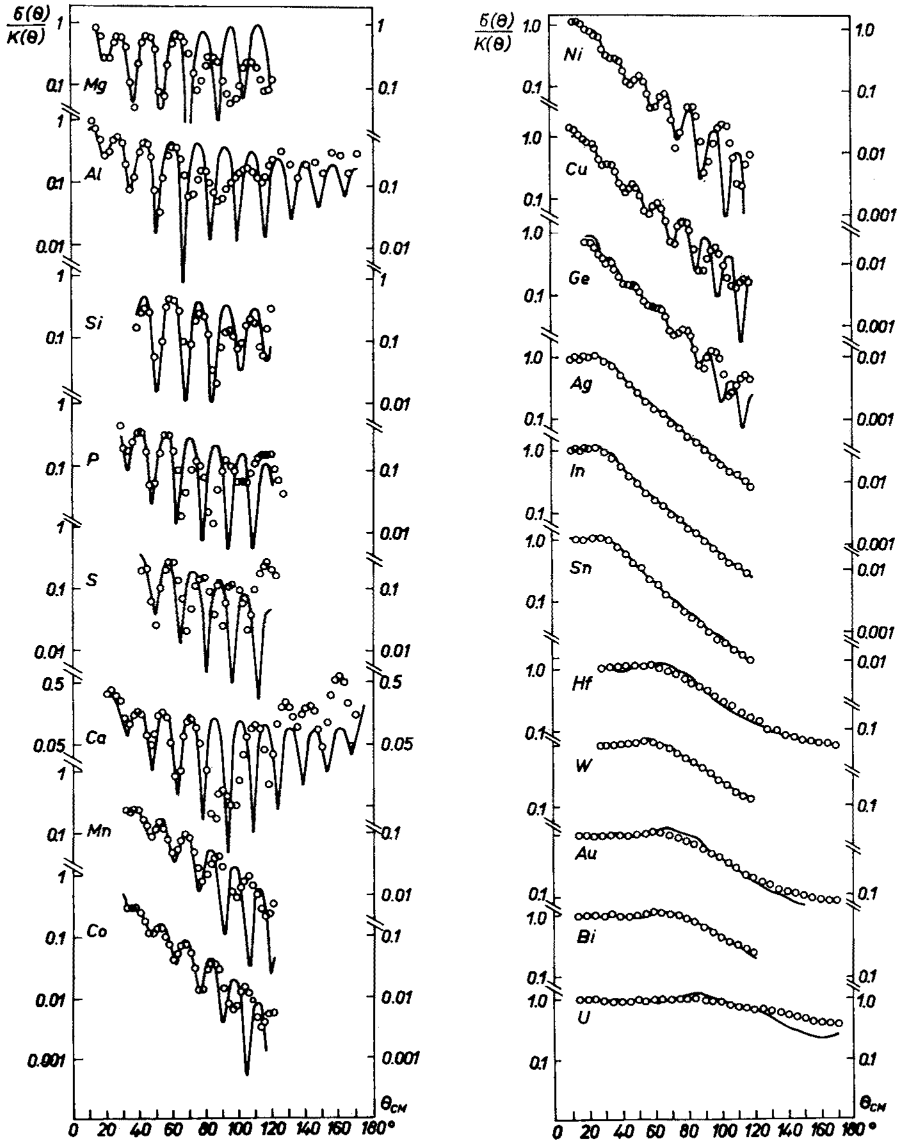


Fig. 3a, b. Frahn-Venter model fits to experimental angular distributions; presented in the fitting ranges. Angular distributions were computed according to the closed-form expressions

The long range Coulomb potential scatters alpha particles outside the nuclear potential range and this scattering is greater when the Coulomb potential is stronger. In consequence, for scattering on heavy elements, the angular momentum of the partial wave for which the real value of nuclear reflection coefficient is equal 1/2 decraeses when the weight of the target

