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MICROSCOPIC AND MACROSCOPIC DESCRIPTIONS OF GAMMA--VIBRATION OF NUCLEI AT EQUILIBRIUM AND SADDLE POINT

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The gamma-vibrational parameters B_γ, C_γ and the energy E_γ of the gamma-vibrational state of the deformed nuclei are calculated as functions of the ε-deformation in the whole region between the equilibrium and the saddle point. Both the microscopic and macroscopic descriptions are used. The following result is obtained: when ε increases from its equilibrium value to the saddle point the nucleus begins to be more stiff with respect to the gamma vibration.

1. Introduction

The first theoretical description of the low-lying collective nuclear oscillation was given in 1952–1953 by A. Bohr and B. Mottelson [1, 2] who treated the collective oscillations of the nuclear shell structure in the adiabatic limit. They assumed that the collective frequency is small in comparison with the single particle excitations.

In the case of the deformed nuclei there are two types of the collective quadrupole vibrations — the axially-symmetric β vibration and the non-axial γ-vibration.

In the present paper we shall discuss the properties of the γ-vibrational characteristics of the deformed nuclei (the energy of the γ-vibrational state E_γ, the stiffness parameter C_γ and the mass parameter B_γ).

The γ-vibrational states have been found in many deformed nuclei in the rare earth and transuranic regions. Their quantum numbers $K\pi = 2+$ (where K is the projection of the angular momentum on the symmetry axis of the nucleus and π is the parity of the γ-vibrational phonon). The energies E_γ have the values 600–1500 keV.

It is well known that the nuclei in the actinide region undergo spontaneous fission. According to the channel theory of fission process [3] the fissioning nucleus in the saddle point of its deformation may exhibit excited states which are analogical to those at equilibrium deformation. Different excited states correspond to different fission channels. The non-axial γ-vibration may be one of those excitations. Hence, it is interesting to investigate the problem of the dependence of the γ-vibrational parameters C_γ, B_γ and E_γ on the value

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of the quadrupole deformation of the nucleus, starting from the equilibrium value of the deformation parameter up to the value which describes the saddle point shape.

In general we can try to solve this problem using two different descriptions. The phenomenological and the microscopic ones. The phenomenological γ-vibrational parameters can be calculated for example, using the hydrodynamical model [4]. In the microscopic description the collective vibrations are treated as a coherent superposition of single particle excitations of the nucleons which move in the single particle potential and interact by the restoring short range pairing forces and the long range multipole-multipole forces. The attractive quadrupole forces cause the β and γ vibrational states. The theoretical microscopic description of the γ-vibrational state was given in 1961 and 1965 by Bès [5, 6] and in 1965 by Soloviev [7] with the use of the old single particle Nilsson potential [8].

In the present paper we shall calculate the energy of the γ vibrational state for even-even nuclei: *i*) in the rare earth region with the use of the experimental values of the quadrupole and hexadecapole deformations $\varepsilon, \varepsilon_4$, *ii*) for even isotopes of Th, U and Pu at equilibrium [9] and for greater values of the quadrupole deformation ε up to the saddle point.

In section 2 we present the microscopic calculations of the γ-vibrational parameters. Section 3 gives the hydrodynamical description. In section 4 we discuss our results.

2. *Microscopic model*

a. General description

At the beginning of this section let us explain the notation for the γ-vibrational characteristics E_γ, C_γ and B_γ.

We shall assume that the nucleus is of ellipsoidal shape which can be described with the use of two deformation parameters β and γ. The semi-axis of such a nucleus are given by:

$$a_1 = R_0 \left[1-\sqrt{\frac{5}{4\pi}}\,\beta \cos(\gamma+\pi/3)\right],$$

$$a_2 = R_0 \left[1-\sqrt{\frac{5}{4\pi}}\,\beta \cos(\gamma-\pi/3)\right],$$

$$a_3 = R_0[1-\sqrt{5/4\pi}\,\beta \cos\gamma]. \tag{1}$$

One may notice that the β parameter gives the measure of the deformation while γ describes the nonaxiality of the ellipsoid. The nuclear oscillations connected with changes in γ are called non-axial γ-vibrations.

