MEASUREMENT OF THE LONGITUDINAL POLARIZATION OF ELECTRONS FROM SOME FIRST FORBIDDEN BETA DECAYS

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The longitudinal polarization of electrons from the first forbidden β transitions in ^{122}Sb, ^{141}Ce, ^{170}Tm, ^{175}Yb, ^{177}Lu isotopes was measured by Mott scattering. The results were compared with theoretical predictions based either on the shell model with spherical potential wave functions or on the Nilsson and Saxon-Woods wave functions. The value of the matrix elements ratio $\langle i\boldsymbol{\alpha}\rangle/\langle \boldsymbol{r}\rangle$ was found to vary considerably depending on the isotope.

1. Introduction

The observables measured in the β decay such as the spectrum shape factor, the comparative half-life of the β transition ft, $\beta-\gamma$ angular correlations, β — circularly polarized γ correlations, and the longitudinal polarization of β particles, are expressed by means of matrix elements which can be found from certain nuclear models. To the first forbidden decay with a spin charge $\Delta I = 0$, in general six matrix elements: $\langle i\gamma_5\rangle$, $\langle \boldsymbol{\sigma r}\rangle$, $\langle \boldsymbol{r}\rangle$, $\langle i\boldsymbol{\alpha}\rangle$, $\langle i\boldsymbol{\sigma r}\rangle$, $\langle B_{ij}\rangle$ contribute, whereas only the last four contribute to a $\Delta I = \pm 1$ transition. The $\Delta I = \pm 2$ transition is unique *i. e.* one matrix element $\langle B_{ij}\rangle$ is involved. The experimental determination of matrix elements for the first forbidden transitions with a change of spin: $\Delta I = 0$, $\Delta I = \pm 1$ is difficult because most of the transitions are dominated by two energy-independent matrix elements combinations as it follows from the ξ approximation (1). In the case of the break down of the ξ approximation caused by a selection rule effect or a cancellation effect, the determination of matrix elements is possible if their number does not exceed the number of the independent observables. The number of free parameters may be reduced by considering the ratios of matrix elements:

$$\Lambda = \langle i\boldsymbol{\alpha}\rangle/\xi\langle \boldsymbol{r}\rangle,\ \Lambda_0 = -\langle i\gamma_5\rangle/\xi\,\langle \boldsymbol{\sigma r}\rangle$$

where $\xi = \dfrac{\alpha Z}{2R}$ has the meaning of Coulomb energy of the electron at the nuclear radius R, α is fine structure constant, and Z is the atomic number.

Several authors have calculated values of Λ and Λ_0. Longitudinal polarization of electrons is one of the observables permitting to find the value of Λ experimentally. One of the

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aims of this work was to study the parameter Λ and its expected dependence on the studied transition. The measurements of longitudinal polarization performed by us permitted also to draw certain conclusions concerning the applicability of Nilsson and Saxon-Woods wave functions.

2. *Theoretical estimation of the parameter* Λ

The usual procedure of the estimation of Λ is to consider the commutator $[H,\boldsymbol{r}_k\tau_k]$ (here τ_k is the plus or minus component of isotopic spin operator). The Hamiltonian H has the form:

$$H = \sum_j H_D(j) + H_C + H_N, \tag{1}$$

where $H_D(j)$ is the free particle Dirac Hamiltonian for particle j, H_C — the Coulomb interaction, H_N — the nucleon-nucleon non-Coulomb interaction. The commutator $[H, \boldsymbol{r}_k\tau_k]$ can be written as a sum of three terms:

$$\langle f|(W_f - W_j)\boldsymbol{r}_k\tau_k|i\rangle = \langle f|[H, \boldsymbol{r}_k\tau_k]|i\rangle$$
$$= \langle f|[H_D(k), \boldsymbol{r}_k\tau_k]|i\rangle + \langle f|[H_C, \boldsymbol{r}_k\tau_k]|i\rangle + \langle f|[H_N, \boldsymbol{r}_k\tau_k]|i\rangle, \tag{2}$$

