EIKONAL MODEL FOR HIGH ENERGY ELASTIC SCATTERING

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The elastic processes pp, $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$ are investigated in terms of a model with Regge poles and cuts generated by multiple scattering. The Born term consists of the Pomeron and degenerate meson trajectories contributions. Predictions of the model are in reasonable agreement with the experimental data for total cross-sections and the slope parameter of elastic differential cross-sections in the energy range from 10 to 70 GeV. The slope of Pomeranchuk trajectory is found to be 0.4 $(\text{GeV}/c)^{-2}$. Some difficulties of the present form of the model connected with antishrinkage phenomenon and the shape of ReF/ImF in the forward direction are discussed. Some predictions are given about the asymptotic values of total cross-sections and their behaviour in the high energy region.

Among many papers dealing with the problem of high energy elastic scattering (see *e. g.* [1–10]) which appeared recently, the paper by Frautschi and Margolis [1] contains a description of a model based on the Pomeron exchange with corrections given by its multiple scattering (in the eikonal sense). We shall generalize the FM model by adding the meson trajectories exchange to that of the Pomeron. Mesons will be taken in the simplest form, namely in the strongly degenerate pairs. The assumption of degeneration, although well justified in this case by duality arguments [11], is an approximate one and is the source of some difficulties with the description of the data. Nevertheless, it simplifies enormously the used formulae and leaves only five unknown parameters for each pair of reactions, involving the scattering of a particle and corresponding antiparticle on protons.

For the sake of simplicity we shall neglect the spin effects and consider for each reaction only one, nonspinflip amplitude $F(s, t)$ with normalization given by the relations [1]

$$\sigma^{\text{tot}} = \frac{4\pi}{k\sqrt{s}} \, \text{Im}\, F(s, t = 0), \tag{1}$$

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2 s} \, |F|^2, \tag{2}$$

where k is the cms momentum.

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Now, the basic assumptions of the model can be listed as follows:

1. The full amplitude $F(s, t)$ is represented by the Glauber relation [12, 1]

$$F(s, t) = ik\sqrt{s}\int \frac{d^2b}{2\pi}\,[1-e^{2i\delta(b)}]e^{i\vec{b}\vec{q}}, \tag{3}$$

where b is the impact parameter, δ the phase shift and $\vec{q}$ the three-momentum transfer.

2. The Born term is represented by the Pomeron and two degenerate mesons exchange

$$F_{\text{pole}} = R_P e^{\beta_P t}\left(\frac{s}{s_0}\,e^{-i\frac{\pi}{2}}\right)^{\alpha_P(t)} + R_M e^{\beta_M t}\left(\frac{s}{s_0}\right)^{\alpha_M(t)}[1+e^{-i\pi\alpha_M(t)}\pm(1-e^{-i\pi\alpha_M(t)})], \tag{4}$$

where R_P, R_M, β_P, β_M represent the couplings and slopes of residue functions of Pomeron and meson respectively, and $\alpha_P(t)$, $\alpha_M(t)$ are their trajectory functions. The constant s_0 will be taken as 1 GeV^2 [1]. The signs in the square bracket depend on the considered reaction.

3. All meson trajectories cont ibuting to (4) are assumed to be strongly degenerate, *i. e.* have equal residua and trajectories.

4. The meson trajectory is taken in the form

$$\alpha_M(t) = 0.5 + t. \tag{5}$$

The amplitude given by (4) can be thought of as a first step on the way of including mesons in the FM model. However, an ambiguity of this procedure requires some comments on the shape of amplitude F_{pole}.

1. The assumption of the strong degeneracy of mesons is necessary to ensure the compact form of a meson contribution. If we had, instead of one meson terms, the sum of two (or more) different terms, the formula for the full amplitude would complicate considerably, resulting in increasing the calculation time by about two orders of magnitude as compared with the non-degeneracy assumption. Of course, trajectories need not have the form (5) but this is the simplest assumption, well justified by experiment.

2. The number of mesons which can be exchanged depends on the considered reaction. In this paper we shall take into account trajectories: P', ω for pp; P', ϱ for πp; P', ω, ϱ, A_2 for Kp. Thus, in each case we have degenerate pairs consisting of trajectories with opposite signatures. Due to the strong degeneracy, even for the case of two pairs, the only change will be the factor 2 in the second term in (4) (for Kp the contant R_M should be replaced by 2 R_M).