The Hamiltonian of the small oscillation of the γ-type arround the axially symmetrical equilibrium ellipsoidal shape has the form:

$$H = 1/2B_\gamma(\beta)\dot{\gamma}^2+1/2C_\gamma(\beta)\gamma^2. \tag{2}$$

The energy of the γ-vibrational state is given by

$$E_\gamma = \hbar\omega_\gamma = \hbar\sqrt{C_\gamma/B_\gamma}, \tag{3}$$

where C_γ and B_γ are called the stiffness and mass parameter respectively.

In order to obtain the values of these parameters in the microscopic model we shall start with the assumption that the nucleons in the nucleus move in the single particle Nilsson deformed potential [8, 10] and that they interact by the attractive restoring forces of pairing and quadrupole types.

The whole Hamiltonian may be written in the form:

$$H = H_0 + H_{\text{pair}} - 1/2\chi_p \hat{S}_p^+ \hat{S}_p - 1/2\chi_n \hat{S}_n^+ \hat{S}_n - 1/2\chi_{np}(\hat{S}_n^+ \hat{S}_p + \hat{S}_p^+ \hat{S}_n), \tag{4}$$

where H_1 is the Nilsson model Hamiltonian, H_{pair} represents the pairing interaction and $\hat{S}_p$ and $\hat{S}_n$ are the non-axial quadrupole operators for protons and neutrons respectively. χ_p, χ_n and χ_{np} are the coupling constants for the non-axial quadrupole interaction between protons, neutrons and protons and neutrons respectively.

Let us introduce the Hamiltonian

$$H_1 = H_0 + H_{\text{pair}}. \tag{5}$$

We can write H_1 and $\hat{S}$ in the form [7, 11]:

$$H_1 = \sum_{\nu\tau} \varepsilon_\nu c_{\nu\tau}^+ c_{\nu\tau} - G \sum_{\substack{\nu\nu' \\ \tau>0}} c_{\nu'\tau}^+ c_{\nu'-\tau}^+ c_{\nu-\tau} c_{\nu\tau} \tag{5a}$$

and

$$\hat{S} = \sum_{\substack{\nu\nu' \\ \tau\tau'}} \langle \nu\tau | 1/\sqrt{2}\ \sqrt{16\pi/5}\ r^2 (Y_{22} + Y_{2-2}) | \nu'\tau' \rangle c_{\nu\tau}^+ c_{\nu'\tau'}$$

$$= \sum_{\nu\nu\tau'} (s_{\nu\nu'} c_{\nu\tau}^+ c_{\nu'\tau} + \bar{s}_{\nu\nu'} c_{\nu\tau}^+ c_{\nu'-\tau}). \tag{6}$$

Here $c_{\nu\tau}^+$ and $c_{\nu\tau}$ are the creation and annihilation operators of the particle in state $|\nu\tau\rangle$ where τ has the meaning of the sign of the projection of the momentum on the symmetry axis of the nucleus and ν is reserved for the remaining quantum numbers, ε_ν is the energy of the particle in the state $|\nu \pm \tau\rangle$, G is the strength of the pairing interaction, $s_{\nu\nu'}$ and $\bar{s}_{\nu\nu'}$ are the matrix elements of the nonaxial quadrupole operator between two single particle states with the same and opposite values of τ respectively.

If the pairing interaction is taken into account in the approximation of the non--interacting qusiparticles Hamiltonian H_1 is given by

$$H_1 = \sum_\nu 2v_\nu^2 \varepsilon_\nu - \Delta^2/G + \sum_\nu E_\nu (\alpha_\nu^+ \alpha_\nu + \beta_\nu^+ \beta_\nu). \tag{7}$$

α_ν^+, β_ν^+ and α_ν, β_ν are the quasiparticle creation and annihilation operators given by the canonical transformation

$$\begin{aligned} \alpha_\nu &= u_\nu c_{\nu+} - v_\nu c_{\nu-}^+ \\ \beta_\nu &= u_\nu c_{\nu-} + v_\nu c_{\nu+}^+ \end{aligned} \tag{8}$$

with the condition

$$u_\nu^2 + v_\nu^2 = 1. \tag{9}$$

The value v_ν^2 is the occupation probability for state $|\nu\rangle$ and

$$v_\nu^2 = 1/2(1-(\varepsilon_\nu-\lambda)/E_\nu) \tag{10}$$

where

$$E_\nu = [(\varepsilon_\nu-\lambda)^2+\Delta^2]^{1/2} \tag{11}$$

is the quasiparticle energy. The values λ and Δ are the Fermi level and the "energy gap" parameters respectively. They are obtained from the superconductivity equations:

$$\sum_\nu 1/E_\nu = 2/G, \tag{12}$$

$$\sum_\nu (1-(\varepsilon_\nu-\lambda)/E_\nu) = N, \tag{13}$$

where N is the number of particles.