where W_f and W_i are the energies in final $\langle f|$ and initial $|i\rangle$ states. The first term can be found easily: $[H_D(k), \boldsymbol{r}_k\tau_k] = i\boldsymbol{\alpha}_k\tau_k$. The conventional way to estimate the Coulomb term used in (2), (3), (4) is:

$$\langle f|[H_C, \boldsymbol{r}_k\tau_k]|i\rangle = \sum_n \{\langle f|H_C|n\rangle\langle n|\boldsymbol{r}_k\tau_k|i\rangle - \langle f|\boldsymbol{r}_k\tau_k|n\rangle\langle n|H_C|i\rangle\}$$
$$\approx \{\langle f|H_C|f\rangle - \langle i|H_C|i\rangle\}\langle f|\boldsymbol{r}_k\tau_k|i\rangle \approx 2.4\,\frac{\alpha Z}{2R}\,\langle \boldsymbol{r}\rangle, \tag{3}$$

here the nondiagonal terms of H_C are neglected.

Some authors give different evaluation of the $[H_N, \boldsymbol{r}_k\tau_k]$ term. Ahrens and Feenberg [2] estimated this term by general physical arguments based on the semi-empirical energy surface and the validity of the shell model considerations. Pursey [3] assumed H_N as a linear combination of short range two-particle ordinary, charge exchange, and spin-orbit coupling interactions. Fujita [4] analysed commutator $[H_N, \boldsymbol{r}_k\tau_k]$ in terms of the Conserved Vector Current (CVC) theory. The values of Λ obtained by these authors are the following:

$\Lambda = 1.0 + (W_i - W_f)A^{1/3}/Z = 1$ Ahrens and Feenberg [2]

$\Lambda = 2.0 + (W_i - W_f)A^{1/3}/Z = 2$ Pursey [3]

$\Lambda = 2.4 + (W_i - W_f)A^{1/3}/Z = 2.4$ Fujita [4] (CVC theory).

Since the experimental confirmation of the CVC theory it was assumed that $\Lambda = 2.4$.

Damgaard and Winther [5] pointed out that the above discussed estimation of the Coulomb term was not correct. Although the nondiagonal matrix elements of H_C are much smaller than the diagonal ones, the matrix elements: $\langle n|\boldsymbol{r}_k\tau_k|i\rangle$, $\langle i|\boldsymbol{r}_k\tau_k|n\rangle$ can also vary by order of magnitude and thus invalidate the approximation. The authors of Ref. [5] assuming the

form of the average Coulomb field found the variation of A by factor of three depending on the isotope studied.

Recently Fujita [6] connected the validity of the approximation (3) with the existence of the well defined isobaric states.

The parameter A is very useful in treating the β-decay data, so it seemed that an experimental test of the A would be valuable.

3. *Apparatus and sources*

The polarization was measured by determining the asymmetry in the electron scattering on a thin gold foil. Since such polarization analyser is sensitive only to the transverse polarization, the longitudinal electron polarization was transformed into a transverse one by means of crossed magnetic and electric fields.

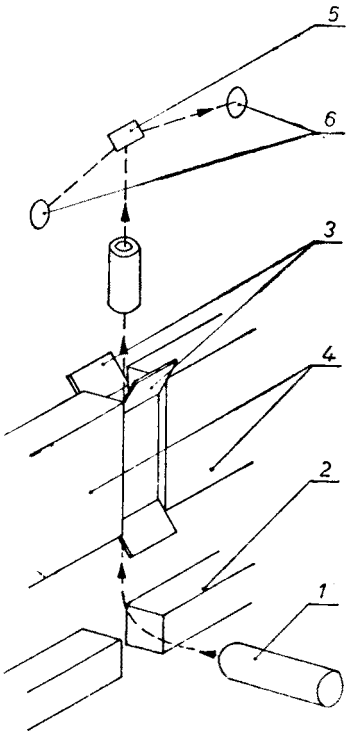

Fig. 1. The scheme of apparatus. *1* – Source holder. *2* – Deflecting electromagnet, *3* – High tension electrodes, *4* – Poles of electromagnet of Wien analyser, *5* – Scatterer, *6* – Si(Li) detectors

A schematic drawing of the instrument is given in Fig. 1. A source was placed out of the symmetry axis of the apparatus in order to diminish the background of γ-rays. The beta particles deflected by a magnet entered the crossed-fields region, where their spins were rotated by 90°. The asymmetry in electron scattering on the gold foil was detected with semiconductor detectors placed at the average scattering angles of 120°. A more detailed description of the used apparatus was given in the Ref. [7]. The main difference between the

present experimental setup and the one described in [7] was the use of Si(Li) detectors for registration of scattered electrons and measurement of the energy distribution of beta particles leaving the crossed-fields region.