Having discussed the Born term, we come now to the formula for the full amplitude (3). Eq. (4) can be written in a shorter form

$$F^{1,2}_{\text{pole}} = \varkappa_P e^{\gamma_P t} + \varkappa^{1,2}_M e^{\gamma^{1,2}_M t}, \tag{6}$$

where

$$\varkappa_P = R_P\left(\frac{s}{s_0}\,e^{-i\frac{\pi}{2}}\right)^{\alpha^0_P},$$

$$\varkappa^1_M = 2R_M\left(\frac{s}{s_0}\right)^{\alpha^0_M},$$

$$\varkappa_M^2 = 2\,R_M \left(\frac{s}{s_0}\, e^{-i\pi}\right)^{\alpha_M^0},$$

$$\gamma_P = \beta_P + \alpha_P' \left(\ln \frac{s}{s_0} - i\,\frac{\pi}{2}\right),$$

$$\gamma_M^1 = \beta_M + \alpha_M' \ln \frac{s}{s_0},$$

$$\gamma_M^2 = \beta_M + \alpha_M' \left(\ln \frac{s}{s_0} - i\pi\right). \tag{7}$$

Index 1 is for the reactions with a positive incident particle, index 2 — for those with a negative one. Now, calculating $\delta(b)$ as the Fourier transform of F_{pole} [1] we get

$$2i\delta(b) = \frac{i\varkappa_P}{2k\sqrt{s}\,\gamma_P}\, e^{-\frac{b^2}{4\gamma_P}} + \frac{i\varkappa_M}{2k\sqrt{s}\,\gamma_M}\, e^{-\frac{b^2}{4\gamma_M}} \tag{8}$$

and the final result after substituting this into (3) and denoting

$$\delta_j = \frac{i\varkappa_j}{2k\sqrt{s}\,\gamma_j} \quad (j = P, M) \tag{9}$$

is

$$F(s,t) = -2\,ik\sqrt{s}\,\gamma_P\gamma_M \sum_{n=1}^{\infty} \sum_{j=0}^{n} \frac{1}{n!} \binom{n}{j} \frac{1}{(n-j)\gamma_M + j\gamma_P} \times$$

$$\times\, \delta_P^{n-j} \delta_M^j e^{t\,\frac{\gamma_P\gamma_M}{(n-j)\gamma_M + j\gamma_P}} \tag{10}$$

(with the indices 1 and 2 dropped for simplicity).

The main trouble with this formula is connected with the infinite sum. However, the direct calculation, similar to that in Ref. [4], shows that for $|t| \leqslant 0.5$ (GeV/c)^2, with the upper limit of the sum $N = 7$, the relative error in the amplitude is less than 10^{-5}. For practical purposes we took $N = 5$, which corresponds to the relative error $\leqslant 0.001$. In terms of the cuts generated by the multiple scattering this means that the smallest non-neglected contribution comes from 5-fold scattering of Pomeron and mesons [2].

Since the Pomeron in the FM model is a moving trajectory, we have altogether five parameters. The first one, the slope of Pomeron, α_P' is common for all reactions, the others — parameters of residua R_P, R_M, β_P, β_M can be different for different pairs of reactions. This means that we need 13 parameters to describe 6 reactions.

Our aim is to reproduce the experimental data for σ^{tot} Re $F(s,0)$/(Im $F(s,0)$, and $\frac{d\sigma}{dt}$ for $|t| \leqslant 0.5$ (GeV/c)^2, for the elastic scattering of pp, $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$. The procedure of evaluating the values of parameters consists of two steps.

