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PREDICTIONS FOR THE DECAY DISTRIBUTIONS OF RESONANCES PRODUCED IN THE REACTIONS $0^-\frac{1}{2}^+ \to 0^+\frac{3}{2}^+$, $0^-\frac{1}{2}^+ \to 1^+\frac{3}{2}^+$ AND $0^-\frac{1}{2}^+ \to 2^+\frac{3}{2}^+$

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Using the additive quark model, the relations between statistical tensors for single and joint decay angular distributions in reactions $0^-\frac{1}{2}^+ \to 0^+\frac{3}{2}^+$, $0^-\frac{1}{2}^+ \to 1^+\frac{3}{2}^+$ and $0^-\frac{1}{2}^+ \to 2^+\frac{3}{2}^+$ are derived.

1. *Introduction*

In this paper we will discuss the consequences of the additive quark model for distributions of resonances, produced in the reactions

$$M+B \to M^*+B^* \tag{1}$$

where M, B, M^*, B^*, denote a 0^+ meson a $\frac{1}{2}^+$ baryon, a 0^+ 1^+ or 2^+ meson, and a $\frac{3}{2}^+$ isobar, respectively.

It is well known that the additivity quark model provides many restrictions on the decay distribution of vector mesons and isobars produced in quasi-two-body reactions at high energies.

We will show that also for 0^+, 1^+ and 2^+ meson production the additivity assumption in the BB^* vertex reduces strongly the number of amplitudes in quasi-two-body processes and gives numerous relations between the statistical tensors which can be experimentally verified.

The plan is the following: in Section 2 we introduce the scalar amplitudes, Sections 3, 4 and 5 contain the amplitudes and relations between the statistical tensors for $0^+\ {}^3/_2{}^+$, $1^+\ {}^3/_2{}^+$, $2^+\ {}^3/_2{}^+$ quasi-two-body final states. Remarks are given in Section 6.

2. *Expansion into scalar amplitudes*

It is found very useful to express the amplitudes for two-body scattering processes in terms of scalar amplitudes [1]. There are, in general, four amplitudes for the production of a 0^+, twelve for a 1^+ and twenty for 2^+ resonances.

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If we suppose that our processes can be described by using the additivity assumption of the quark model, the number of independent complex amplitudes reduces from four to two, from twelve to four and from twenty to eight.

The additivity assumption is equivalent to the statement that the change of spin in a BB^* vertex cannot exceed one unit. We must emphasize, that the additivity assumption is assumed in the BB^* vertex only. In the MM^* vertex arbitrary spin can be exchanged (in our case zero, one or two units).

We present below the expansion into scalar amplitudes. The amplitudes for reaction $M_0+B_\alpha = M^*_\beta + B^*_\gamma$ denoted $f_{0\alpha\beta\gamma}$ ($0, \alpha, \beta, \gamma$ denote spin projections for M, B, M^* and B^* respectively) are defined in terms of the scalar amplitudes as follows:

$$f_{0\alpha\beta\gamma} = \sum_{LM} C^L_{M(0\alpha\beta\gamma)} \cdot (-1)^M \cdot B^L_{-M} \tag{2}$$

This general expression for coupling the spin factor with the momentum factor B^L_{-M} was given *e. g.* by Joos [2] C^L_M is the product of Clebsch-Gordan coefficients (see Fig. 1).

$$C^L_{M(0\alpha\beta\gamma)} = \sum_{\substack{m_1 m_2 \\ J_1 J_2}} C\left(\frac{1}{2}\,\alpha;\, J_2 m_2 \,\middle|\, \frac{3}{2}\,\gamma\right) \cdot C(J_1 m_1;\, J_2 m_2 | LM) \tag{3}$$