If one assumes that the number of quasiparticles in the ground state due to the quadrupole forces is small then the $\hat{S}$ operator is given by the formula

$$\hat{S} = \sum_{\nu\nu'} (u_\nu v_{\nu'}+u_{\nu'}v_\nu)\ [s_{\nu\nu'}\alpha_\nu^+\beta_{\nu'}^+ + \bar{s}_{\nu\nu'}\alpha_\nu^+\alpha_{\nu'}^+]. \tag{14}$$

In order to obtain the energy of this state we shall use the first order time dependent perturbation theory treating the restoring long range interaction as the perturbation [12]. If we assume that $\chi_p = \chi_n = \chi_{np} = \chi$ [13] our whole Hamiltonian takes the form

$$H = H_1 - 1/2\chi(\hat{S}_n^+ + \hat{S}_p^+)(\hat{S}_n+\hat{S}_p). \tag{15}$$

Let us now introduce the single-particle Hamiltonian

$$H_g = H_1 - \chi(S_n+S_p)(\hat{S}_n+\hat{S}_p), \tag{16}$$

where

$$S_p+S_p = \langle\psi|(\hat{S}_n+\hat{S}_p)|\psi\rangle \tag{17}$$

and the wave function $|\psi\rangle$ is obtained from the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H_g|\psi\rangle. \tag{18}$$

If we assume that S_p+S_n has periodic time-dependence

$$S_n+S_p = (S_n+S_p)_0 \cos\omega t, \tag{19}$$

the self-consistency condition (17) leads us to the formula:

$$1/2\chi = \sum_{pp'} \frac{(|s_{pp'}|^2+|\bar{s}_{pp'}|^2)\,(u_p v_{p'}+v_p u_{p'})^2\,(E_p+E_{p'})}{(E_p+E_{p'})^2-(\hbar\omega_\gamma)^2} + \\ + \sum_{nn'} \frac{(|\bar{s}_{nn'}|^2+|s_{nn'}|^2)\,(u_{n'}v_n+v_{n'}u_n)\,(E_n+E_{n'})}{(E_n+E_{n'})^2-(\hbar\omega_\gamma)^2} \tag{20}$$

p, p' and n, n' denote the states of protons and neutrons respectively.

The energy of the collective vibrational state is given by the lowest solution of this equation. In the adiabatic limit where one assumes that the collective frequency is much lower than the single particle frequency

$$\hbar\omega_\gamma \ll E_p + E_{p'}$$

and

$$\hbar\omega_\gamma \ll E_n + E_{n'}$$

the stiffness parameter and mass parameter introduced in (4) are given by

$$C_\gamma = 1/2\Sigma_1 - \chi \tag{21}$$

$$B_\gamma = \hbar^2\Sigma_3/2\Sigma_1^2 \tag{22}$$

where

$$\Sigma_1 = \Sigma_{1p} + \Sigma_{1n}$$

$$= \sum_{pp'} \frac{(|s_{pp'}|^2 + |\bar{s}_{pp'}|^2)\,(u_p v_{p'} + v_p u_{p'})^2}{E_p + E_{p'}} + \sum_{nn'} \frac{(|s_{nn'}|^2 + |\bar{s}_{nn'}|^2)\,(u_n v_{n'} + u_{n'} v_n)^2}{E_n + E_{n'}} \tag{23}$$

and

$$\Sigma_3 = \Sigma_{3p} + \Sigma_{3n}$$

$$= \sum_{pp'} \frac{(|s_{pp'}|^2 + |\bar{s}_{pp'}|^2)\,(u_p v_{p'} + u_{p'} v_p)^2}{(E_p + E_{p'})^3} + \sum_{nn'} \frac{(|s_{nn'}|^2 + |\bar{s}_{nn'}|^2)\,(u_n v_{n'} + v_n u_{n'})^2}{(E_n + E_{n'})^3}. \tag{24}$$