The measurements were performed with the aid of a simple automatic system synchronizing the measurement on a particular scatterer with a corresponding counting unit.

The ^{122}Sb, ^{170}Tm, ^{175}Yb, ^{177}Lu sources were prepared by irradiation of the samples in a thermal neutron flux of 2×10^{13} n cm^{-2} sec^{-1} in the reactor of the Institute of Nuclear Research in Świerk. The ^{141}Ce source was supplied from the Radiochemical Center Amersham. The activities of the sources varied from 5 to 20 mC. All sources were prepared by an evaporation of the radioactive solutions on thin mylar aluminum coated foils. The thickness of the sources varied from 4.5 mg/cm^2 for ^{141}Ce to 1.8 mg/cm^2 for the majority of the isotopes studied. The purity of the sources was checked by means of a scintillation NaJ(Tl) spectrometer.

The depolarization in the sources was estimated by means of theoretical relations given by Mühlschlegel [8], which have been confirmed experimentally in the Ref. [9] and [10].

4. *Measurements and results*

Four runs corresponding to the gold scatterer thicknesses: 0.297, 0.590, 0.790 and 1.074 mg/cm^2 were made for each isotope investigated. The linear dependence of the Mott asymmetry on a scatterer thickness for foils up to 1.5 mg/cm^2 [11], [12] permitted to find the asymmetry for a scatterer of "zero" thickness.

Since the asymmetry of the Mott scattering on low Z-material is close to zero the instrumental asymmetry was accounted for by means of replacing gold by aluminum after each measurement of the effect. The scatterers were fixed on a rotated disc. The measurements of the effect, the instrumental asymmetry and the background were made in cycles. Each run consisted of 60–100 one-hour cycles. The periodic measurement of the instrumental asymmetry enabled to account for its possible slow changes in time (they did not exceed a few percent for each run).

The measurements of the polarization were preceded by some control experiments such as measurements of the detector efficiency, of the background and of the instrumental asymmetry for different geometrical setups, of the transmission of the apparatus, of the effective length of the crossed-fields region and of the energy distribution of electrons depending upon the values of the electric and magnetic fields.

The following formula was used for the determination of the electron polarization, P_L:

$$P_L = \left(\frac{A_0}{S \cdot v/c} + P''\right) \frac{1}{1-P'} \tag{4}$$

where:

A_0 is the asymmetry for a scatterer of zero thickness (corrected for background and instrumental asymmetry),

S is the Mott function [13]

P' is the depolarization in the source material.

P'' is the depolarization due to the multiple scattering on the fragments of apparatus, v/c is the ratio of the velocity of electrons to the velocity of light.

The results of the polarization measurements for the energy 250 keV are as follows:

Isotope	^{122}Sb	^{141}Ce	^{170}Tm	^{175}Yb	^{177}Lu
$P_L \Big/ \left(-\frac{v}{c}\right)$	0.94±0.03	0.90±0.04	0.99±0.03	0.98±0.02	0.95±0.02

5. *Discussion of results*

The longitudinal polarization of β particles from isotopes measured by us was previously investigated only for ^{170}Tm [14] and ^{177}Lu [15]. The results for ^{177}Lu were inconsistent.

5.1. ^{122}Sb

The analysis of the first forbidden $2^- \rightarrow 2^+$ transition in ^{122}Sb was performed by Pipkin *et al.* [16] who have found nine sets of nuclear matrix elements fitting the experimental data. Later, some other measurements permitted to diminish the amount of sets. The spectrum shape factor [17] was found to be in agreement with V, VI, VII and IX sets given in Ref. [16], but sets I, II, III cannot be totally eliminated. The results of the β-circularly polarized γ correlations [18] are best fitted with the sets I and IX and the measurements of electron — neutrino angular correlations [19] with sets I and III.