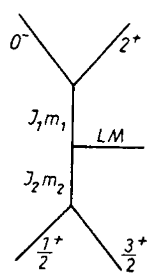

Fig. 1

The functions B^L_{-M} depend on the scalar amplitudes $A^L_{l_1 l_2}$ and the momenta $\vec{p}_1$, $\vec{p}_2$ of the initial and the final baryon, in the following way:

$$B^L_{-M} = \sum_{\substack{l_1 l_2 \\ W_1 W_2}} A^L_{l_1 l_2} \cdot Y^{l_1}_{W_1}(\vec{p}_1) \cdot Y^{l_2}_{w_2}(\vec{p}_2)\; C(l_1, w_1;\, l_2 w_2 | L, -M) \tag{4}$$

where l_1 and l_2 satisfy the conditions

$$l_1 + l_2 = L \qquad \text{for} \quad \prod_{i=1}^{4} \eta_i (-1)^L = 1$$

$$l_1 + l_2 = L+1 \quad \text{for} \quad \prod_{i=1}^{4} \eta_i (-1)^L = -1$$

(η_i denotes the intrinsic parity of the particle). The quark additivity assumption is equivalent to $J_2 = 1$ only (in the MM^* vertex arbitrary J_1 may be exchanged, see Fig. 1). For this case we obtain less scalar amplitudes than for the general case of $J_2 = 1, 2$. If we substitute $J_2 = 1$ into (2) we obtain eight amplitudes for 2^+, four for 1^+ and two for 0^+. These amplitudes $f_{0\alpha\beta\gamma}$ were calculated in a spin reference frame with the z axis normal to the reaction plane. An ambiguity in the choice of the spin reference frame for which the additivity assumption holds, is discussed in reference [1], [7]. The relations derived in this paper belong to the class *a* and do not depend on the choice of reference frame [3].

3. *Statistical tensor for* $0^+ \, {}^3/_2^+$ *two-body final states*

In this section we will discuss reactions of the type

$$0^- + \frac{1}{2}^+ \to 0^+ + \frac{3}{2}^+ .$$

The number of independent amplitudes reduces from four to two

$$f_{0\,{}^1/_2\,0\,{}^3/_2} = \sqrt{3} \cdot f_{0-{}^1/_2\,0\,{}^1/_2} = R, \qquad \sqrt{3} f_{0\,{}^1/_2\,0-{}^1/_2} = f_{0-{}^1/_2\,0-{}^3/_2} = S$$

For statistical tensors we obtain a single relation

$$T^{02}_{00} = T^2_0\left(\frac{3}{2}^+\right) = \frac{1}{4}. \tag{5}$$

4. *Relations between statistical tensors for* $1^+ \, {}^3/_2^+$ *two-body final states*

Next, we will discuss the relation

$$0^- + \frac{1}{2}^+ \to 1^+ + \frac{3}{2}^+ .$$

All of the amplitudes $f_{0\alpha\beta\gamma}$ for 1^+ production can be expressed by four independent complex amplitudes K, L, M, N, which depend on energy and on the momentum transfer.

$$f_{0\,{}^1/_2\,1\,{}^1/_2} = f_{0-{}^1/_2\,1-{}^1/_2} = K$$

$$\frac{1}{\sqrt{3}} f_{0\,{}^1/_2\,0\,{}^3/_2} = f_{0-{}^1/_2\,0\,{}^1/_2} = L$$

$$f_{0\,{}^1/_2\,0-{}^1/_2} = \frac{1}{\sqrt{3}} \cdot f_{0-{}^1/_2\,0-{}^3/_2} = M$$

$$f_{0\,{}^1/_2\,-1\,{}^1/_2} = f_{0-{}^1/_2\,-1-{}^1/_2} = N.$$

The remaining scattering amplitudes vanish. Expressing statistical tensors by the density matrix elements, and next by K, L, M, N we obtain the following relations:

$$T^{22}_{22} = T^{22}_{2-2} = 0 \tag{6}$$

$$T^{22}_{20} = -T^{20}_{20} = -NM^* \tag{7}$$

$$T^{22}_{02} = -\sqrt{2}\, T^{02}_{02} = -2LK^* \tag{8}$$

$$T^{20}_{00} + \sqrt{2}\, T^{02}_{00} = -\frac{1}{2\sqrt{6}} = -\frac{1}{2\sqrt{6}}\left(2|M|^2 + 4|K|^2 + 4|L|^2 + 2|N|^2\right) \tag{9}$$

$$T^{22}_{00} = -\frac{1}{2\sqrt{6}}\,(2|M|^2+4|K|^2+4|L|^2+2|N|^2) = -\frac{1}{2\sqrt{6}} \tag{10}$$

$$T^{22}_{11} = -\frac{\sqrt{3}}{2}\,(LM^*+NK^*) \tag{11}$$

$$T^{22}_{1-1} = -\frac{\sqrt{3}}{2}\,(KM^*+NL^*) \tag{12}$$

$$T^{00}_{00} = \frac{1}{2\sqrt{3}}\,(2|M|^2+4|K|^2+4|L|^2+2|N|^2) = \frac{1}{2\sqrt{3}}. \tag{13}$$

