ON THE ENERGY DEPENDENCE OF THE POLARIZATION IN ^{9}Be(d, p) ^{10}Be REACTION

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In our paper we give an analysis of the polarization of protons from the reaction ^{9}Be$(d,p)^{10}$Be. It is shown that the predictions based on the invariance of the density matrix under the exchange of distorted waves (when $p_d r_d = p_p r_p$) are in qualitative agreement with the experimental data. The usefulness of the DWBA for the describing of the polarization in the energy range 2–10 MeV is also discussed.

Recently a lot of experimental data on polarization of particles in direct reaction is available [1]. The most striking feature of these data is the strong dependence of polarization of outgoing particles on the energy of projectiles. From ten point of view of the orthodox theory of direct reaction it is an unexpected fact that the polarization has strong energy dependence, while the angular distribution for the outcoming particle is smooth function of energy.

If we are going to interpret the polarization data on the basis of a definite theoretical model, *e. g.* the DWBA, we have to have the certainty that the mechanism which is governing the reaction is unique. Of course, this is rather a rare situation in experimental low energy physics (1–10 MeV). However it is possible to imagine the situation where that is the case. In our paper we intend to propose a certain explanation of the energy dependence of polarization in direct reaction. Our approach is to some extend nondynamical. This means that we start from the general structure of direct reaction amplitude written in the well known form [2], [3]

$$f(\vec{p}_a, \vec{p}_b) = \langle \varphi_2^{(-)}(\vec{p}_b, \vec{r}_b) || V_{aA;bB} || \varphi_1^{(+)}(\vec{p}_a, \vec{r}_a) \rangle$$

$$V_{aA;bB} = \int \psi_B^* \psi_b^* V \psi_A \psi_a d\xi \tag{1}$$

where

$\varphi_i(\vec{r}_i)$ $(i = 1, 2)$ denote the distorted waves for incoming and outgoing particles, $\psi_j (j = a, A, b, B)$ are the internal wave functions for particles and nuclei

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and look the symmetry relation of the density matrix of one of the particles which are produced in the reaction of the type

$$a+A \rightarrow B+b. \tag{2}$$

It is obvious that this symmetry relation is reflected in the polarization of outgoing particles. The symmetry relations can be devided into two groups: according to their dependence or independence on the energy involved in the reaction. To the first group belong symmetry properties which arise on the ground of the conservation laws which operate in the reaction, *e. g.* conservation of parity, conservation of total angular momentum[1]. The second group of symmetry of density matrix can be deduced from the structure of the amplitude (1) — Satchler [2]. The content of Satchler approach can be formulated as follows: The amplitude (1) can be divided into two parts: the outside part constructed from the distorted waves and the inside part $V_{aA;bB}$ which is dependent on residual nuclear interaction. If the outgoing distorted waves are "equal" (in the Satchler's meaning [2]), then the amplitude (1) and the density matrix elements must be invariant under exchange of these two waves. Let us neglect the spin orbit dependence of the distorting potential. The condition for the exchange wave symmetry can be stated as follows [2]. The amplitude for the direct reaction (2) is invariant under the exchange distorted waves $\varphi_1 \rightleftarrows \varphi_2$ when $r_b p_b = r_a p_a$. The p_i denote the relative momenta of particle a and b and r_i corresponds to their relative distances from the target nucleus and residual nucleus respectively.

Let us look for the mathematical formulation of the effect of this symmetry on the polarization of particle b produced in the reaction (2). It is well known that the polarization

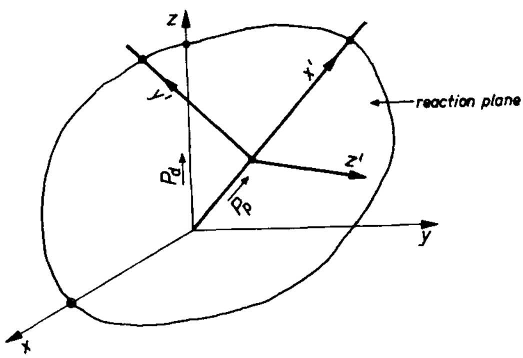

Fig. 1. Orientation of the frame of reference when the transversity representation is used

phenomena can be described in reference frames with direction of axes dependent on the representation which is used for describing the projection of spin. In our approach we use the transversity quantum number [4]. In our representation the direction of axes is shown in the Fig. 1. We assume that the particle a is unpolarized and that in the experiment we detect the polarization of particle b only. Then in the transversity representation the density matrix

[1] These are discussed by R. Johnson, *Nuclear Phys.*, **A90**, 289 (1967).