Longitudinal polarization of β particles found by us to be $P_L = (0.94\pm0.03)\left(-\frac{v}{c}\right)$, is in agreement with sets I, V and VIII (Fig. 2). The remaining solutions may be excluded. Our result together with measurements presented above permits to choose set I given in Ref. [16], which yields $A = 1.5$.

Recently Manthuruthil and Poirier [20] using the experimental data on the spectrum shape factor [17], $\beta-\gamma$ directional correlations [18] and angular distribution of γ-rays from oriented nuclei [19], obtained two sets of matrix elements describing first forbidden β-decay of ^{122}Sb. The values of longitudinal polarization resulting from those matrix elements are $1.3\left(-\frac{v}{c}\right)$ for both sets. It is an unexpected result because polarization exceeding the $\left(-\frac{v}{c}\right)$ value to such an extent has never been yet found, and such a large value of polarization seems to be improbable.

5.2. ^{141}Ce

Both ^{141}Ce β transitions are first forbidden. In the parent nucleus ^{141}Ce, a single neutron outside of the closed shell $N = 82$ is in the $f_{7/2}$ configuration. The β decay branch to the ground state is a single particle transition from a $f_{7/2}$ to $d_{5/2}$ proton state. The proton configuration of the excited state is mainly $(g_{7/2})^7\,(d_{5/2})^2$. Thus, transition to this state will be $f_{7/2} \rightarrow g_{7/2}$, but there exists a possibility of a $d_{5/2}$ admixture.

For the transition to the first excited state the measurement of the angular correlation yields a small anisotropy, $\beta-\gamma$ circular polarization correlation is small or zero [21], the shape of the spectrum deviates slightly from the statistical one [22]. The asymmetry of the β-particle emission from the polarized ^{141}Ce nuclei was investigated by Hoppes *et al.* [23]. They made an attempt to find A and obtained a value close to 2.3. For the $7/2^- \rightarrow 5/2^+$

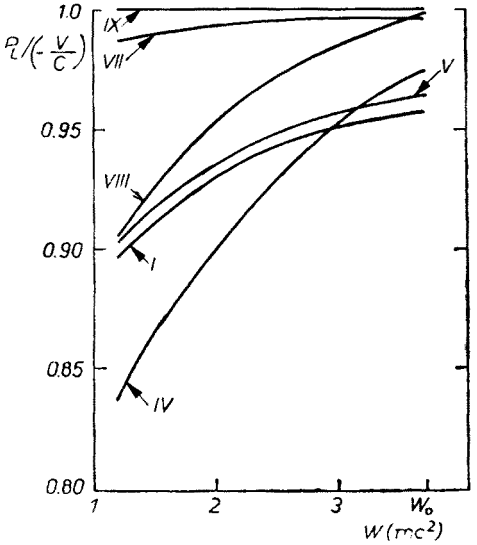

Fig. 2. The longitudinal β-polarization P_L as a function of electron energy W for different sets of nuclear matrix elements according to Ref. [16]

transition in ^{141}Ce a strong cancellation of matrix elements occurs. In this case P_L is very sensible to the changes of the parameter A. The analysis of first forbidden transitions in ^{141}Ce was made with the use of nuclear matrix elements found from the shell model with $j-j$ coupling given by Lipnik and Sunier [24]. If the first excited state were pure $g_{7/2}$ the P_L for the $7/2^- \rightarrow 7/2^+$ branch would be equal to $\left(-\frac{v}{c}\right)$. Under this assumption the value of P_L closest to the experimental value is obtained by means of nuclear matrix elements given in Ref. [24] for $A = 2.5$. In the presence of a $d_{5/2}$ admixture in the first excited state one still obtains $A > 2$. Our result is in good agreement with the value of $A = 2.37$ calculated by Spector [25] from the shell model with a harmonic oscillator potential.