Hence the energy of the collective vibrational state is equal to

$$E_\gamma = \hbar((1/2\Sigma_1 - \chi)/\hbar^2\Sigma_3/2\Sigma_1^2)^{1/2}. \tag{25}$$

b. Details of calculations

As was mentioned above the single particle states are given by the Nilsson potential [10] of the form:

$$H_0 = 1/2\hbar\omega_0(\varepsilon,\varepsilon_4)\,(-\Delta^2 + \varrho^2) - \varkappa\hbar\mathring{\omega}_0 \Bigg[-4/3\,\frac{\varepsilon}{\varkappa}\,\frac{\omega_0(\varepsilon,\varepsilon_4)}{\mathring{\omega}_0}\,\sqrt{\pi/5}\,\varrho^2 Y_{20} -$$

$$-2\mathbf{ls} - \mu(l^2 - \langle l^2\rangle) + \sqrt{\pi}/3\,\frac{2\,\varepsilon_4}{\varkappa}\,\varrho^2\omega_0(\varepsilon,\varepsilon_4)/\mathring{\omega}_0 Y_{40}\Bigg], \tag{26}$$

where

$$\varrho^2 = \xi^2 + \eta^2 + \zeta^2,$$

and

$$\xi = x(M\omega_0(\varepsilon)(1+\varepsilon/3)/\hbar)^{1/2},$$

$$\eta = y(M\omega_0(\varepsilon)(1+\varepsilon/3)/\hbar)^{1/2},$$

$$\zeta = z(M\omega_0(\varepsilon)(1-2/3\varepsilon)/\hbar)^{1/2}. \tag{27}$$

ε and ε_4 are the quadrupole and hexadecapole deformation parameters of the nucleus, $\omega_0(\varepsilon, \varepsilon_4)$ is obtained from the condition of constant volume closed by an equipotential surface.

In order to obtain the position of the γ-oscillation in the rare earth region the Hamiltonian (26) was diagonalized [14] with the use of the experimental values of the quadrupole and hexadecapole deformation for few isotopes of Sm, Dy, Er and Yb for every nucleus separately. The parameters of the Nilsson model in this region are [10]:

$$\varkappa_{\text{protons}} = 0.0637 \qquad \varkappa_{\text{neutrons}} = 0.0637$$

$$\mu_{\text{protons}} = 0.60 \qquad \mu_{\text{neutrons}} = 0.42.$$

In the actinide region the corresponding parameters are [10]:

$$\varkappa_{\text{protons}} = 0.0577 \qquad \varkappa_{\text{neutrons}} = 0.0635$$

$$\mu_{\text{protons}} = 0.65 \qquad \mu_{\text{neutrons}} = 0.325.$$

In order to find the position of the Fermi level and the value of the energy gap the superconductivity equations (12) have been solved with the use of 24 Nilsson levels and with the following strength of the pairing interaction:
in the rare earth region:

$$G_{\text{proton}} = 32.2/A \text{ MeV} \qquad G_{\text{neutron}} = 26.5/A \text{ MeV},$$

in the actinide region:

$$G_{\text{proton}} = 32.2/A \text{ MeV} \qquad G_{\text{neutron}} = 26.04/A \text{ MeV}.$$

In order to obtain the values of Σ_1 and Σ_3 we took into account 120 double degenerated states of protons and 165 states of neutrons in the rare earth region and 165 proton states and 220 neutron states for the actinide region.

The only free parameter in this calculation is the strength of the restoring non-axial quadrupole interaction; its dependence on the mass number A has the form:

$$\chi = \chi_0(M\omega_0(\varepsilon)/\hbar)^2 A^{-4/3}\hbar\omega_0, \tag{28}$$

where

$$\hbar\omega_0 = 41/A^{1/3} \text{ MeV}. \tag{29}$$

The value of χ_0 was fitted to the experimental energies E_γ for all the nuclei in both regions seperately.

3. *γ-oscillations of the uniformly charged ellipsoidal drop*

As was mentioned in section 1 we can find the γ-vibrational characteristics using two different descriptions — the microscopic and the phenomenological ones. It seems to be interesting to compare the results obtained by these different methods.