for the particle b can be written in standard notation as follows [6]:

$$\varrho^b = \sum_{LM} q_M^L(b) Q_M^L \tag{3}$$

where

$$\frac{d\sigma}{d\omega} q_M^L(b) = \frac{1}{4(2s_B+1)} \sum_{L_1M_1} \sum_{\mu_b} \sum_{\mu_{b'}} \sum_{\mu_B} \sum_{\mu_a} \sum_{\mu_A} \sum_{\mu_{A'}} \varepsilon_1 \varepsilon_2 q_{M_1}^{L_1} \times$$

$$\times (Q_{M_1}^{L_1})_{\mu_A,\mu_{A'}} (Q_M^{L+})_{\mu_b,\mu_{b'}} f_{\mu_b\mu_B;\mu_a\mu_A} \bar{f}_{\mu_{b'}\mu_B;\mu_A\mu_{A'}}; \tag{4}$$

$$\varepsilon_1 = (1+(-1)^{\mu_b-\mu_{b'}+\mu_A-\mu_{A'}});$$

$$\varepsilon_2 = (1+\eta(-1)^{\mu_b+\mu_B-\mu_a-\mu_A}).$$

The coefficients $q_{M_1}^{L_1}$ describe the polarization of particle A. The ε_1, ε_2 denote the projection operators which selected only those amplitudes $f_{\mu_b\mu_B;\mu_a\mu_A}$ for which the transversities fulfil the selection rules

$$\mu_b - \mu_{b'} + \mu_A - \mu_{A'} = \text{even}$$

$$\eta(-1)^{\mu_b+\mu_B-\mu_a-\mu_A} = 1. \tag{5}$$

According to [2], [5] when the exchange waves symmetry is fulfilled then [5]

$$f_{\mu_B\mu_\mu;\mu_A\mu_a} = (-1)^{i\pi(S_B-S_a)} f_{-\mu_B-\mu_b;-\mu_A-\mu_a}. \tag{6}$$

When the formula (6) is taken into account the q_M^L coefficients satisfy the relation

$$\frac{d\sigma}{d\omega} (q_M^L(b) - (-1)^L q_{-M}^L(b)) = \frac{1}{4} \frac{1}{(2s_B+1)} \sum_{L_1M_1} \sum_{\mu_b} \sum_{\mu_{b'}} \sum_{\mu_B} \sum_{\mu_a} \sum_{\mu_A} \sum_{\mu_{A'}} \varepsilon_1 \varepsilon_2 q_{M_1}^{L_1} \times$$

$$\times (Q_{M_1}^{L_1})_{\mu_A\mu_{A'}} (Q_M^{L+})_{\mu_{b'}\mu_b} f_{\mu_b\mu_B;\mu_a\mu_A} \bar{f}_{\mu_{b'}\mu_B;\mu_a\mu_{A'}} (q_{M_1}^{L_1} - (-1)^{L_1} q_{-M_1}^{L_1}). \tag{7}$$

From the formula (7) we conclude that the symmetry of q_M^L is related to the symmetry of the density matrix for incoming particle A. Suppose that the density matrix for particle A fulfil the relation

$$q_{M_1}^{L_1} = (-1)^{L_1} q_{-M_1}^{L_1}. \tag{8}$$

The physical content of the relation (8) is very clear if the density matrix for particle A contains only even M_1. It simply means, when M_1 is even, that the density matrix for particle A is invariant under rotation π about the axis Y from Fig. 1. For unpolarized particle A relation (8) holds and the q_M^L satisfy the equality

$$q_M^L(b) = (-1)^L q_{-M}^L(b), \quad M = \text{even}. \tag{9}$$

For M even we also can write (9) as follows

$$q_M^L(b) = (-1)^L \bar{q}_M^L(b), \tag{10}$$

where we have used the well known relation for the complex conjugation of the coefficient q_M^L [6]

$$\bar{q}_M^L = (-1)^M q_{-M}^L. \tag{11}$$

From (10) we conclude that when the exchange symmetry is fulfilled then

$$\text{for } L \text{ even } q_M^L \text{ is real}$$

$$\text{for } L \text{ odd } q_M^L \text{ is pure imaginary.} \qquad (12)$$

The selection rule (12) imposed very strong restriction on the polarization tensors q_M^L. In conclusion we can say that when the reaction

$$a+A \to B+b$$

is performed on the unpolarized particle A and we measure only polarization for particle b then the vector polarization is equal to zero. Let us look for the experimental evidence for our last statement. Recently, a paper was published by Blue *et al.* [1] from Ohio State University who measured the polarization for protons from the reaction ^{9}Be (d, p) ^{10}Be for the energy range 1–6 MeV. This particular reaction was sel cted because the cross-section for (d, p) is quite free from resonant structure above about 2 MeV. It was hoped that the lack of resonance could be taken as an indication that the ^{9}Be (d, p) ^{10}Be reaction proceeds predominantly *via* the direct process. In that paper the polarization at the laboratory angle of 30° was measured in 0.2 MeV steps from 1.0 to 6.0 MeV. These results shown in Fig. 2