5.3. ^{170}Tm

The interpretation of the ^{170}Tm decay is not easy because of the high log ft values (9.0 and 9.3 for the first excited state and the ground state respectively) whereas the other observables have close to allowed characteristics. The value of the asymmetry of the $\beta-\gamma$ angular correlations is low [26], shape of β spectrum was supposed to be statistical until recent very precise measurements [27]. The longitudinal polarization of β-particles measured

in our experiment is equal to $(0.99 \pm 0.03)\left(-\frac{v}{c}\right)$. This result is in agreement with the value of polarization calculated by us with the use of matrix elements derived in [28] and [29] by means of the Nilsson wave functions. However, it was shown by Bogdan *et al.* in Ref. [30] that it is not possible to obtain the correct log ft values by means of these wave functions. The Saxon-Woods wave functions are more suitable to describe β-decay of ^{170}Tm; namely they give a better account of the log ft values.

In their calculations based on Saxon-Woods wave functions, Bogdan *et al.* [30] used $A=1$ obtained by means of an average Coulomb potential suggested by Damgaard and Winther [5]. The values of P_L found from the matrix elements given by Bogdan *et al.* in Ref. [30] are: in the absence of configuration mixing $0.961\left(-\frac{v}{c}\right)$ and $0.924\left(-\frac{v}{c}\right)$ with the admixture of $K=0$ level to the ground state. The second value differs considerably from the experimental value of polarization. It is interesting that also the discrepancy between calculated and experimetal value of the angular correlation coefficient is larger under the assumption of configuration mixing. The experimental results tend rather to the smaller values [30].

It seems that the case of ^{170}Tm needs further theoretical investigation.

5.4. ^{175}Yb, ^{177}Lu

The ground state β-transitions of ^{175}Yb and ^{177}Lu have a lot in common. For the ^{175}Yb decay the initial and the final states are characterised by the orbitals: 7/2 [514] and 7/2 [404] respectively. The decay of ^{177}Lu is just the opposite, *i. e.* the neutron in the 7/2 [404] state transforms into a proton in the 7/2 [514] state. In the Nilsson model the relevant matrix elements differ only in phase for the equal deformations. Since the deformations of ^{175}Yb and ^{177}Lu nuclei are very similar, the values of observables are very close.

The nuclear matrix elements have been calculated for ^{175}Yb by Bogdan [28] and for ^{175}Yb and ^{177}Lu by Berthier and Lipnik [29] by means of the Nilsson wave functions. The values of the observables obtained with the use of the matrix elements given in Ref. [28] and [29] by means of the well-known Kotani [1] relations are very similar although the matrix elements differ.

According to the Ref. [28] $|\langle r\rangle|$ is the largest matrix element, whereas according to the [29] the largest one is $|\langle B_{ij}\rangle|$. The longitudinal polarization calculated by means of the matrix elements given in [28] and [29] is the same for both isotopes: $P_L = 0.97$ assuming $A = 1$ and $P_L = 0.99$ assuming $A = 2.4$.

The experimental value $P_L = (0.98 \pm 0.02)\left(-\frac{v}{c}\right)$ obtained by us for ^{175}Yb does not permit to choose one particular value of A from the above given. In the case of ^{177}Lu, $A = 1$ seems to be more probable.

The results of our measurements for deformed nuclei ^{175}Yb, ^{177}Lu are in agreement with the theoretical values obtained by means of the Nilsson wave functions; in the case of ^{170}Tm the Nilsson model does not account well for the experimental values of observables. Our measurements seem to indicate that also the Saxon-Woods wave functions do not give satisfactory agreement with experiment for all the observable quantities.

From our experiment it follows that the value of A varies from 2.5 to 1 depending on the transition studied. This result emphasizes the value of the Damgaard and Winther [5] remark against the uncritical use of the Fujita [4] evaluation of $\langle i\alpha\rangle/\langle r\rangle$ ratio. Namely, in usual procedure of A estimation one should rather use average Coulomb potential, than neglect the nondiagonal terms in relevant matrix elements. The value of A cannot be fixed as 1 or 2.4 but should be calculated for each investigated isotope.

