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THEORETICAL ESTIMATES OF LIFETIMES OF SUPERHEAVY NUCLEI

By A. Łukasiak, A. Sobiczewski and W. Stępień-Rudzka

Institute of Nuclear Research, Warsaw*

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The spontaneous fission and alpha decay half-lives of even superheavy nuclei are calculated microscopically. The calculations are based on the single-particle Nilsson scheme with the parameters fitted to the scheme obtained in the Woods-Saxon potential at zero deformation. The largest total half-lives of the order of 10^{10} years are obtained for the nuclei with $Z = 108$–110 and $N = 184$. The strength of the pairing forces used is smaller than in previous investigations. The use of a larger strength would lead to half-lives smaller by a few orders. The sensitivity of the half-lives to different factors, in particular to the pairing forces strength, is discussed.

1. *Introduction*

The heaviest elements, known up to now, are the isotopes of the elements 104 and 105. Their half-lives are of the order of seconds and the efficiency of techniques by which they were obtained is of the type of "one atom per hour".

According to the liquid drop model the half-lives of heavy nuclei decrease quickly with increasing atomic number Z (or mass number A following, let us say, the beta stability line). Thus according to this model we could not expect to go much further, in the production and detection of heavy elements.

The liquid drop model does not take, however, into account the shell effects in the nuclear structure which are of the basic importance for the half-lives of heavy nuclei, especially for nuclei close to a magic nucleus. For example, the shell correction to the mass of the double magic nucleus ^{208}Pb is of the order of 10 MeV, while the total height of the spontaneous fission barrier of the usual fissionning nucleus (like uranium) is of the order of only 5 MeV. Thus, if the closed shells for a proton number $Z > 82$ and a neutron number $N > 126$ realize in nature, one may expect the lifetimes of the corresponding double magic and neighbouring nuclei (superheavy nuclei) to be extra long, due to the shell effect.

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* Address: Instytut Badań Jądrowych, Warszawa, Hoża 69, Poland.

The first systematic analysis of the shell correction to the liquid drop mass has been performed by Myers and Swiatecki [1]. Their phenomenological, deformation dependent, three-parameter shell correction allowed one to decrease significantly the discrepancy between the experimental masses and those given by the liquid drop model and to describe relatively well the equilibrium quadrupole moments of nuclei.

The consequences of the shell correction on the problem of superheavy nuclei are of particular interest. If one assumes, for example, the nucleus $^{312}126$ to be double magic, one obtains a fission barrier for it as high as 9 MeV when one takes the shell correction into account, while there is no fission barrier for this nucleus in the pure liquid drop model. This illustrates the reason why a considerable interest in the problem of superheavy nuclei arose after publishing paper [1].

The calculations of the single-nucleon spectra have been performed using realistic nuclear potentials. In these spectra, obtained for both non-local [2, 3] and local [4–6] potentials, considerable energy gaps at proton number $Z = 114$ and neutron number $N = 184$ have been obtained, suggesting these numbers as the magic ones. Candidates for higher magic numbers have also been found [5, 7, 8]. Detailed estimations of the half-lives of superheavy nuclei neighbouring the nucleus $^{298}114$ have been performed [9–13]. They based on the microscopic method of the calculation of the shell correction proposed by Strutinsky [14]. The estimations have shown that total half-lives as large as 10^8 years may be expected. This gave a chance not only to detect such nuclei as products of nuclear reactions but also to find them in the primary cosmic rays or even on the earth.

The aim of the present paper is to estimate the lifetimes of superheavy nuclei on the basis of the single-particle spectra different from these used up to now. It is also aimed at discussing a few other effects, such as the effect of the deformation dependence of the mass parameter, the effect of the pairing forces strength increasing with the increase of the nuclear surface area, and others, on these lifetimes.

In Section 2 we describe the method and specify the details of the calculations. In Section 3 we present the results and discuss various factors affecting these results.

2. *Description of the calculations*

2.1. Method of the calculations

We aim here at estimating the spontaneous fission T_{sf} and the alpha decay T_α half-lives of even superheavy nuclei in their ground state.

To estimate T_{sf} we base on the formula

$$T_{sf} = \frac{\ln 2}{n} \frac{1}{P}, \tag{1}$$

where n is the number of assaults of a nucleus on the fission barrier per unit time and P is the probability of penetration of the nucleus through the barrier for a given assault. For n one usually uses the reciprocal of the frequency of the vibration (in the one-dimentional case, considered here, it is the beta vibration) leading to fission.

