A REGGEIZED PION EXCHANGE MODEL FOR SINGLE PION PRODUCTION PROCESSES

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Single pion production processes $\pi^- p \to n\varrho^0, pp \to n\,\Delta^{++}$ are investigated using a Reggeized one-pion exchange amplitude suggested by the generalized Veneziano model. A good description of the experimental momentum transfer (t) distributions for $|t| < 1$ GeV^2 and laboratory momentum between 1.6 GeV/c and 10 GeV/c is found with only one normalizing parameter fitted to the data for each process.

1. Introduction

In this paper we propose a Reggeized one-pion exchange model describing three-body reactions with resonance production and one-pion exchange domination. The amplitudes of this model are the Veneziano amplitudes [1] constructed by means of standard rules with the following two additional conditions:

i) the amplitudes should be applied in the resonance region,

ii) the dominating graphs are those with one pion exchange (we neglect the remaining graphs).

The amplitudes obtained in this way have the same form as for the Reggeized pion exchange with one additional important feature: the residuum functions have no free parameters. We present a general procedure for constructing the amplitude for the above class of reactions and apply it to processes $\pi^- p \to n\varrho^0$ and $pp \to n\Delta^{++}$ which are likely to be dominated by one-pion exchange. We calculate momentum transfer distibutions for these reactions and compare our results with experiment. The agreement is surprisingly good.

2. Notation

We use the following notation: p_i — the four-momentum vectors, s — square of the total centre-of-mass energy, t — square of the four-momentum transfer,

$$s_{ij} = (p_i+p_j)^2, \quad t_{ij} = (p_i-p_j)^2, \quad u_{ij} = (p_i-p_j)^2$$

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m — $\pi^+\pi^-$ invariant mass, m_π — pion mass,
M — $p\pi^+$ invariant mass, $\alpha(t)$ — Regge trajectory.

The trajectories used in this paper (see table) are taken from Törnqvist [2].

The two reactions in question are schematically represented in Figs 1*a*, *b*.

3. *Details of the model*

We present step-by-step the procedure of constructing the scattering amplitude in our model.

a) Each reaction in question is described by the Veneziano amplitude which is given as a linear combination of 12 terms corresponding to 12 non-equivalent orderings of external lines. It can readily be checked that among these, only few are free from exotic channels

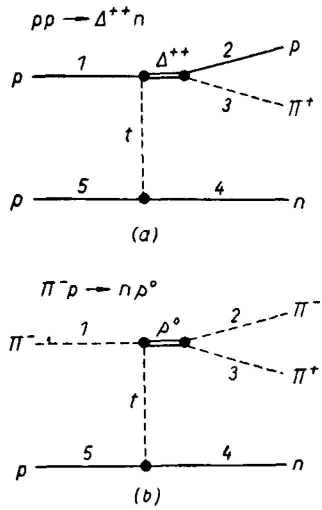

Fig. 1. Scheme for reaction. *a*) $pp \to \Delta^{++}n$; *b*) $\pi^+p \to n\varrho^0$

and are dominated by one pion exchange. Only these graphs are taken into account. The contributions of the remaining graphs are supposed to be very small and are therefore neglected.

b) In practice it is usual to take into account only one trajectory in each channel. In each case we have to insert therefore what we believe to be the dominant trajectory. In the

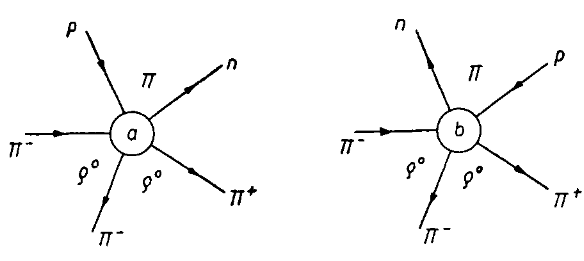

Fig. 2. Dual graphs, which contribute to the amplitude for the reaction $\pi^-p \to n\varrho^0$

special case of reactions $\pi^- p \to n\varrho^0$ and $pp \to n\Delta^{++}$ we arrive at the result shown in Figs 2 and 3. An exception is made in the case of channels in which trajectories are degenerate in pairs. Then we have to allow for both trajectories. Thus, in the case of $p\pi$ channel (reaction $pp \to n\Delta^{++}$), we took into account both N and Δ trajectories (Figs 3*b* and 3*c*) because of the assumed $N\Delta$ degeneracy.