First, basing on the comparison of the slope of differential cross-sections in the forward direction given by the formula

$$b(s) = \left[\frac{d}{dt}\left(\frac{d\sigma}{dt}\right) \Big/ \frac{d\sigma}{dt}\right]_{t=0} \tag{11}$$

with experiment, we get the starting values of α'_P, β_P, and β_M, and then, from total cross-sections, those of R_P and R_M. Unfortunately, the experimental data on $b(s)$ for Serpukhov energies are published only for pp [13]. Using them we get the value of Pomeron slope

$$\alpha'_P = 0.4\ (\text{GeV}/c)^{-2} \tag{12}$$

which agrees with the previous calculations [3, 6, 9, 10]. For the other reactions the data are somewhat ambiguous with different procedures of fitting used by different authors. Thus,

TABLE I

Values of parameters and asymptotic values of total cross-sections

Reaction	β_P [GeV^{-2}]	β_M [GeV^{-2}]	R_P	R_M	σ^{tot}_{∞} [mb]
$pp \to pp$ $\bar{p}p \to \bar{p}p$	3.4	0.5	−4.4	−5.7	43.0
$\pi^+p \to \pi^+p$ $\pi^-p \to \pi^-p$	2.4	0.5	−2.75	−0.8	26.9
$K^+p \to K^+p$ $K^-p \to K^-p$	1) 1.8 2) 0.3	0.5 0.5	−1.9 −2.2	−0.8 −0.85	18.6 21.5

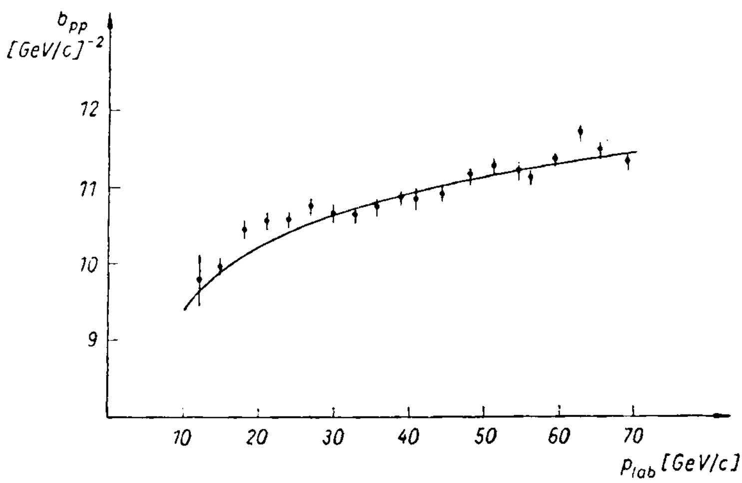
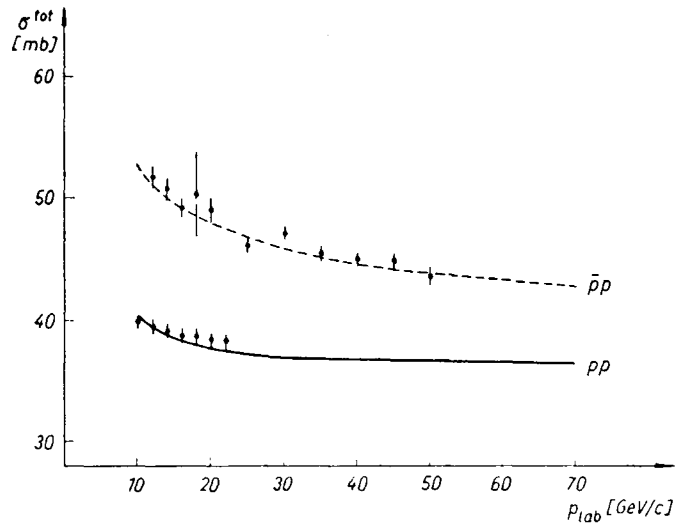


Fig. 1. Slope of the eleastic pp differential cross-section as a function of laboratory momentum of incident proton. Experimental data are taken from Ref. [13]

we took only their average values [6] to evaluate β_P, while β_M had a very small influence on all the experimental quantities. (Except that for very large values of β_M the shape of σ^{tot} disagrees with experiment, what will be discussed later).