In the present section we shall calculate the phenomenological parameters with the use of the hydrodynamical model. We shall assume here that the nucleus is a uniformly charged ellipsoidal drop the semi-axis of which have the form [15]:

$$a_1 = R_0 \exp(\sigma \cos \delta),$$
$$a_2 = R_0 \exp(\sigma \cos(\delta - 2\pi/3)),$$
$$a_3 = R_0 \exp(\sigma \cos(\delta + 2\pi/3)). \tag{30}$$

The correspondence between the deformation parameters σ and δ and the β and γ parameters introduced in (1) is given by the formulas:

$$\sigma = 0.631\beta(1 + 0.045\beta \cos \delta + \ldots)$$
$$\cos 3\gamma = \cos 3\delta - 3/14\sigma \sin^2 \delta. \tag{31}$$

In order to obtain the stiffness parameter we shall calculate the surface and Coulomb energies of the nucleus. The surface energy U_{surf} of such an ellipsoid is given by the product of the surface of the nucleus $2\pi R_0^2 S$ and the surface tension coefficient η [16].

$$u_{surf} = 2\pi R_0^2 S\eta = 2\pi R_0^2 \eta \left[\frac{1}{a_1^3 a_2 \sin\varphi} F_1 + \frac{1}{a_1 a_2 \sin^3\varphi \cos^2\vartheta \sin^2\vartheta} \times \right.$$
$$\left. \times \left(\frac{\cos^2\varphi \sin^2\vartheta}{a_1^2} + \frac{\cos^2\vartheta}{a_3^2} - \frac{a_3^2}{a_2^4} \right) F_2 + \frac{a_3 \cos\varphi}{a_1 a_2^2 \sin^2\varphi \cos^2\vartheta} \left(\frac{1}{a_2^2} - \frac{1}{a_1^2} \right) \right] \tag{32}$$

where

$$\sin\varphi = \frac{a_3}{a_1},$$
$$\sin^2\vartheta = \frac{1-(a_3/a_2)^2}{1-(a_3/a_1)^2},$$
$$F_1 = \int_0^\varphi (1 - \sin^2\vartheta \sin^2\psi)^{-1/2} d\psi, \tag{32a}$$
$$F_2 = \int_0^\varphi (1 - \sin^2\vartheta \sin^2\psi)^{1/2} d\psi$$

and η is the surface tension coefficient.

The Coulomb energy of the ellipsoid has the form [17]:

$$u_c = 1 - 1/5\delta^2 - 1/105\sigma^3 \cos 3\delta + 1/28\sigma^4 + 13/4620\sigma^5 \cos 3\delta -$$
$$-203/28600\sigma^6 + 379/900900\sigma^6 \cos 3\delta = \langle r_1^2 \rangle\, u_c^{sphere}. \tag{33}$$

It is easy to notice that both the surface and the Coulomb energies increase with the non-axial deformation parameter δ. One can also notice that the harmonic approximation may be used here. Thus the energy of the γ-vibrational level is given by the formula:

$$E_\gamma = \hbar(C_\gamma/B_\gamma)^{1/2}, \tag{34}$$

where the stiffness parameter C_γ is equal to

$$C_\gamma = \left(\frac{\partial^2(u_{surf}+u_c)}{\partial\delta^2}\right)_{\delta=0} = 2\pi R_0^2\eta\left(\frac{\partial^2 S}{\delta\partial^2}\cdot+4x\frac{\partial^2\langle r_1^2\rangle}{\partial\delta^2}\right)_{\delta=0}. \tag{35}$$

The fissility parameter

$$x = \frac{\text{Coulomb energy of sphere}}{2\text{ (surface energy of sphere)}} = \frac{Z^2/A}{51.77\,(1-1.79\,((N-Z)/A)^2)}\ [18]. \tag{36}$$

The mass parameter B_γ can be obtained by calculating the kinetic energy of the potential flow

$$T = 1/2\varrho\iiint \boldsymbol{v}^2 d\tau \tag{37}$$

where ϱ is the density of the liquid and $\boldsymbol{v}$ is the velocity of the potential flow. We have then

$$\text{rot}\,\boldsymbol{v} = 0 \tag{38}$$

and

$$\boldsymbol{v} = -\text{grad}\,\varphi \tag{39}$$

so the potential of this velocity φ may be calculated from the equation

$$\Delta\varphi = 0, \tag{40}$$

with the condition

$$\text{grad}\,\varphi\,\text{grad}\,F + \frac{\partial F}{\partial t} = 0, \tag{41}$$

where

$$F(x, y, z, t) = \frac{x^2}{[a_1(t)]^2} + \frac{y^2}{[a_2(t)]^2} + \frac{z^2}{[a_3(t)]^2} - 1 = 0 \tag{42}$$

is the equation of the ellipsoid.

