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ON THE GAUGE TRANSFORMATION OF THE LEPTONIC WEAK INTERACTION

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By using the compensation field method we will show that it is possible that three intermediate vector bosons of the leptonic weak interaction with nonvanishing masses exist. These vector bosons are the compensation fields of the transformation of the Touschek type and of its generalization.

In this note, basing on the Yang-Mills theory [1] which is used successfully for the strong interaction, we shall study the gauge transformation of the Touschek type and its generalization in the weak interaction of leptons.

1

1. It is well known that the Lagrangian of lepton weak interaction is invariant under the transformation of the form

$$\psi_l \rightarrow \exp i \frac{\alpha}{2} (1-\gamma_5)\, \psi_l \,. \tag{1.1}$$

According to Yang-Mills theory, we suppose that this transformation is non-local, that is $\alpha = \alpha(x)$ to be a function of Space-time coordinates. The invariant condition of the Lagrangian demands hence the existance of a compensation vector field A_μ which transforms according to the following law:

$$A_\mu \rightarrow A_\mu + \frac{i}{\sqrt{2}} (1-\gamma_5) \frac{\partial \alpha}{\partial x \mu} \tag{1.2}$$

It is clear that A_μ must be a multicomponent field. The invariant condition of combination

$$S_{\mu\nu} = A_\mu A_\nu - A_\nu A_\mu$$

gives us

$$[\gamma_5, A_\mu] = 0 \tag{1.3}$$

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In the linear approximation, the linear equation which is satisfied by A_μ is of the form:

$$(\Box+m^2)\,A_\mu=0$$

where m^2 is a mass operator. The invariant condition of this equation under (1–2) gives us

$$m^2(1-\gamma_5)=0 \tag{1.4}$$

which means that m^2 must have the following form

$$m^2=\frac{\mu^2}{\sqrt{2}}\,(1+\gamma_5) \tag{1.5}$$

Hence, the above mentioned equation of A_μ can be rewritten in the following form:

$$\left(\Box+\frac{\mu^2}{2}\,(1+\gamma_5)\right)A_\mu=0 \tag{1.6}$$

Outside of (1–2) there exists the following transformation under which this equation is invariant

$$A_\mu\to\frac{1}{\sqrt{2}}\,(1+\gamma_5)\,A_\mu \tag{1.7}$$

In the diagonal representation of γ_5-matrix we have

$$m^2=\frac{\mu^2}{\sqrt{2}}\begin{pmatrix}1 & & & 0\\ & 1 & & \\ & & 0 & \\ 0 & & & 0\end{pmatrix}$$

and

$$A_\mu=\begin{pmatrix}A_\mu^{11} & A_\mu^{12} & & 0\\ A_\mu^{21} & A_\mu^{22} & & \\ & & A_\mu^{33} & A_\mu^{34}\\ 0 & & A_\mu^{43} & A_\mu^{44}\end{pmatrix}$$

Resuming, it is possible that there exist three intermediate vector bosons of lepton weak interaction and their masses are different from zero. The transformation (1.7) proves that we may always eliminate four components A^{33}, A^{34}, A^{43}, and A^{44} of A-operator and therefore, in reality, there are only four components A^{11}, A^{12}, A^{21} and A^{22} that have physical sense. For convenience, we represent A_μ in the following form

$$A_\mu=\begin{pmatrix}A_\mu^{11} & A_\mu^{12}\\ A_\mu^{21} & A_\mu^{22}\end{pmatrix}.$$

In this form of A_μ, the equation (1–6) becomes

$$\left(\Box+\frac{\mu^2}{2}I\right)A_\mu=0$$

where I is a two-dimensional unit matrix.

Hence the transformation (1–2) becomes:

$$A_\mu \to A_\mu = 0$$

2

Let us specify the transformation (1.1) by considering it as a "rotation" about a 3[rd]-axis in an abstract space. In this space, we suppose that one may classify four leptons into two spinors[1]

$$l_1 = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \cdot l_2 = \begin{pmatrix} e \\ \mu \end{pmatrix}. \tag{2.1}$$

These spinors transform according to the following form

$$\psi_{l_i} \to \exp i\,[(1-\gamma_5)\,\alpha_i \sigma^i] \cdot \psi_{l_i} \tag{2.2}$$

where σ^i are the Pauli matrices and α_i are the "rotation" parameters.

The above mentioned abstract space is in analogy with the isotopic spin space of strong interaction, we call it the isoleptonic spin space. Thus, in this terminology, four leptons have isoleptonic spin equal $\frac{1}{2}$.Let L_e and L_μ be the e-leptonic and μ-leptonic charge and let L_z be the 3[rd]-component of the isoleptonic spin vector, we have then the following formula which is in analogy with Gell-Mann-Nishijima's:

$$Q = L_z + \frac{L_e + L_\mu}{2} \tag{2.3}$$

Because vector bosons A_μ are three intermediate bosons of the weak interaction of leptons, therefore we may suppose that they have isoleptonic spin equal 1, so that they form a vector in the isoleptonic spin space:

$$A_\mu = \begin{pmatrix} A^3_\mu & A^1_\mu + iA^2_\mu \\ A^1_\mu - iA^2_\mu & -A^3_\mu \end{pmatrix}. \tag{2.4}$$

Alternatively, we may write A_μ in the following form

$$A_\mu = \vec{A}_\mu \cdot \vec{\tau} \tag{2.5}$$

where $\vec{\tau}$ have three components $\tau_1 = \sigma^1$, $\tau_2 = \sigma^2$ and $\tau_3 = \sigma^3$. From the law of conservation of electrical charge in the weak interaction, we see that one of the three bosons is neutral and two others have electrical charges equal 1: A°, A^+.