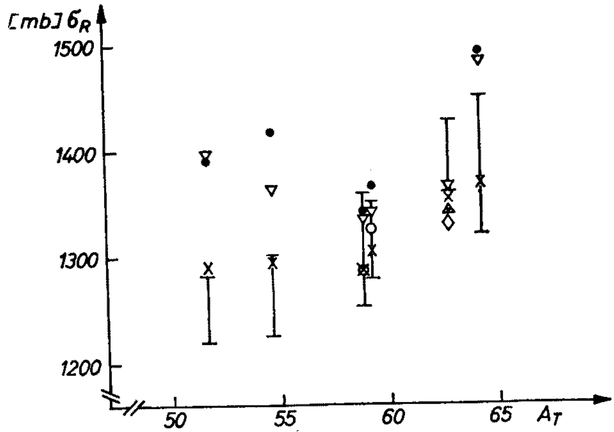


Fig. 4. Total reaction cross-sections for the interaction of 24.7 MeV alpha particles with Cr, Fe, Ni, Co, Cu, and Zn nuclei. The bars show the experimental data with their errors, × — the optical model predictions, ● — Springer-Harvey five parameter model, ▽ — Springer-Harvey three parameter model, ○ — Frahn-Venter three parameter model. For Cu predictions of the σ_R value with the optimum parameters obtained from fitting a non-normalized angular distribution are presented as ◇ (Springer-Harvey three parameter model) or △ (Frahn-Venter three parameter model)

nuclei increases (Fig. 5*c*). Nevertheless, the radius of interaction (formula (11)) still increases when the weight of the target nuclei increases (Fig. 8).

The optical model, as opposed to the Springer-Harvey model, predicts the experimental angular distributions of alpha particle scattering even in the extreme backward direction. As an example, fits to K(α, α)K data are shown in Figs 1 and 2. The importance of the backward scattering for the optical model analysis has been discussed previously [5]. The procedure of fitting the predictions of the Springer-Harvey model to the angular distribution measured for $\Theta \geqslant 150°$ led to the reflection coefficient of some partial waves having a value greater than one (non-physical value). The calculated angular distribution does not reproduce the differential cross-section measured for $\Theta < 150°$. Since the Springer-Harvey model is not able to reproduce angular distributions in the region of scattering angles near 180°, it was necessary to find the angular range for which the model is applicable. It was found by fitting the experimental data of the scattering of 25 MeV alpha particles in different angular ranges that the limit of the angular region for which the Springer-Harvey model is applicable increases when the charge of the target nucleus increases. Generally, in the case of 25 MeV alpha particle scattering, the angular limit of the range applicable for the Springer-Harvey model is 75° when the angular distributions show the diffraction pattern or 120° when they do not. The comparison of these results with the results of other works [3], [4], [17], [22]