It can be noticed that φ has the form [19]

$$\varphi = -1/2\left(\frac{\dot a_1}{a_1}x^2 + \frac{\dot a_2}{a_2}y^2 + \frac{\dot a_3}{a_3}z^2\right), \tag{43}$$

and for the kinetic energy we obtain the formula:

$$T = 1/10M(\dot a_1^2 + \dot a_2^2 + \dot a_3^2) \tag{44}$$

where M is the mass of the drop. Now with the use of Eq. (30) we can write the last formula as the sum of three terms

$$T = T_{\sigma\sigma} + T_{\delta\delta} + T_{\sigma\delta} \tag{45}$$

$T_{\sigma\sigma}$ has the meaning of the kinetic energy of β-vibrations. $T_{\delta\delta}$ is the corresponding energy for the vibrations of the γ-type and $T_{\sigma\delta}$ describes the interaction of both types of the vibrational motion. For the non-axial vibration arround the axially symmetrical ellipsoidal shape $T_{\sigma\delta} = 0$ and

$$T_{\delta\delta} = 3/20MR_0^2\sigma^2e^{-\sigma}\dot{\delta}^2. \tag{46}$$

For the mass parameter we obtain

$$B_\gamma = 2T_{\delta\delta}/\dot{\delta}^2 = 0.3MR_0^2\sigma^2e^{-\sigma} \tag{47}$$

where

$$MR_0^2 \approx \frac{0.033\,A^{5/3}}{1-1.79((N-Z/A)^2}\,\hbar^2\,\mathrm{MeV}^{-1}. \tag{48}$$

Using (35) and (37) we get

$$E_\gamma = \hbar\sqrt{\frac{C_\gamma}{B_\gamma}} = \left(\frac{\frac{\partial^2}{\partial\delta^2}\,(S/4\pi R_0^2) + 2x\,\frac{\partial^2}{\partial\delta^2}\,\langle r_1^2\rangle}{0.3\,\sigma^2e^{-\sigma}}\right)^{1/2}\hbar\left(\frac{4\pi R_0^2\eta}{MR_0^2}\right)^{1/2}, \tag{49}$$

where

$$\hbar\left(\frac{4\pi R_0^2\eta}{MR_0^2}\right)^{1/2} = 23.12\left(\frac{1-1.79\,((N-Z)/A)^2}{A}\right)^{1/2}\mathrm{MeV}.$$

4. *Results and discussion*

The results for the rare earth region are given in Table I. The calculations are performed *i*) with the value of the hexadecapole deformation parameter ε_4 taken from experiment [20] and *ii*) with the value $\varepsilon_4 = 0$. One can notice that the case *i*) gives much better values of energies than the case *ii*).

TABLE I

Energy E_γ of the γ-vibrational state in the rare earth region. First column gives the nucleus, the next three columns the experimental values for E_γ, the quadrupole ε and the hexadecapole ε_4 deformation parameters. Two remaining columns present calculated values for E_γ obtained with the corresponding value of ε_4 and the result for $\varepsilon_4 = 0$

Nucleus	E_γ exper. (MeV)	ε	ε_4	E_γ theor. (MeV)	E_γ theor. ($\varepsilon_4 = 0$) (MeV)
$_{62}Sm^{152}$	1.09	0.25	−0.02	0.79	0.83
$_{62}Sm^{154}$	1.45	0.29	−0.02	1.60	1.60
$_{66}Dy^{160}$	0.97	0.26	−0.02	1.12	1.21
$_{68}Er^{164}$	0.84	0.27	0.02	0.97	1.26
$_{68}Er^{170}$	0.95	0.27	0.04	1.04	1.18
$_{70}Yb^{170}$	1.23	0.27	0.06	1.20	1.10
$_{70}Yb^{172}$	1.47	0.27	0.06	1.27	1.21
$_{70}Yb^{176}$	1.26	0.27	0.07	0.98	0.82

The value of χ_0 in this region is $\chi_0 = 1.748$.