Having obtained the starting values of the parameters, we made a fit to the total cross-sections. The results are listed in Table I and compared with experimental data for $b(s)$ [13], σ^{tot} [14, 15], and $\frac{d\sigma}{dt}$ [16–19] on Figs 1–3. The agreement is quite good considering such a small number of parameters. The main trouble is with Kp scattering. The flat or even increasing K^+p total cross-section requires very small β_P and, consequently, a too small slope of differential cross-sections. On the other hand, a value of β_P taken to obtain a correct



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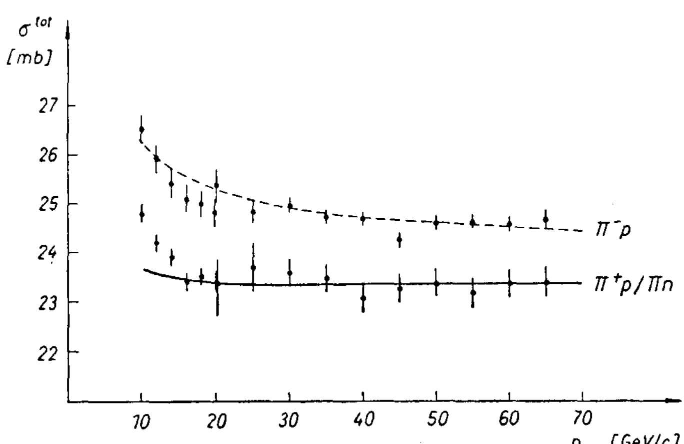

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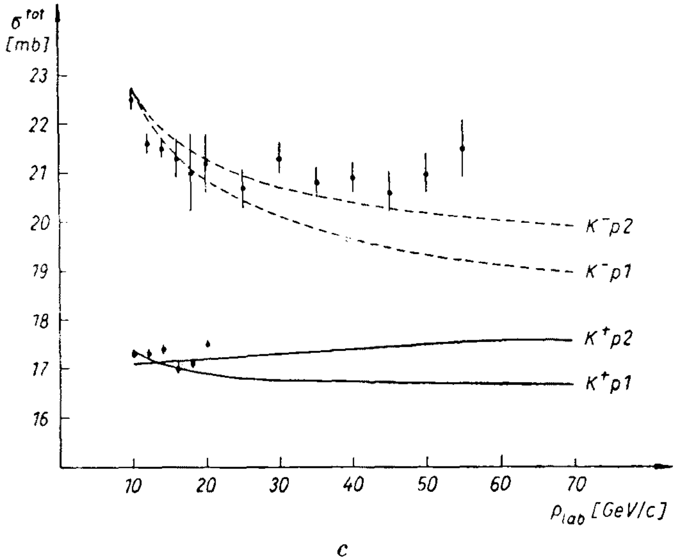

Fig. 2. Total cross-sections for: *a*) pp and $\bar{p}p$, *b*) π^+p and π^-p, *c*) K^+p and K^-p. Full lines correspond to reactions with positive incident particles, dashed ones — to those negative. Numbers *1*, *2*, on *c*) correspond to the two sets of parameters in Table I. Data from Ref. [14, 15]

slope, gives much worse fit to σ^{tot}. Both possibilities are presented in Table I and on Figs 2*c*, 3*c*, and 4. The most probable explanation of this disagreement is based on the breaking of degeneracy. For Kp we have four instead of two degenerate trajectories, which is obviously a much stronger assumption.

There are two other features of experimental data which cannot be reproduced by our model in its present form — the functional shape of Re $F(s, 0)$/Im $F(s, 0)$ and the anti-shrinkage of pp differential cross-sections. Since we have too few degrees of freedom to include Re/Im in the fit, this was a prediction. For pp and π^+p we get a disagreement in shape (ours tend to zero slower than the experimental ones), and for π^-p we get the wrong sign, although the absolute value is very small ($\leqslant 0.1$). Changing the meson trajectory (5) we could improve slightly this situation but this would require an unphysical value of intercept ($\alpha_M^0 < 0.2$). The proper antischrinkage effect can be obtained in our model by means of a very large value of β_M [10], but this makes worse the fit to total cross-sections. Perhaps in both cases some further improvement can be obtained by abandoning assumptions (3) and (4).