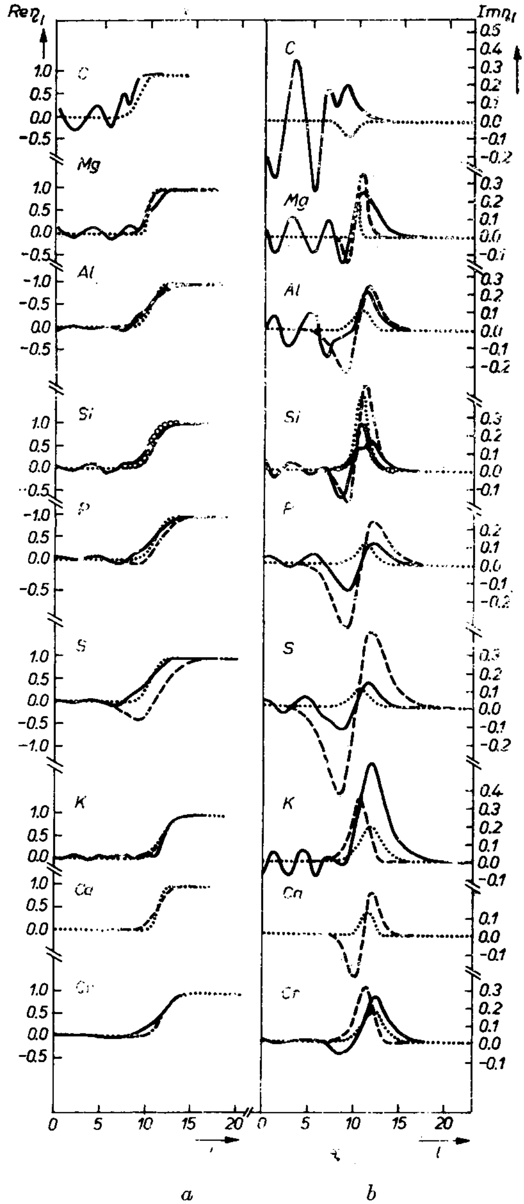


Fig. 5*a*, *b*. Shapes of reflection coefficients corresponding to angular distributions from Figs 1 and 2. Solid curves — optical model; broken curves — Springer-Harvey five parameter model; dotted curves — Springer-Harvey three parameter model. For Mg(α, α)Mg the best physically significant reflection coefficients obtained in the Springer-Harvey five parameter model analysis are shown. For Si(α, α)Si the open-circle curves correspond to the fit drawn as a broken curve in Fig. 2. For K(α, α)K the solid curves were obtained in the optical model analysis up to 177.5°LAB

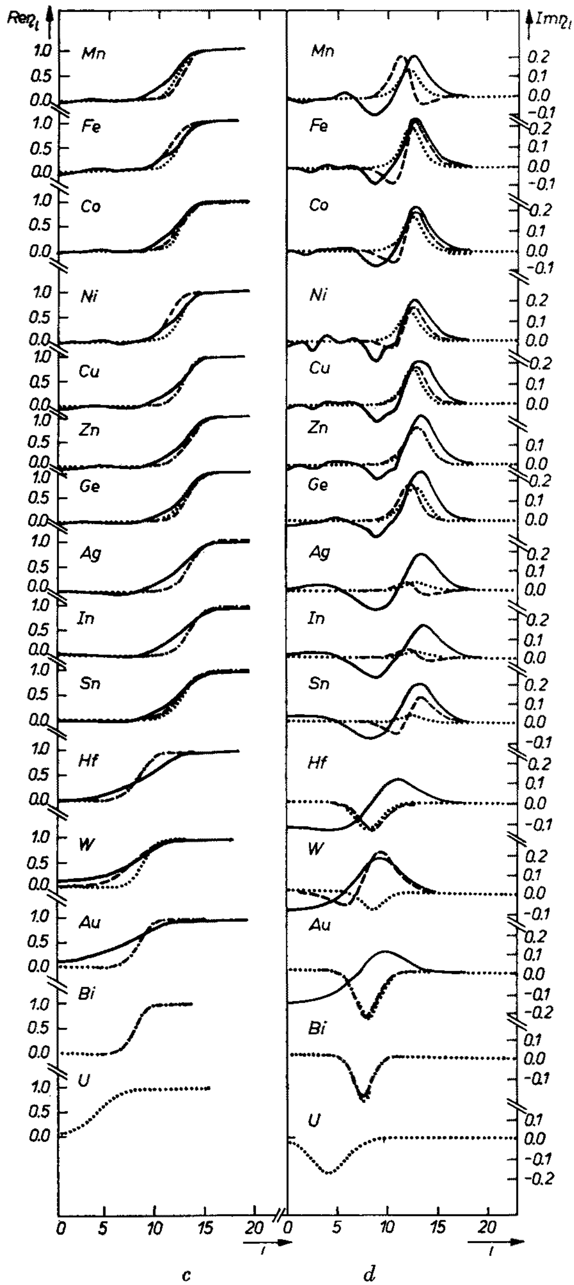


Fig. 5*c*, *d*. Shapes of reflection coefficients corresponding to angular distributions from Figs 1 and 2. Solid curves — optical model; broken curves — Springer-Harvey five parameter model; dotted curves — Springer-Harvey three parameter model