In Table II we present the results obtained for the even isotopes of Th, U and Pu at the equilibrium deformation $\varepsilon, \varepsilon_4$. Here $\chi_0 = 1.64$.

TABLE II

Energy E_γ of the γ-vibrational state in the actinide region. For description of columns, see Table I, with the exception of the last column

Nucleus	E_ε exper. (MeV)	ε	ε_4	E_γ theor. (MeV)
$_{90}Th^{228}$	0.96	0.177	−0.026	0.64
$_{90}Th^{230}$	0.78	0.184	−0.023	0.72
$_{90}Th^{232}$	0.79	0.193	−0.021	0.81
$_{92}U^{232}$	0.87	0.200	−0.024	0.92
$_{92}U^{234}$	0.92	0.209	−0.022	0.98
$_{92}U^{236}$	—	0.212	−0.018	1.06
$_{92}U^{238}$	1.06	0.218	−0.016	1.18
$_{94}Pu^{238}$	1.03	0.216	−0.019	1.14
$_{94}Pu^{240}$	0.94	0.230	−0.015	1.35
$_{94}Pu^{242}$	—	0.235	−0.012	1.37

In both regions the agreement with the experimental results is quite good, therefore we use the same method of calculation for larger values of the quadrupole deformation parameter ε — up to the saddle point.

The microscopic results for E_γ for U^{232}, U^{236} and Pu^{240} and the hydrodynamical results for U^{236} are shown in Fig. 1. It may be noticed that in the microscopic results the energy of the γ-vibrational state decreases with the deformation up to the value $\varepsilon \sim 0.3$–0.4, then increases, and at the saddle point it is about twice as large as that at the equilibrium.

In Fig. 2 and 3 we present the stiffness parameter C_γ and the mass parameter B_γ for the microscopic and hydrodynamical methods of calculation. One may notice that the microscopic C_γ has its minimum for $\varepsilon \sim 0.3$–0.4 and then increases very rapidly with the deformation ε. The mass parameter B_γ increases nearly monotonically with the deformation of the nucleus.

As a general conclusion from Figs 1, 2 and 3 we see that the strongly deformed nucleus is more stiff with respect to the non-axial vibration in comparison with the nucleus at its equilibrium state. We also notice that the same result is obtained in the hydrodynamical model. The decreasing of C_γ and E_γ for $\varepsilon \sim 0.3$–0.4 is a microscopical effect connected with the changes of the density of the Nilsson single-particle levels in this deformation region. All the microscopic results for the γ-vibrational parameters for different deformations of the nuclei in the actinide region are calculated with the use of the same value of the pairing force strength G. We assume also that the quadrupole coupling constant χ_0 does not depend on the deformation parameter.

There are some experimental suggestions that for U^{234}, U^{236} and Pu^{240} the γ-vibrational state in the saddle point has the smaller energy value than at equilibrium [21].

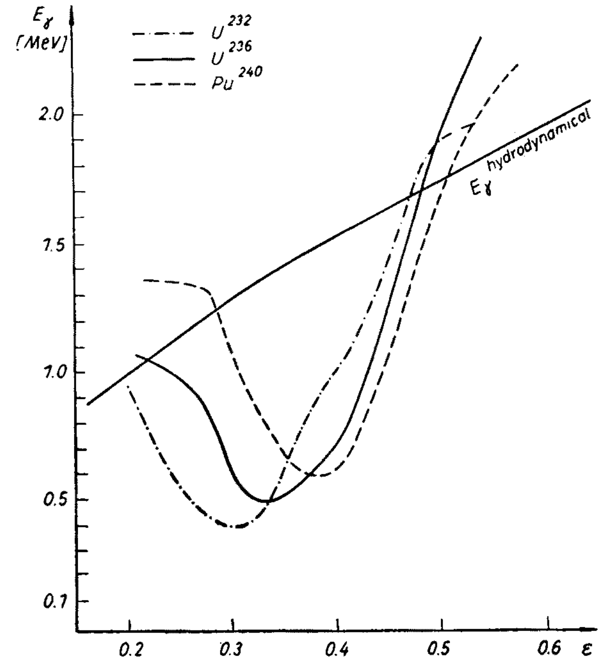

Fig. 1. Microscopic values of energy of γ-vibrational state for U^{232}, U^{236}, Pu^{240} and hydrodynamical results for U^{236} as the functions of the deformation parameter ε

