EFFECTIVE INTERACTION OF VALENCE NEUTRONS IN ^{58}Ni

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In the case of ^{58}Ni the matrix elements of the effective interaction of the last two nucleons are calculated using Migdal's method and compared with those given by Kuo and empirical values determined by Auerbach.

1. Introduction

A method of calculating the effective interaction was given by Brown and Kuo in 1966 [1]. They started with the Hamada-Johnston two nucleon potential and assumed that the single particle wave functions are those of a harmonic oscillator. In order to avoid the infinite values of integrals, the potential V is replaced by Brueckner's reaction matrix G for the scattering of two nucleons. The important point in their approach is the separation method in which the two-particle potential V is divided into the short range part V_s and the long range part V_L in such a manner that the attractive part V_s balances the repulsive core. Using this method, Kuo [2] obtained the matrix elements of the effective interaction in nickel isotopes. In his calculations he took into account the core polarization of the type

$$G_{HJ}+G_{HJ}\frac{Q_{3p1h}}{\Delta E}G_{HJ}$$

and stated that this leads to an essential renormalization of the reaction matrix.

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As we know, the idea of the particle-hole core polarization pertains to Migdal's theory of effective interaction in nuclei. But in his method the primitive interaction is postulated as a short range, $\delta(\vec{r}_1-\vec{r}_2)$ type, two-nucleon force. It seemed it would be interesting to calculate the matrix elements of the effective two-nucleon interaction in ^{58}Ni using Migdal's method and compare them with those given by Kuo for two reasons:

1. In the Brown and Kuo approach the long-range part of the realistic Hamada-Johnston potential plays the major role, while in the theory of Migdal one starts with a short-range force only.

2. This comparison will give us a possibility of ascertaining how sensitive Migdal's method is to the choice of the numerical parameters of the primitive force F.

2. Calculations

We consider Migdal's equation for the effective intraction of two nucleons in the partly filled shell n_0 [3]:

$$(\lambda_1\lambda_2|\Gamma|\lambda_3\lambda_4) = (\lambda_1\lambda_2|F|\lambda_3\lambda_4) + \sum_{\substack{\lambda,\lambda' \\ n\neq n_0}} (\lambda_1\lambda'|F|\lambda_3\lambda)\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}+\omega}(\lambda\lambda_2|\Gamma|\lambda'\lambda_4). \tag{1}$$

Here, λ_i is the set of quantum numbers describing the one-particle state (in our case $\lambda_i = n_i, l_i, j_i, t_{z_i}$), $(\lambda_i\lambda_j|F|\lambda_k\lambda_l)$ means that λ_i, λ_k and λ_j, λ_l are coupled to the total angular momentum J and isospin T, the summation over λ, λ' goes over particle and hole states, respectively, excluding the n_0 shell, n_λ, $n_{\lambda'}$, ε_λ, $\varepsilon_{\lambda'}$ are the occupation numbers and the one-particle energy, respectively, and ω is excitation energy (we shall omit it, as it is small compared to $\varepsilon_\lambda - \varepsilon_{\lambda'}$). Furthermore,

$$\begin{aligned} F &= c\delta(\vec{r}_1-\vec{r}_2)\{f_0+f'_0(\vec{\tau}_1\vec{\tau}_2)+(g_0+g'_0(\vec{\tau}_1\vec{\tau}_2))(\vec{\sigma}_1\vec{\sigma}_2)\} \\ &= cf(r)\delta(\vec{r}_1-\vec{r}_2), \end{aligned} \tag{2}$$

where f_0, f'_0 depend on $|\vec{r}| = r$ as follows:

$$\begin{aligned} f_0(r) &= f_{0\text{in}}+(f_{0\text{ex}}-f_{0\text{in}})\frac{1}{1-e^{-\alpha(r-R_0)}}, \\ f'_0(r) &= f'_{0\text{in}}+(f'_{0\text{ex}}-f'_{0\text{in}})\frac{1}{1-e^{-\alpha(r-R_0)}}, \end{aligned} \tag{3}$$