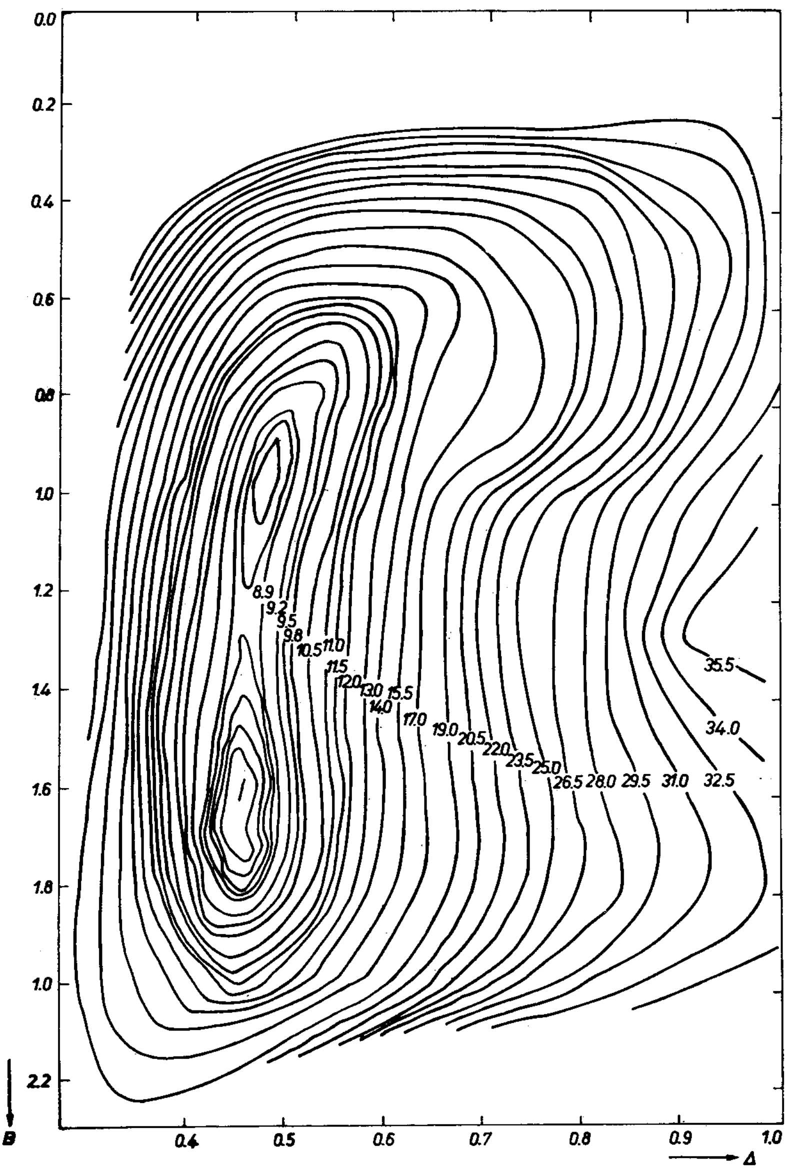


Fig. 6. Map of the $\chi^2/(100 \cdot N)$ values obtained in the Springer-Harvey three parameter model analysis of the Si(α,α)Si data. Computations made for the L parameter equal 10.7

regarding the scattering of alpha particles of energy higher than 25 MeV permits the supposition that the angular limit of the region for which the Springer-Harvey model is applicable decreases when the energy of the scattered particles increases.

The ambiguities of the optical model, consisting in the presence of a few minima of χ^2/N in the space of parameters to be fitted, are well known. Evaluations of the depth of the optical potential for alpha particle scattering by means of nucleon interaction are available [6],[16],[21]. According to these evaluations $U \approx 200$ MeV and this result considerably reduces the ambiguities of the optical model.

The Springer-Harvey model used for analysis of experimental data also gave ambiguities. One of these ambiguities, that of the B parameter, is shown in Fig. 6. Fitting procedures started from the two minima of χ^2/N shown in Fig. 6 brought to a common minimum in the space of five parameters of the Springer-Harvey model. In Table I are presented the