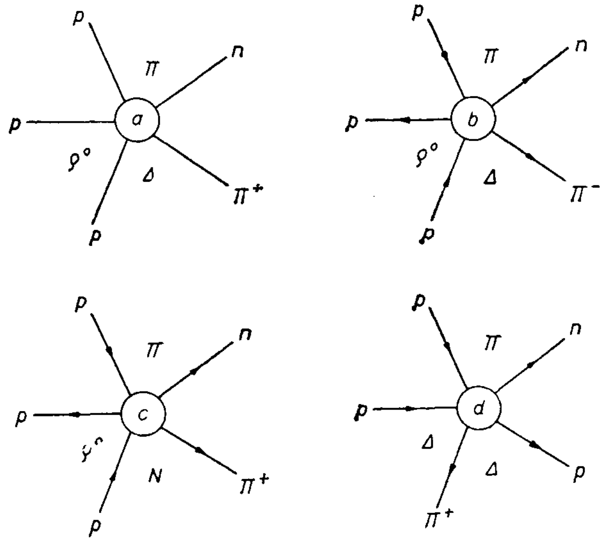

Fig. 3. Four of eight dual graphs, which contribute to the amplitude for the reaction $pp \to \Delta^{++}n$. The remaining graphs are obtained by the interchange of the n and p lines. The label between the pair of adjacent lines represent the trajectory assumed to be dominant in that channel. Only those trajectories are labeled, which enter the amplitudes used in the numerical calculations (formulae 2 and 3)

c) We connect each graph in question with a B_5 function by means of standard rules.

d) We allow for the fact that our model should be applied in the resonance region where the B_5 function can be well approximated by the product of two B_4 functions[1], one of which describes the one-pion exchange, whereas the other contributes to the off-shell amplitude for a two-body process with one pion off the mass shell. In particular, for the diagram in Fig. 1*a* with s_{23} in the resonance region we obtain:

$$B_5 = B_4(-\alpha_{15}(s)_1 - \alpha_{45}(t))B_4(-\alpha_{12}(t_{12}), -\alpha_{23}(s_{23})) \tag{1}$$

e) We fix the relative weights and signs of various graphs contributing to the total amplitude as follows. We divide the graphs into a number of groups (two graphs in each group). In each group two graphs differ only by the interchange of the n and p lines. If we wish the pion trajectory to have a definite and correct signature we should require equal weights and signs of these two graphs. The relative weights and signs of these groups of graphs are established by the requirement that at the pole $t = m_\pi^2$ the correct on-shell

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[1] As it was shown in Ref. [3], the B_5 function can be expressed as a product of two B_4 functions and a generalized hypergeometric function. The latter function can be replaced in the resonance region by 1.

amplitude for a two-body process should be obtained. These are the $\pi\pi$ and πN elastic processes in the case of the two reactions in question, respectively. The amplitudes for these processes are known from other studies in which they were carefully investigated, in the case of $\pi\pi$ elastic scattering, from Ref. [4], and in the case of πN scattering, from Ref. [5]. Since we fixed the relative weights and signs of all graphs contributing to our amplitudes, the only remaining free parameters are the normalizings constants (one per each reaction) which are to be fitted to the experimental data.

f) To take some account of spin we multiply the full amplitude by a factor $G\bar{u}(p_5)\gamma_5 u(p_4)$ (G is the πNN coupling constant, $G^2 = 29.2$) to be able to sum over final spins and average over initial spins of nucleons.