Combined with the conditions $T^{J_1 0}_{M_1 0} = \frac{1}{\sqrt{2S_2+1}}\,T^{J_1}_{M_1}$ and $T^{0 J_2}_{0 M_2} = \frac{1}{\sqrt{2S_1+1}}\,T^{J_2}_{M_2}$ Eq. (8) implies the following relation for the single statistical tensors

$$\frac{1}{2}\,T^2_0(1^+) + \sqrt{\frac{2}{3}}\,T^2_0\left(\frac{3}{2}^+\right) = -\frac{1}{2\sqrt{6}}. \tag{14}$$

5. *Relations between statistical tensors for $2^+ {}^3/_2{}^+$ two-body final states*

A calculation analogous to that which allowed to find relations (5)—(14) gives corresponding relations for double statistical tensors for 2^+ and ${}^3/_2{}^+$ particles. In this case the number of independent amplitudes reduces to eight.

$$\frac{1}{\sqrt{3}} f_{0\,\frac{1}{2}\,2\,\frac{3}{2}} = f_{0\,-\frac{1}{2}\,2\,\frac{1}{2}} = A$$

$$f_{0\,\frac{1}{2}\,2\,-\frac{1}{2}} = \frac{1}{\sqrt{3}} f_{0\,-\frac{1}{2}\,2\,-\frac{3}{2}} = B$$

$$f_{0\,\frac{1}{2}\,1\,\frac{1}{2}} = f_{0\,-\frac{1}{2}\,1\,-\frac{1}{2}} = C$$

$$\frac{1}{\sqrt{3}} f_{0\,\frac{1}{2}\,0\,\frac{3}{2}} = f_{0\,-\frac{1}{2}\,0\,\frac{1}{2}} = D$$

$$f_{0\,\frac{1}{2}\,0\,-\frac{1}{2}} = \frac{1}{\sqrt{3}} f_{0\,-\frac{1}{2}\,0\,-\frac{3}{2}} = E$$

$$f_{0\,\frac{1}{2}\,-1\,\frac{1}{2}} = f_{0\,-\frac{1}{2}\,-1\,-\frac{1}{2}} = F$$

$$\frac{1}{\sqrt{3}} f_{0\,\frac{1}{2}\,-2\,\frac{3}{2}} = f_{0\,-\frac{1}{2}\,-2\,\frac{1}{2}} = G$$

$$f_{0\,\frac{1}{2}\,-2\,-\frac{1}{2}} = \frac{1}{\sqrt{3}} f_{0\,-\frac{1}{2}\,-2\,-\frac{3}{2}}.$$

Substituting these amplitudes into the expression for the statistical tensors, we obtain the following relations (which hold in the reference frame with the z axis perpendicular to the reaction plane).

$$\sqrt{\frac{5}{14}}\, T^{02}_{00} = \frac{1}{2}\, T^{20}_{00} - T^{22}_{00} + \frac{1}{4\sqrt{14}} = \frac{1}{4\sqrt{14}}\,(\alpha+\beta-\gamma) \tag{15}$$

$$\frac{\sqrt{5}}{7}\, T^{42}_{00} = \frac{13}{84}\, T^{22}_{00} - \frac{1}{6}\, T^{20}_{00} + \frac{1}{8\sqrt{14}} = \frac{1}{7\sqrt{14}}\,(\alpha+6\beta+4\gamma) \tag{16}$$

$$\frac{2}{\sqrt{5}}\, T^{40}_{00} = -\frac{7}{3}\, T^{22}_{00} + \frac{2}{3}\, T^{20}_{00} + \frac{21}{30\sqrt{14}} = \sqrt{\frac{8}{175}}\,(\alpha+6\beta-2\gamma) \tag{17}$$