TABLE I

Ambiguities of the Springer-Harvey models

Fit to:	L	Δ	A	B	C	χ^2/N	σ_R	Remark
Si(α, α)Si	10.669	0.455	0	1.665	0	822.5	1127.6	
$E = 24.7$ MeV	10.688	0.477	0	0.969	0	863.0	1161.8	
$40° \div 75°$								
Hf(α, α)Hf	8.09256	0.83994	0.15384	−0.55031	0.01749	1.81	603.06	*
$E = 24.7$ MeV	7.36029	1.42642	0.24091	0.10360	−0.69904	2.93	578.8	*
$30° \div 120°$								
	11.765	1.060	−2.910	0.765	−2.195	301.1	1474.8	
Co(α, α)Co	12.270	1.145	−2.120	0.025	−2.685	79.8	1589.07	
$E = 27.5$ MeV	12.36364	1.03479	−1.24433	0.09443	−2.07638	35.7	1491.19	*
$17.5° \div 75°$	13.01739	0.72732	0.02148	0.61975	−0.17227	99.2	1368.12	*

* Results of Miss A. Radwańska obtained using her programme written for the ODRA-1204 computer.

ambiguities of the Springer-Harvey models which occurred in our analysis of experimental data. In previous Springer-Harvey model analysis performed by others [12], [17], [20] ambiguities were found in the Springer-Harvey five parameter model but not in the Springer-Harvey three parameter model.

In the Springer-Harvey three parameter model analysis of the C(α, α) C data two sets of the best parameters were found, but the problem of ambiguity did not arise as one of these sets could not be taken into consideration, since corresponding reflection coefficients contained at least one coefficient of nonphysical value. The sets of parameters without physical significance were found in the Springer-Harvey five parameter model analysis of C(α, α)C data and Mg(α, α)Mg data. The procedure of fitting to angular distribution as well as σ_R may possibly prevent the Springer-Harvey model analysis from entering the region of parameters without physical significance.

The Frahn-Venter five parameter model assuming partial transparency of target nuclei was used for the analysis of the alpha particle scattering data on carbon as a light element. The results (σ_R values and fit to angular distribution) are comparable with those of the optical model analysis (Fig. 7).

It is well known that the foundations and assumptions of the Springer-Harvey three parameter model are the same as those in the Frahn-Venter three parameter model. Only the mathematical formalism of the models differs. It was twice proved for the scattering of alpha particles that the Frahn-Venter three parameter model is equivalent to the Springer-

-Harvey three parameter model in the region for which the models are applicable: a) this was shown by the similar values of parameters of the Springer-Harvey model and the Frahn--Venter model obtained phenomenologically from our independent fits to the same experimental data, b) values of the geometrical parameters (sizes) obtained by Frahn and -Venter [25] are consistent with values obtained in Springer-Harvey model analysis of different experimental data carried out by many authors (Table X, of Ref. [18] or [19]).

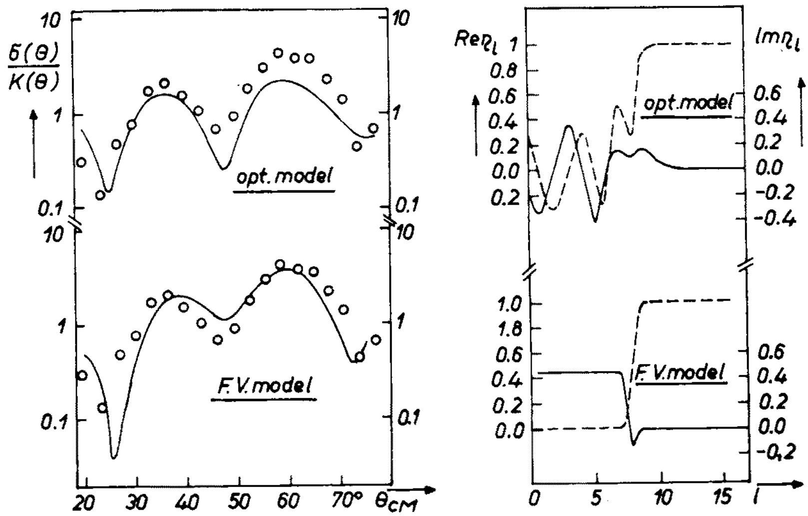


Fig. 7. Optical model and Frahn-Venter five parameter model fits to the C(α, α)C data at 24.7 MeV and corresponding shapes of reflection coefficients. Optical model fit: $U = 251.8222$ MeV, $W = 125565$ MeV, $r_0 = 1.4439$ fm, $a = 0.4839$ fm. $\sigma_R = 859$ mb, $\chi^2/N = 320$. Frahn-Venter model fit: $L = 7.9725$, $\Delta = 0.1095$, $\mu/\Delta = -1.3065$, $\varepsilon_1 = 0.0056$, $\varepsilon_2 = 0.4416$, $\sigma_R = 736$ mb, $\chi^2/N = 363$