$f_{0\text{in}}$, $f_{0\text{ex}}$, $f'_{0\text{in}}$, $f_{0\text{ex}}$ being constants, $\alpha = 1.49 f^{-1}$, $R_0 = 1.279\, A^{1/3} f$, $c = 2/3T_F/\varrho_0$, T_F is the Fermi energy and $\varrho_0 = 3Z/4\pi R_0^3$. Passing to the relative and centre-of-mass oscillator coordinates we obtain

$$(\lambda_1\lambda_2|F|\lambda_3\lambda_4) = C\sum_{\substack{L,S,n,n' \\ N,N'}} X\begin{Bmatrix} \frac{1}{2} & l_1 & j_1 \\ \frac{1}{2} & l_2 & j_2 \\ S & L & J \end{Bmatrix}\cdot X\begin{Bmatrix} \frac{1}{2} & l_3 & j_3 \\ \frac{1}{2} & l_4 & j_4 \\ S & L & J \end{Bmatrix}\cdot(1-(-1)^{-S-T})\times$$
$$\times\langle n0, NL, L|n_1l_1n_2l_2, L\rangle\langle n'0, N'L, L|n_3l_3n_4l_3, L\rangle\cdot\varphi^*_{n00}(0)\varphi_{n'00}(0)\Xi_{NN'L}. \tag{4}$$

C is a new constant, X denotes the transformation coefficients from j—j to L—S coupling, $\langle nl, N\mathscr{L}, L|n_a l_a, n_b l_b, L\rangle$ is the Brody-Moshinsky transformation bracket between the laboratory frame and the centre-of-mass and relative frame

$$\varphi_{nlm}(\vec{r}) = R_{nl}(r)\, Y_{lm}(\vartheta, \varphi)$$

are the normalized harmonic oscillator eigenfunctions, and

$$\Xi_{NN'L} = \int R_{NL}(\varrho)\varrho^2 f\left(\frac{\sqrt{2}}{2}\varrho\right) R_{N'L}(\varrho)d\varrho.$$

TABLE I

Comparison of two-body matrix elements

	$1p^2_{3/2}$			$0f^2_{5/2}$			$1p^2_{1/2}$		
	I	II	k—B	I	II	k—B	I	II	k—B
$1p^2_{3/2}$	−1.129	−0.604	−0.967 −0.878 −0.92	−0.549	−0.320	−0.558 −1.259 −1.12	−0.595	−0.347	−1.241 −0.961 −0.97
$0f^2_{5/2}$				−1.182	−0.678	−0.268 −1.249 −1.74	−0.390	−0.189	−0.201 −0.952 −0.56
$1p^2_{1/2}$							−0.625	−0.274	−0.089 −0.241 −0.89

$J = 0,\ T = 1$

	$1p_{3/2}\,0f_{5/2}$			$1p_{3/2}\,1p_{1/2}$		
	I	II	k—B	I	II	k—B
$1p_{3/2}\,0f_{5/2}$	1.146	0.521	−0.222 0.403 1.10	0.071	0.017	−0.089 −0.042 0
$1p_{3/2}\,1p_{1/2}$				−0.001	−0.001	−0.133 0.055 1.24