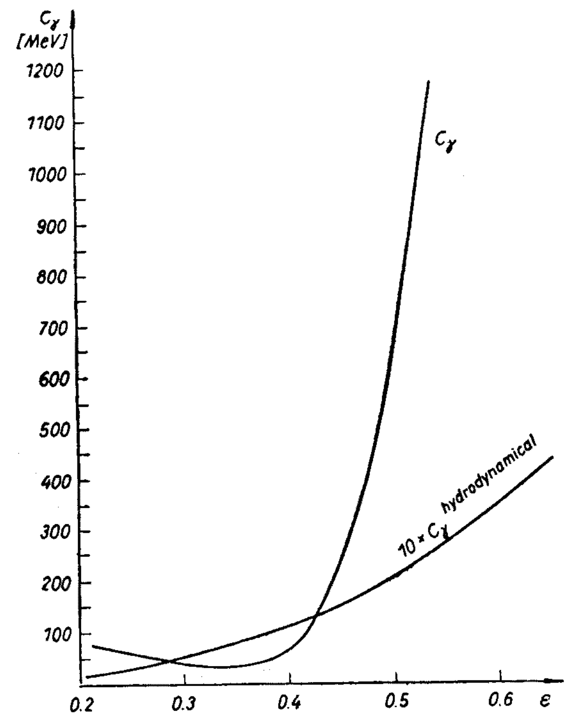

Fig. 2. Microscopic and hydrodynamical stiffness parameter C_γ as function of the deformation parameter ε

Our results are not in agreement with these experiments. However if the value of χ_0 in the saddle point is increased by about 30% the agreement is quite good in all the cases. It may be interesting to notice that the equilibrium results for the rare earth and the actinide

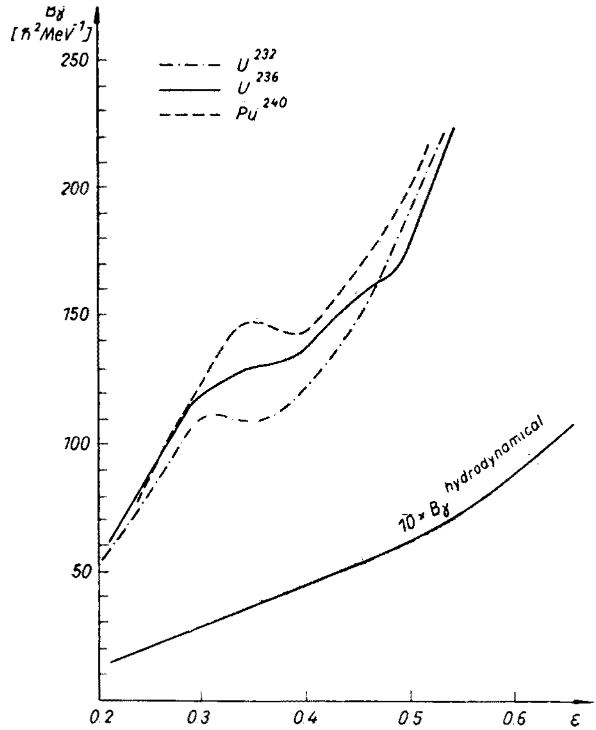

Fig. 3. The microscopic mass parameter B_γ for U^{232}, U^{236}, Pu^{240} and hydrodynamical mass parameter for U^{236} as functions of the deformation parameter ε

regions suggest that the value of χ_0 increases with the deformation of the nucleus. This seems to support the conclusion of Ref. [22].

One may also try to change the strength of the pairing force for different deformations. If one assumes that G increases proportionally to the surface of the nucleus [23] then the stiffness parameter does not change appreciably in comparison with the case of constant G value. However the mass parameter decreases and as a result the energy increases by about 5% in comparison with the result obtained for $G_{\text{saddle}} = G_{\text{equilibrium}}$.

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