Beyond the angular region for which the Springer-Harvey three parameter model and the Frahn-Venter three parameter model are applicable, differences occur in the calculated angular distributions. It was to be expected that the Frahn-Venter model would not predict scattering data for angles near 180°, because the Frahn-Venter formulae were obtained under the assumption that:

$$4(L+1/2)(\pi-\Theta) \gg 1. \tag{10}$$

It is possible to calculate the geometrical parameters (sizes) of nuclei using the values of parameters of the strong absorption model. The radius of interaction between the scattered particle and the target nucleus is:

$$R_{\alpha A} = \frac{1}{k}\left[n + \sqrt{n^2 + L(L+1)}],\right] \tag{11}$$

and the width of the surface zone is:

$$d = \frac{L\Delta}{k\sqrt{n^2+L(L+1)}}. \tag{12}$$

Values $R_{\alpha A}$ calculated from the phenomenological values of L found in the Springer-Harvey three or five parameter model analysis of the data obtained in Cracow, treated as a function of the third root of atomic weight A_T, appear to show behaviour represented by the straight line (Fig. 8) described by:

$$R_{\alpha A} = (1.42 A_T^{1/3} + 2.28)\ \mathrm{fm}. \tag{13}$$

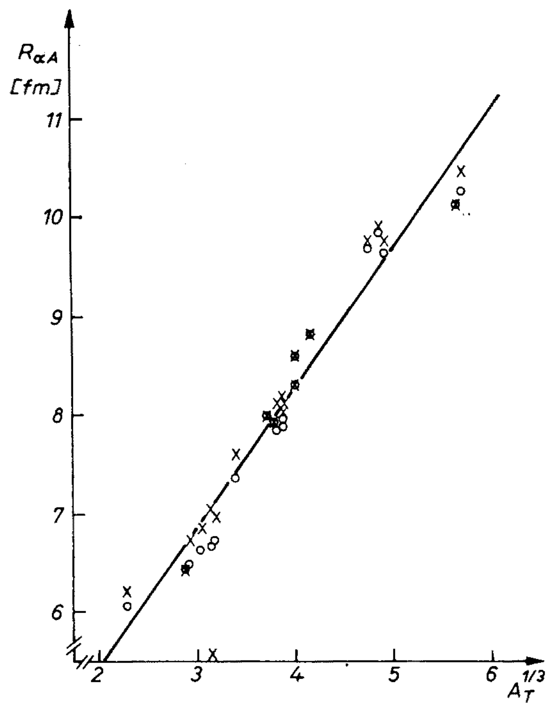


Fig. 8. Interaction radius *versus* $A_T^{1/3}$ evaluated from the Springer-Hervey model analysis; × — three parameter model, o — five parameter model

A slightly different straight line was found from the analysis [12] of scattering data of 44 MeV alpha particles:

$$R_{\alpha A} = (1.523 A_T^{1/3} + 2.135)\ \mathrm{fm}, \tag{14}$$

The value of the coefficient of $A_T^{1/3}$ is similar to the mean value of the parameter of the optical potential r_0, equal 1.439 fm according to our results [18], [19] (sets of optimum parameter values with $U \approx 200$ MeV).