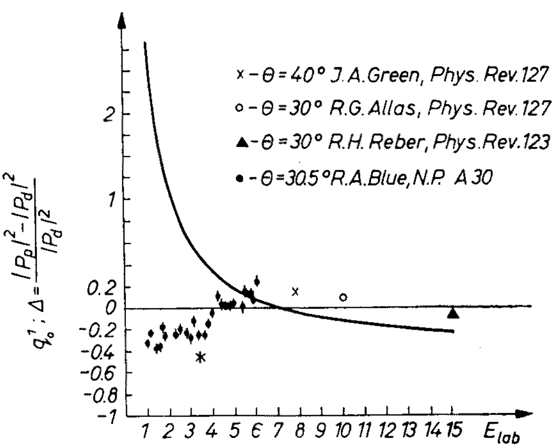

Fig. 2. Smooth curve denotes the function $\Delta = f(E_d(\text{lab}))$ for the reaction ^{9}Be$(d, p)^{10}$Be. Experimental points are denoted by dots

indicate that the change in the polarization with energy occurs most rapidly in the vicinity of 4 MeV. This transition from negative to positive polarization at forward angles was also observed at θ (lab) $\cong$ 10.7° in the measurements at 3.40 and 4.40 MeV as indicated by the crosses in Fig. 2. We want to explain such behaviour of polarization by assuming that in the reaction investigated by Blue *et al.*, for certain energy region (4–5 MeV), the exchange wave symmetry is fulfilled. First of all we define the quantity Δ by the formula

$$p_b^2 = p_a^2(1+\Delta).$$

From the conservation of energy for the reaction (2) we obtain

$$E_a(\text{lab}) = \frac{m_a+m_A}{m_A} Q\left[(1+\Delta)\frac{(m_B+m_b)\,m_a m_A}{m_b m_B(m_a+m_A)} - 1\right]^{-1} \tag{13}$$

or

$$\Delta = \frac{Q(m_a+m_A)+\left(1-\dfrac{(m_B+m_b)\,m_a m_A}{m_b m_B(m_A+m_a)}\right) m_A E_a(\text{lab})}{\dfrac{m_a m_A(m_B+m_b)}{(m_A+m_a)\,m_B m_b}\, m_A E_a(\text{lab})}. \tag{14}$$

The function $\Delta = f(E_a\,(\text{lab}))$ for the reaction $^9\text{Be}(d,p)^{10}\text{Be}$ is shown in Fig. 2. It is easily to be seen that for the energy range (4–10 MeV) Δ is small. This means that for the above energy range $p_a = p_b$ holds. When we take into account that for zero range approximation the relative distances $\vec{r}_a$, $\vec{r}_b$ fulfil the relation

$$\vec{r}_a = \vec{r}_b \frac{M_B}{M_A} \tag{15}$$

then from the equality $p_a r_a = p_b r_b$ we obtain for Δ, $\Delta = \left(\frac{M_B}{M_A}\right)^2 - 1$. For the reaction ^9Be (d,p) ^{10}Be, $\Delta = 0.235$ which when substituted into the formula (13) gives E_d (lab) $= 4.62$ MeV. Hence for that energy, q_0^1, the vector polarization of protons, must be equal zero. From Fig. 2 we conclude that this is indeed true. For the energy range 4–5 MeV the experimental value of vector polarization is equal zero. It is also easily seen that the region, where Δ is rapidly change with energy (only due to pure kinematic effects), shows also a marked change of polarization. On the other hand, for the energy E_d (lab) near 10 MeV the polarization is smooth function of energy. From our analysis of polarization data for the reaction ^9Be (d,p) ^{10}Be we can conclude that there a corelation exists between the parameter Δ and the gross feature of polarization. Bearing in mind the fact that the function $f(E_d\,(\text{lab}))$ for the energy E_d (lab) > 2 MeV does not exhibit resonances, it is difficult to imagine that the pure DWBA can explain such a rapid change of polarization near 4 MeV. On the other hand, the lack of rapid change of polarization for energy range above 10 MeV suggests that for this range of E_d(lab), DWBA can be well suited to explain the behaviour of polarization. Our conclusions are in agreement with the commonly adopted opinion that the DWBA calculations can satisfactorily reproduce the features of only those direct reactions for which $p_a \cong p_b$, *i. e.* $\Delta \ll 1$, [2], [3].

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