4. *The amplitudes for reactions* $\pi^- p \to n\varrho^0$ *and* $pp \to n\Delta^{++}$

a) $\pi^- p \to n\varrho^0$

The graphs contributing to the amplitude for this reactions are shown in Fig. 2. The amplitude for this reaction takes the form[2]

$$T_\varrho = \beta_\varrho G\bar{u}(p_5)\gamma_5 u(p_4)\,\frac{(1+e^{-i\pi\alpha_\pi})}{\sin\pi\alpha_\pi(t)}\,\frac{\pi e^{\alpha_\pi \ln s}}{\Gamma(1+\alpha_\pi(t))}\,(1-\alpha_\varrho(t_{12})-\alpha_\varrho(s_{23}))B_4(1-\alpha_\varrho(t_{12}),\,1-\alpha_\varrho(s_{23})), \tag{2}$$

where β_ϱ is the normalizing constant.

b) $pp \to n\Delta^{++}$

The graphs contributing to the amplitude for this reaction are shown in Fig. 3. The amplitude for this reaction takes the form

$$T_\Delta = \beta_\Delta G\bar{u}(p_5)\gamma_5 u(p_4)\,\frac{(1+e^{-i\pi\alpha_\pi})}{\sin\pi\alpha_\pi(t)}\,\frac{\pi e^{\alpha_\pi \ln s}}{\Gamma(1+\alpha_\pi(t))}\,A_{p\pi^+}, \tag{3}$$

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[2] We use the well-known relations for B_4 and Γ functions to write the above amplitudes in the form presented in formulae (2) and (3). Below are given some relations for B_4 and Γ functions which are used in this work

$$B_4(x,y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$$

$$\Gamma(x) = \frac{\pi}{\sin\pi x}\,\frac{1}{\Gamma(1-x)}$$

$$\frac{\Gamma(y)}{\Gamma(x+y)} = \exp(-x\ln y) \text{ for small } x \text{ and large } y.$$

By means of these three equations we can write the B_4 function, which describes the one-pion exchange, in the form presented in formula (2).

where

$$A_{p\pi^+} = 3\left[C\left(\frac{3}{2}-\alpha_\Delta(s_{23}),\ \frac{3}{2}-\alpha_\Delta(u_{13})\right)-C\left(\frac{3}{2}-\alpha_\Delta(s_{23}),\ 1-\alpha_\varrho(t_{12})\right)\right]+$$
$$+C\left(1-\alpha_\varrho(t_{12}),\ \frac{3}{2}-\alpha_\Delta(u_{13})\right)+C\left(\frac{3}{2}-\alpha_N(u_{13}),\ 1-\alpha_\varrho(t_{12})\right) \quad (4)$$

and β_Δ is the normalizing constant.

The function C is defined by

$$C(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y-1)}. \quad (5)$$

5. *Comparison with experiment. Discussion*

The differential cross-sections $\frac{d\sigma}{d|t|}$ for both reactions in question were obtained experimentally[3] by considering all events in certain ϱ^0 and Δ^{++} mass intervals which typically are 200 MeV wide. The Veneziano cross-sections obtained on the basis of (2) and (3) are integrated over exactly the same ϱ^0 and Δ^{++} mass regions. By means of only one normalizing constant for each reaction a good description of the experimental data for $|t| < 1\ \mathrm{GeV}^2$