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REFERENCES

[1] T. Kotani, *Phys. Rev.*, **114**, 795 (1959).
[2] T, Ahrens, E. Feenberg, *Phys. Rev.*, **86**, 64 (1952).
[3] D. L. Pursey, *Phil. Mag.*, **42**, 1193 (1951).
[4] J. I. Fujita, *Prog. Theor. Phys.*, **28**, 338 (1962); *Phys. Rev.*, **126**, 202 (1962).
[5] I. Damgaard, Aa. Winther, *Phys. Letters*, **23**, 345 (1966).
[6] J. I. Fujita, *Phys. Letters*, **24B**, 123 (1967).
[7] Z. Wilhelmi, J. Wołowski, S. Chojnacki, A. Korman, A. Płochocki, Rep. IBJ. Nº 798/I/PL (1967).
[8] B. Mühlschlegel, *Z. Phys.*, **155**, 69 (1959).
[9] A. Brossi *et al.*, *Nuclear Phys.*, **33**, 353 (1962).
[10] J. Van Klinken, *Nuclear Phys.*, **75**, 145 (1966).
[11] H. Wegener, *Z. Phys.*, **151**, 252 (1958).
[12] J. Van Klinken, *Nuclear Phys.*, **75**, 161 (1963).
[13] Shin-R-Lin, *Phys. Rev.*, **133**, A965 (1964).
[14] A. J. Alichanov, G. P. Eliseiev, W. A. Lubimov, *Nuclear Phys.*, **7**, 655 (1958).
[15] L. A. Mikaelian, P. E. Spiwak, *Zh. Eksper. Teor. Fiz.*, **37**, 1168 (1959); P. E. Spiwak, L. A. Mikaelian, *Zh. Eksper. Teor. Fiz.* (USRR), **39**, 574 (1960); P. E. Spiwak, L. A. Mikaelian, I. Kutikov, W. W. Apalin, *Nuclear Phys.*, **23**, 169 (1961).
[16] M. Pipkin, J. Sanderson, W. Weyhman, *Phys. Rev.*, **129**, 2626 (1963).
[17] S. T. Hsue *et. al. Nuclear Phys.*, **A117**, 686 (1968).
[18] Z. W. Grabowski, R. S. Raghvan, R. N. Steffen, *Nuclear Phys.*, **70**, 170 (1965).
[19] I. C. Palanthigal, *Phys. Rev. Letters*, **14**, 983 (1965).
[20] J. C. Manthuruthil, C. P. Poirier, *Nuclear Phys.*, **A118**, 657 (1968).
[21] J. P. Deutsch, L. Grenacs, P. Lipnik, *J. Phys. Radium*, **22**, 662 (1961).
[22] H. Beekhuis, R. J. Van Duinen, *Nuclear Phys.*, **A108**, 382 (1968).
[23] D. D. Hoppes, E. Ambler, R. W. Hayward, R. S. Kaeser, *Phys. Rev. Letters*, **6**, 115 (1961).
[24] P. Lipnik, J. W. Sunier, *Nuclear Phys.*, **65**, 21 (1965).
[25] R. M. Spector, *Nuclear Phys.*, **39**, 464 (1962).
[26] H. Dulaney, C. H. Braden, E. T. Patronis, L. D. Wyly, *Phys. Rev.*, **129**, 283 (1963); K. Runge, *Z. Phys.*, **183**, 195 (1965); H. Daniel, B. Martin, P. Schmidlin, H. Schmitt, *Z. Phys.*, **220**, 181 (1969).
[27] A. André, P. Depommier, P. Liaud, J. C. Millies-Lecroix, *CR Acad. Sci.* (France), **264**, 819 (1967); S. André, P. Liaud, J. C. Millies-Lecroix, *CR Acad. Sci.* (France), **267**, 836 (1968); S. André, P. Liaud, *CR Acad. Sci.* (France), **268**, 270 (1969); S. Y. Van der Werf, H. de Ward, H. Beekhuis, *Nuclear Phys.*, **A134**, 215 (1969).
[28] D. Bogdan, *Nuclear Phys.*, **81**, 120 (1966); *Z. Phys.*, **206**, 49 (1967).
[29] J. Berthier, P. Lipnik, *Nuclear Phys.*, **78**, 448 (1966).
[30] D. Bogdan, I. Vâtă, *Nuclear Phys.*, **A109**, 347 (1968); D. Bogdan, C. Protop, I. Vâtă, *Nuclear Phys.*, **A119**, 113 (1968).