The square value of the mean square radius of the optical potential $\langle r^2 \rangle$ was calculated from the phenomenological values of the radius r_0 and diffuseness a. The least square method was used to obtain parameters of the linear dependence $\langle r^2 \rangle$ *versus* $A_T^{2/3}$, after which neglecting the range of the elementary interaction, the values of the mean square radius of the alpha particle ($\approx$ 1.76 fm) and coefficient ($\approx$ 1.30 fm) designating the radius of the target were found. These values are greater than the R. Hofstadter [14] values, obtained from electron scattering:

$$\langle r_\alpha^2 \rangle_H^{1/2} = 1.68 \text{ fm} \quad \text{and} \quad r_H = 1.16 \text{ fm}.$$

The values of the width of the surface zone d were calculated from the phenomenological values of L and Δ parameters found in the Springer-Harvey model analysis of the data obtained in Cracow. The mean value obtained in the Springer-Harvey three parameter model (Fig. 9) is equal 0.28 fm or, if it is obtained in the Springer-Harvey five parameter model analysis, 0.37 fm (Fig. 10). The value $d = 0.36$ fm was found in the analysis [12]

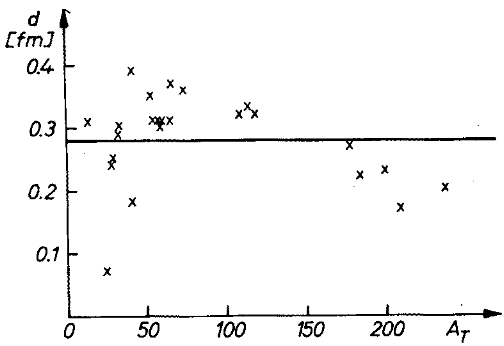


Fig. 9. Width of the surface zone calculated with the optimum parameters of Springer-Harvey three parameter model *versus* atomic weight of target nuclei

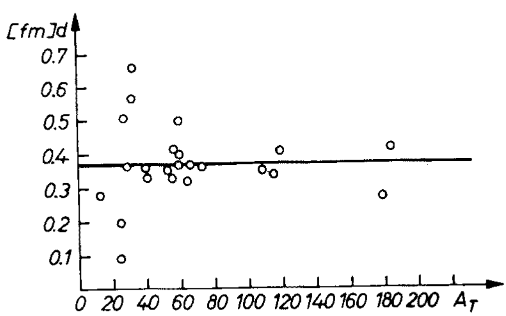


Fig. 10. Width of the surface zone calculated with the optimum parameters of Springer-Harvey five parameter model *versus* atomic weight of target nuclei

of scattering data of 44 MeV alpha particles. To recapitulate: the value of the width of the surface zone ($d \leqslant 0.37$ fm) obtained from the Springer-Harvey model analysis differs from the value of the diffuseness parameter of the optical potential, the mean value of which is equal 0.519 fm according to our results [18], [19] (sets of optimum parameters values with $U \approx 200$ MeV).

When the shapes of η_l are nearly identical in the Springer-Harvey three and five parameter models, then the A and C parameters may be considered as additional ones contributing only slightly to the η_l values. In such a case the values of the width of the surface zone d calculated in three or five parameter model analysis are identical and the well-known physical significance of L, Δ, B phenomenological parameters of the Springer--Harvey three parameter model is valid for the Springer-Harvey five parameter model.

Description of the scattering of the alpha particles of higher energies requires more partial waves than scattering at lower energies (Fig.11). As expected, the values of the L parameter of the strong absorption models increase when the energy of the scattered alpha particles increases. A similar energy dependence of the Δ parameter may be seen from the list of the phenomenological values of the parameter [18], [19].

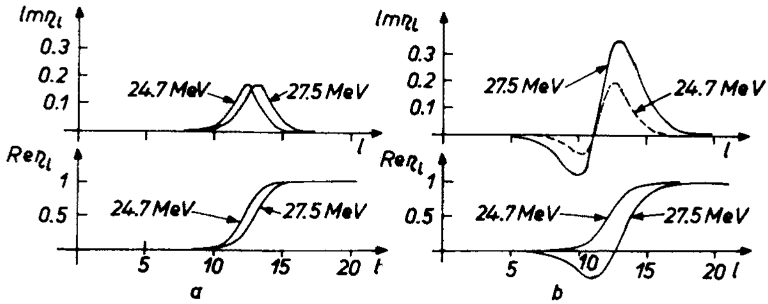


Fig. 11*a*, *b*. Variation of the reflection coefficients of Springer-Harvey three (Fig. 11*a*) parameter and five (Fig. 11*b*) parameter models with energy of alpha particles scattered on potassium nuclei

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