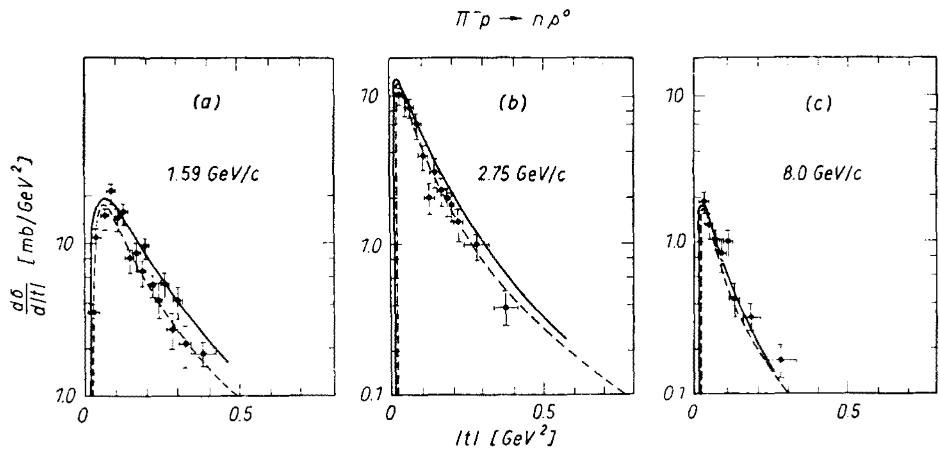

Fig. 4. Differential cross-section $\frac{d\sigma}{d|t|}$ for events of reaction $\pi^-p \to n\varrho^0$ in the ϱ^0 region. The solid curves give the present results. The dashed curves give the Wolf's results.
a — at 1.59 GeV/c (0.616 GeV < m < 0.85 GeV)
b — at 2.75 GeV/c (0.65 GeV < m < 0.85 GeV)
c — at 8.0 GeV/c (0.675 GeV < m < 0.875 GeV)

at several energies of the incoming particle is achieved. The results are presented in Figs 4a, b, c and 5a, b. Mass intervals used in the calculations are explicitly given in figure captions.

[3] A review of the experimental data can be found in Ref. [6].

The presented model describes also the experimentally known fact that the width of the forward peak in the differential cross-sections $\frac{d\sigma}{d|t|}$ shrinks with increasing momentum of the incident particle. The apparent schrinkage in this model originates from the propor-

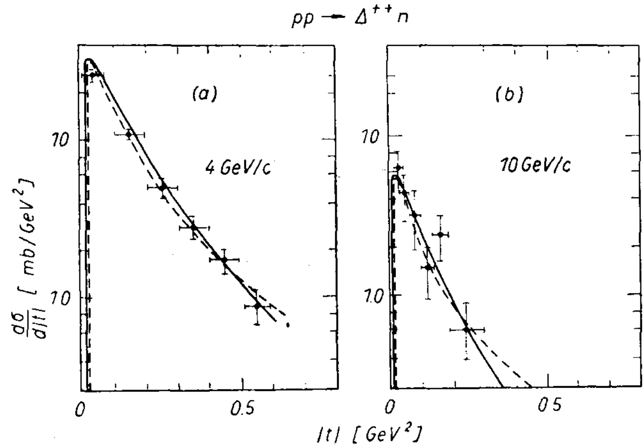

Fig. 5. Differential cross-sections $\frac{d\sigma}{d|t|}$ for events of reaction $pp \rightarrow \Delta^{++}n$ in the Δ^{++} region. The meaning of curves as in Fig. 4.

a — at 4.0 GeV/c (1.08 GeV < M < 1.40 GeV)

b — at 10.0 GeV/c (1.125 GeV < M < 1.325 GeV)

tionality of $\frac{d\sigma}{d|t|}$ to $\exp(2\alpha_\pi \ln s)$. If it is so, the assumption that the exchange of Reggeized pion is the dominant production mechanism for the processes in question, is a correct one.

We also compare our results with those of Wolf [6], who used the OPE model with Benecke-Dürr [7] parametrization. Wolf's results are in slightly better agreement with experiment than ours. However, while in the OPE model one has to fit one parameter for each

Table of trajectories

Below threshold	Above threshold
$\alpha_\varrho(t) = 0.48+0.9\,t$	$\alpha_\varrho(s) = 0.48+0.9\,s+i\,0.13\,\sqrt{s-0.08}$
$\alpha_N(t) = -0.30+0.9\,t$	$\alpha_N(s) = -0.37+1.0\,s+i\,0.13\,(s-1.0)$
$\alpha_\Delta(t) = 0.13+0.9\,t$	$\alpha_\Delta(s) = 0.13+0.9\,s+i\,0.13\,(s-1.0)$
$\alpha_\pi(t) = 0.9\,(t-m_\pi^2)$	

vertex to describe the two above discussed reactions, and in addition it is necessary to take from experiment the values of the elastic scattering cross-sections $\sigma_{\pi^+\pi^-}$ and σ_{π^+p}, in our model we have to fit only one normalizing constant for each reaction.

The final conclusion is as follows: the Reggeized one-pion exchange amplitude in the form suggested by the generalized Veneziano model (B_5) gives a rather good description of $\pi^- p \to n\varrho^0$ and $pp \to n\Delta^{++}$ reactions. It would be interesting to check if this model holds also for other single-pion production processes.

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