Finally, we would like to give some predictions about asymptotic values of total cross--sections. As is easily seen from (10), the only contribution which does not vanish for $s \to \infty$ is that of the Pomeron itself, which gives us

$$\sigma_\infty^{\text{tot}} = -9.78\, R_P. \tag{13}$$

The values of $\sigma_\infty^{\text{tot}}$ for different reactions are given in Table I and the shapes of σ^{tot} for higher energies are presented on Fig. 4. Unfortunately, the asymptotic value can be reached

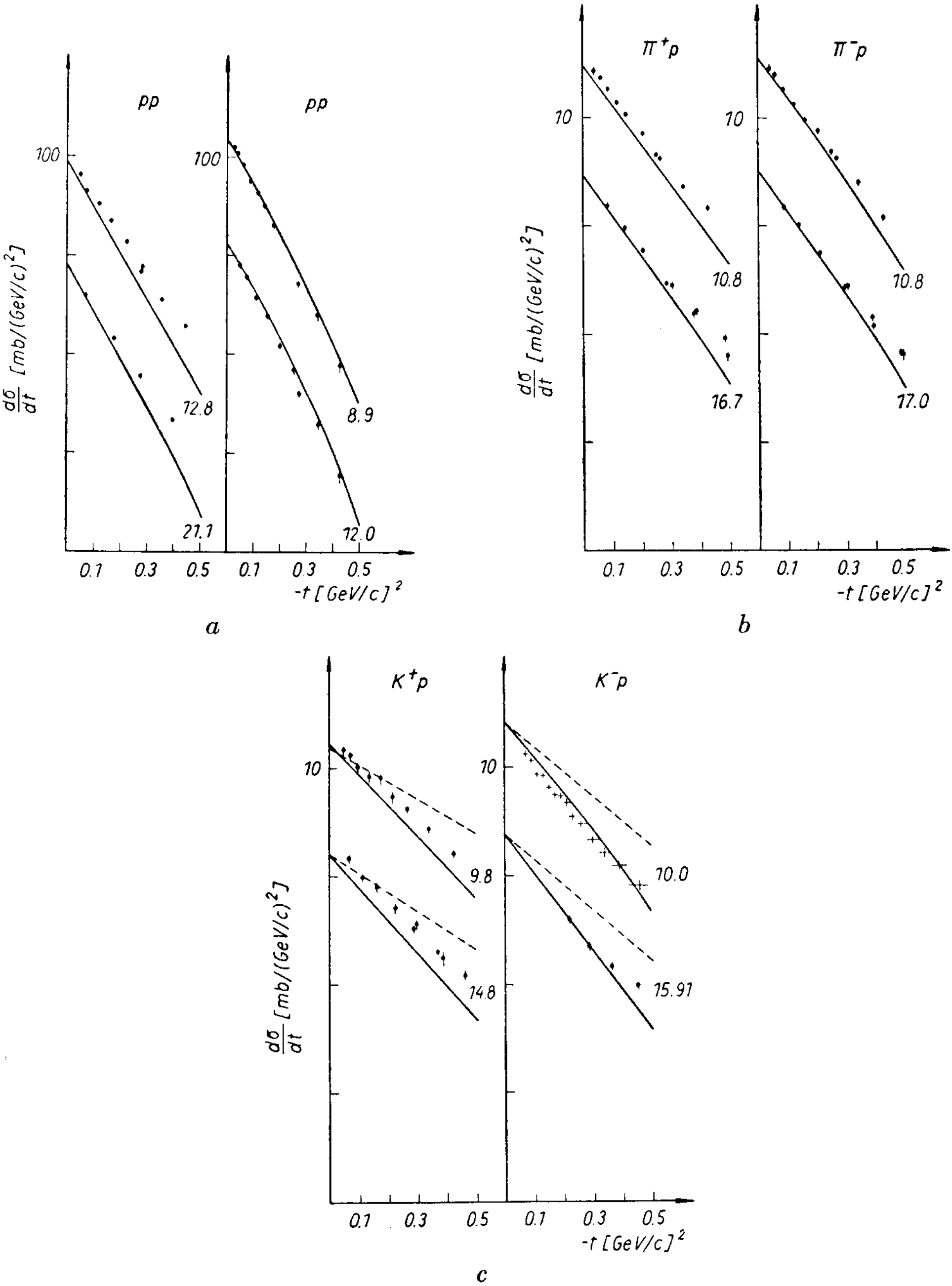

Fig. 3. Predicted elastic differential cross-sections for $|t| \leqslant 0.5\ (\mathrm{GeV/c})^2$ for reactions:

a) $pp \to pp$ at $p_{\mathrm{lab}} = 12.8$, 21.1 GeV/c; $\bar{p}p \to \bar{p}p$ at 8.9, 12.0 GeV/c;

b) $\pi^+p \to \pi^+p$ at 10.8, 16.7 GeV/c; $\pi^-p \to \pi^-p$ at 10.8, 17 GeV/c;

c) $K^+p \to K^+p$ at 9.8, 14.8 GeV/c; $K^-p \to K^-p$ at 10.0, 15.91 GeV/c;

The full line at each energy corresponds to solution 1 and the dashed one to solution 2 from Table I. Data from Ref. [16–19] only for energies much larger than those available at present. The main reason is the double Pomeron term which tends to zero very slowly, being of the shape

$$\sigma^{\text{tot}}_{P\otimes P} \approx -\frac{2\pi R_P^2}{s_0^2 \alpha'_P \ln \frac{s}{s_0}}. \tag{14}$$

Thus, *e. g.* for pp, to obtain the value of σ^{tot} differing less than 1 mb from $\sigma^{\text{tot}}_{\infty}$, we must have $s \geqslant 10^{20}$ GeV^2 (this agrees with the prediction in Ref. [7, 9]).

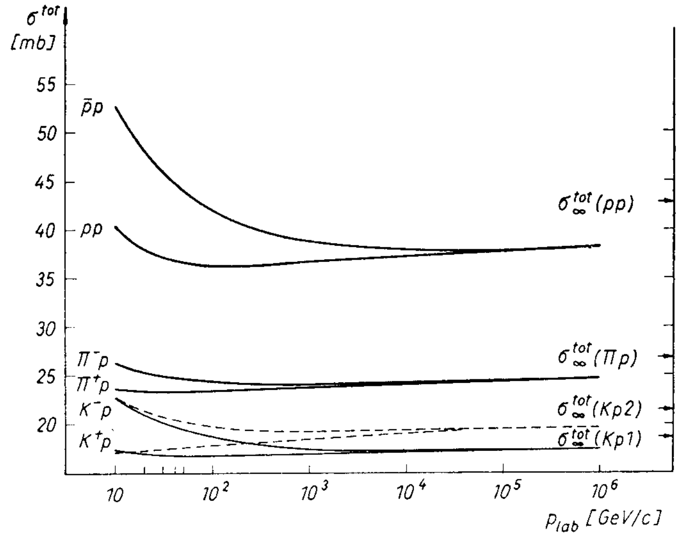

Fig. 4. Predictions for the asymptotic shape of total cross-sections. Arrows mark asymptotic values. Full lines for Kp correspond to the solution 1 and the dashed lines to the solution 2. Note that the energy range where the both σ^{tot} in the pair are practically equal is many orders of magnitude less than the energy where the asymptotic value is reached

Now, to gather all the basic features of the model, we shall list the most important implications of the results.

a) In spite of the strong assumptions and the small number of parameters, the model reproduces reasonably the experimental data for total and differential elastic cross-sections.

b) The breaking of degeneracy of meson trajectories, which should be the next step on the way to improve the model, can improve the fit for Re/Im and Kp but will considerably complicate the calculations.

c) If the phenomenon of antishrinkage in higher energies exists, it will cause the next problem for the model, unless the degeneracy breaking can help also in this case.

d) According to this model, the total cross-sections should very slowly rise to their asymptotic values, the asymptotic energy region for them being far beyond the present possibilities, namely in the range of $s > 10^{20}$ GeV^2.

e) When the data on slopes of $\frac{d\sigma}{dt}$ in Serpukhov energy range become available, we shall be able to determine more exactly the slopes of residua.

After completing this paper we were informed of a related work done independently by Frautschi, Hamer and Ravndal [10]. The physical features of both models are nearly identical. There are, however, some important differences in parametrization and the source of experimental data taken as a constraint in the fit, which led the mentioned authors to somewhat different results.

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