$$T^{42}_{02} = \sqrt{\frac{7}{8}}\, T^{02}_{02} - \sqrt{\frac{5}{4}}\, T^{22}_{02} = \sqrt{\frac{3}{35}}\,(BA^*+6ED^*+HG^*) \tag{18}$$

$$T^{02}_{02} = 2\sqrt{\frac{3}{10}}\,(BA^*+ED^*+HG^*) \tag{19}$$

$$T^{22}_{20} = \frac{3\sqrt{3}}{7}\, T^{40}_{20} - \frac{1}{7}\, T^{20}_{20} = \sqrt{\frac{2}{7}}\,(\xi-32\eta) \tag{20}$$

$$T^{42}_{20} = \frac{3\sqrt{3}}{7}\, T^{20}_{20} - \frac{5}{14}\, T^{40}_{20} = \sqrt{\frac{3}{14}}\left(\xi+\sqrt{\frac{8}{3}}\,\eta\right) \tag{21}$$

where

$$\alpha = |A|^2+|B|^2+|G|^2+|H|^2, \quad \beta = |D|^2+|E|^2, \quad \gamma = |C|^2+|F|^2$$

$$\xi = GD^*+HE^*+DA^*+EB^*, \quad \eta = FC^*$$

$$T^{42}_{2-2} = \sqrt{\frac{3}{4}}\, T^{22}_{2-2} = \frac{6}{\sqrt{28}}\,(GE^*+DB^*) \tag{22}$$

$$T^{42}_{22} = \sqrt{\frac{3}{4}}\, T^{22}_{22} = \frac{6}{\sqrt{28}}\,(HD^*+EA^*) \tag{23}$$

$$T^{40}_{40} = 2\cdot T^{42}_{40} = 2\cdot(GA^*+HB^*) \tag{24}$$

$$T^{42}_{42} = \sqrt{6}\, HA^* \qquad T^{42}_{4-2} = \sqrt{6}\, GB^* \qquad (25),\ (26)$$

$$T^{42}_{31} = \frac{\sqrt{3}}{2}\,(FA^*+HC^*) \qquad T^{42}_{3-1} = \frac{\sqrt{3}}{2}\,(FB^*+GC^*) \qquad (27),\ (28)$$

$$T^{42}_{11} = \frac{3}{\sqrt{14}}\left[\frac{1}{\sqrt{6}}\,(HF^*+CA^*)+FD^*+EC^*\right] \tag{29}$$

$$T^{22}_{11} = \frac{3}{\sqrt{14}}\left[-\frac{1}{\sqrt{6}}\,(FD^*+EC^*)+HF^*+CA^*\right] \tag{30}$$

$$T^{42}_{1-1} = \frac{3}{\sqrt{14}}\left[\frac{1}{\sqrt{6}}(CB^* + GF^*) + FE^* + DC^*\right] \tag{31}$$

$$T^{22}_{1-1} = \frac{3}{\sqrt{14}}\left[\frac{1}{\sqrt{6}}(FE^* + DC^*) + CB^* + GF^*\right]. \tag{32}$$

From relations (15), (17) we can find the following relation for single statistical tensors

$$T^4_0(2^+) = \frac{7}{3}\sqrt{\frac{5}{14}}\, T^2_0\left(\frac{3}{2}^+\right) - \frac{\sqrt{5}}{4}\, T^2_0(2^+) + \frac{7\sqrt{5}}{60\sqrt{14}}. \tag{33}$$

6. *Conclusions*

Using the quark additivity assumption we have shown that the decay distribution for spin-zero meson is described by two, for spin-one meson by four, and for spin two meson by eight independent complex amplitudes, respectively.

This implies one linear relation between statistical tensors in the first case, five linear relations in the second and nine in the third case.

The experimental data on $M+B = M^*+B^*$ (M^* denotes 0^+, 1^+, 2^+ meson) allow to check the additivity assumption and to determine the remaining tensors. This will be done in next paper. Experimentally, the most important case seems to be that of 2^+ meson production which has a reasonably large cross-section in the few GeV region. We hope that it will be possible to verify our relations in the near future.

The 0^+ and 1^+ meson production is probably more difficult but would be also very interesting.

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