$J = 1,\ T = 1$

	$1p^2_{3/2}$			$1p_{3/2}\,0f_{5/2}$			$1p_{3/2}\,1p_{1/2}$			$0f^2_{5/2}$			$0f_{5/2}\,1p_{1/2}$		
	I	II	k−B	I	II	k−B	I	II	k−B	I	II	k−B	I	II	k−B
$1p^2_{3/2}$	−0.329	−0.157	−0.424 −0.248 −0.24	−0.024	−0.003	−0.029 −0.144 −0.12	0.137	0.086	−0.493 −0.266 −0.83	−0.130	−0.073	−0.070 −0.239 −0.16	−0.110	−0.022	−0.092 −0.064 −0.23
$1p_{3/2}$ $0f_{5/2}$				−0.146	−0.071	0.003 0.303 0.30	−0.062	−0.003	−0.127 −0.167 −0.18	0.084	0.048	0.013 −0.143 −0.17	−0.151	−0.084	−0.178 −0.205 −0.39
$1p_{3/2}$ $1p_{1/2}$							−0.221	−0.118	−0.772 −0.355 −0.60	−0.100	−0.061	−0.189 −0.460 −0.22	0.039	0.027	−0.211 −0.332 −0.33
$0f^2_{5/2}$				$J=2,\ T=1$						0.390	−0.197	−0.316 −0.053 −0.40	−0.137	−0.083	−0.053 −0.510 −0.31
$0f_{5/2}$ $1p_{1/2}$													−0.281	−0.156	−0.257 −0.167 −0.20

	$1p_{3/2}\,0f_{5/2}$			$0f_{5/2}\,1p_{1/2}$		
	I	II	k–B	I	II	k–B
$1p_{3/2}\,0f_{5/2}$	−0.073	−0.026	−0.190 0.370 0.80	−0.01	−0.002	−0.017 0.120 0
$0f_{5/2}\,1p_{1/2}$	$J=3,\ T=1$			−0.011	−0.006	−0.118 0.64 0.62

	$1p_{3/2}\,0f_{5/2}$			$0f_{5/2}^2$		
	I	II	k–B	I	II	k–B
$1p_{3/2}\,0f_{5/2}$	−0.245	−0.140	−0.366 −0.070 −0.38	0.099 0.099	0.059 0.059	−0.024 −0.351 −0.25
$0f_{5/2}^2$	$J=4,\ T=1$			−0.268	−0.119	−0.061 0.461 0.32

The transition from $(\lambda_1\lambda_2|F|\lambda_3\lambda_4)$ to $(\lambda_1\lambda_2|F|\lambda_3\lambda_4)$ gives the formula [4]

$$\sum_{J'T'} (\lambda_1\lambda_2|F|\lambda_3\lambda_4)\sqrt{2J'+1}\sqrt{2T'+1}\cdot U(j_1j_3j_2j_4;JJ')\times$$

$$U\left(\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2};TT'\right)(-1)^{J+T+J'+T'+j_1+j_4} = (\lambda_1\lambda_2|F|\lambda_3\lambda_4)\,.$$

In ^{58}Ni the n_0 shell consists of the levels: $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$. We assume the levels $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $0f_{7/2}$ to be the hole states (λ), and $0g_{9/2}$, $1d_{5/2}$, $0g_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, $0h_{11/2}$ to be the particles states (λ).

Apart from Migdal, Bochnacki *et al.* [5] and Mikhailov [6] obtained a different set of parameters describing the two-particle force F. They took for g_0 and g_0' the same formula as (3) and assumed the following values of the parameters: $f_{\text{in}} = 0.15$; $f'_{\text{in}} = 0.62$; $f_{\text{ex}} = -1.4$; $f'_{\text{ex}} = -0.22$; $g_{\text{in}} = 0.1$; $g'_{\text{in}} = 0.45$; $g_{\text{ex}} = 0.3$; $g'_{\text{ex}} = -0.05$.

The results of our calculations are given for both cases; the Migdal problem: $f_{\text{in}} = 0.5$; $f'_{\text{in}} = 0.7$; $f_{\text{ex}} = -2$; $f'_{\text{ex}} = 1$; $g_0 = g_0' = 0.5$ (denoted I), and that of Mikhailov and

others (denoted II). The first and second columns are the values we received. The third column, first row, constains the results of Kuo for G_{HJ}, the second row gives his results with core polarization, and the third row presents the values used by Auerbach [7].

3. *Conclusions*

Our results, for the case I, oscillate near the Auerbach values with the same accuracy as the numbers given by Kuo. The variant II gives, as a rule, too small values of the matrix elements of the Migdal effective interaction. From comporison of the variants I and II one can see that the matrix elements of the effective interaction are sensitive to changes of the numerical parameters entering F.

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