To simplify, we denote by μ the masses of vector bosons and by $\vec{f}_{\mu\nu}$ the field tensor defined as:

$$\vec{f}_{\mu\nu} = \frac{\partial \vec{A}_\mu}{dx^\nu} - \frac{\partial \vec{A}_\nu}{\partial x^\mu} + 2G\,(\vec{A}_\mu \times \vec{A}_\nu) \tag{2.6}$$

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[1] By using the Fierz identity we may obtain also the same result for the following classification

$$l_1 = \begin{pmatrix} e \\ \overline{\nu_e} \end{pmatrix} \quad \text{and} \quad l_2 = \begin{pmatrix} \mu \\ \overline{\nu_\mu} \end{pmatrix}$$

(see [4]).

Let us suppose the Leptonic weak interaction to be invariant under (2–2), then Lagrangian of the system of two interacting fields has the form:

$$L = -\frac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}_{\mu\nu} - \mu^2\vec{A}_\mu^+\vec{A}_\mu - \bar{\psi}\gamma_\mu(\partial_\mu - iG\vec{\tau}\vec{A}_\mu)\psi \tag{2.7}$$

(in the limit $m_e = m_\mu = 0$).

From (2–7) we obtain the following movement equations:

$$\frac{\partial\vec{f}_{\mu\nu}}{\partial x^\nu} + 2G(\vec{A}_{\nu'}\cdot\vec{f}_{\mu\nu}) + \mu^2\vec{A}_\mu + J_\mu = 0$$

and

$$\gamma^\mu(\partial_\mu - iG\vec{\tau}\cdot\vec{A}_\mu)\psi = 0 \tag{2.8}$$

where

$$\vec{J}_\mu = iG\bar{\psi}\gamma_\mu(1-\gamma_5)\,\vec{\tau}\psi \tag{2.9}$$

We may show easily that $\vec{J}_\mu$ satisfies the following equation

$$\frac{\partial\vec{J}_\mu}{\partial x^\mu} = -2G\vec{A}_\mu\times\vec{J}_\mu \tag{2.10}$$

Let us define:

$$D_0 = \int d\vec{x}\, J_0(\vec{x}, x^0)\; D_\pm = \int d\vec{x}\, J_\pm(\vec{x}, 0)$$

Then $D_0, D_\pm$ satisfy the three following relations:

$$\begin{aligned}[D_+, D_-] &= 2D_0 \\ [D_0, D_+] &= D_0 \\ [D_0, D_-] &= -D_0\end{aligned} \tag{2.11}$$

That is D_0, $D_\pm$ form a Lie algebra of $SU(2)$-group, we call it the current algebra of Lepton weak interaction.

The study of this algebra together with the relation (2.10) is very interesting in view of intermediate boson theory.

This will be presented in the next paper.

3

Lagrangian of Lepton weak interaction

Let us now clear the above mentioned weak current $\vec{J}_\mu$:

$$\vec{J}_\mu = iG\{\bar{l}_1\gamma_\mu(1-\gamma_5)\vec{\tau}l_2 + \text{h. c.} + \sum_{i=1}^{2} l_i\gamma_\mu(1-\gamma_5)\vec{\tau}l_i\} \tag{3.1}$$

From which we have

$$J_\mu^+ = iG\{\tilde{\nu}_e O_\mu \mu + \tilde{\mu} O_\mu \nu_e + \tilde{\nu}_e O_\mu \nu_\mu + \tilde{e} O_\mu \mu\}$$

$$J_\mu^- = iG\{\tilde{\nu}_\mu O_\mu e + \tilde{\nu}_e O_\mu + \tilde{\nu}_\mu O_\mu \nu_e + \tilde{\mu} O_\mu e\}$$

$$J_\mu^0 = iG\{\tilde{\nu}_e O_\mu e - \tilde{\nu}_\mu O_\mu \mu + \tilde{\nu}_e O_\mu \nu_e - \tilde{\nu}_\mu O_\mu \nu_\mu + \tilde{e} O_\mu e -$$

$$- \tilde{\mu} O_\mu \mu + \tilde{e} O_\mu \nu_e - \tilde{\mu} O_\mu \nu_\mu\}$$

$$O_\mu = \gamma_\mu (1 - \gamma_5)$$

And the interaction Lagrangian in view of the hypothesis on the current interaction of Gell-Mann-Feynman [3] is of the form:

$$\mathscr{L}_w = G^2 \vec{J}_\mu \cdot \vec{J}_\mu \tag{3.2}$$

which is analogous to the Lagrangian given by [4] in $SU(4)$-symmetry model of leptons.

However, in our theory, there exist three intermediate vector bosons, thereby, the interaction Lagrangian in given by

$$\mathscr{L}_w = G^2 \{J_\mu^0 A_\mu^0 + J_\mu^+ A_\mu^+ + J_\mu^- A_\mu^-\} \tag{3.3}$$

Hence, we obtain:

$$G^2 = \frac{g_w}{\sqrt{2}}$$

To end, we note that in the Gell-Mann-Feynman theory cross-section of the following processes:

$$\nu_e + \mu \rightarrow \nu_e + \mu$$

$$\nu_\mu + e \rightarrow \nu_\mu + e$$

. . . .

are equal to zero, but in our theory they have definite values different from zero. It would be interesting to verify this fact.

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