In the WKB approximation, the probability of penetration through the fission barrier is [15]

$$P = \left\{1+\exp\left(2\int\limits_{\varepsilon_1}^{\varepsilon_2}\sqrt{2\frac{B(\varepsilon)}{\hbar^2}\,[W(\varepsilon)-E]\;d\varepsilon}\right)\right\}^{-1} \equiv (1+\exp K)^{-1}. \tag{2}$$

For $K \gg 1$

$$P \approx e^{-K}.$$

In eq. (2) $B(\varepsilon)$ is the mass parameter describing the inertia of a nucleus with respect to its deformation which is characterized by the parameter ε. The quantity $W(\varepsilon)$ describes the

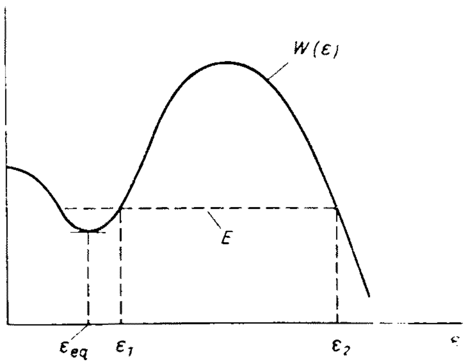

Fig. 1. Schematic fission barrier. For nuclei close to a magic nucleus $\varepsilon_{eq} = 0$

fission barrier and E is the energy of a nucleus in the state characterized by the equilibrium deformation ε_{eq}, as shown in Fig. 1.

We calculate the mass parameter B microscopically. In the adiabatic approximation the corresponding formula is [16]

$$\frac{1}{\hbar^2}\,B = \frac{2\Sigma_3}{(2\Sigma_1)^2}\left(\frac{\partial Q}{\partial\varepsilon}\right)^2, \tag{3}$$

where

$$\sum\nolimits_n = \sum_{\mu,\nu}\frac{(q_{\mu\nu})^2\,(u_\mu v_\nu + u_\nu v_\mu)^2}{(E_\mu+E_\nu)^n}$$

with $n = 1$ or 3. Here $q_{\mu\nu}$ is the quadrupole moment matrix element between states $|\mu\rangle$ and $|\nu\rangle$, u_μ and v_μ are the coefficients of the BCS wave function, E_μ is the energy of a quasi-particle and Q is the mass quadrupole moment of the nucleus. The details of the microscopic calculations of the mass parameter B are described in Ref. [16].

To estimate the alpha decay half-lives we use two alternative phenomenological formulae: the one by Taagepera and Nurmia [17]

$$\log T_\alpha(y) = 1.61\left[\frac{Z-2}{\sqrt{Q_\alpha}} - (Z-2)^{2/3}\right] - 28.9 \tag{4}$$

and the formula by Viola and Seaborg [18]

$$\log T_\alpha(\text{sec}) = A_Z Q_\alpha^{-\frac{1}{2}} + B_Z \tag{5}$$

with

$$A_Z = 2.11329\,Z - 48.9879$$

$$B_Z = -0.390040\,Z - 16.9543$$

used by the authors [18] to predict alpha decay half-lives for elements heavier than those known experimentally. In both formulae Z is the atomic number of a parent nucleus and Q_α is the alpha decay energy.

We see that in order to calculate T_α we have to know the masses of parent and daughter nuclei at their equilibrium deformations and to calculate T_{sf} we have to know the dependence of the mass on deformation, as shown in Fig. 1. Thus to calculate both T_α and T_{sf} we have to know the nuclear mass as a function of the deformation.

For the calculation of the nuclear mass or the total nuclear energy we adopt the formula representing the nuclear energy as composed of two parts: one given by the liquid drop model and the other corresponding to the shell correction, *i.e.*

$$E(Z, N, \text{def.}) = E_{LD}(Z, N, \text{def.}) + \Delta E_{\text{SHELL}}(Z, N, \text{def.}) \ldots \tag{6}$$

The most microsopic method, among the practical methods elaborated up to now, for the calculation of the shell correction is the one proposed by Strutinsky [14]. It consists in representing this correction as the difference between the sum of real single-particle energies of a nucleus and the sum when these real energies are smeared out to give a continuous single-particle spectrum instead of the discrete one. Such a difference is expected to give all shell structure effects on mass which are not present in the liquid drop mass formula and which were found so important for nuclear masses (*cf. e.g.* Ref. [1]).

After inclusion of the pairing interaction by the BCS formalism, the formula for the shell correction is [11]

$$\Delta E_{\text{SHELL}}(Z, N, \text{def.}) = \Delta E_{\text{SHELL}}(Z, \text{def.}) + \Delta E_{\text{SHELL}}(N, \text{def.}), \tag{7}$$

where

$$\Delta E_{\text{SHELL}}(X, \text{def.}) = \{\sum_\nu e_\nu\, 2v_\nu^2 - \Delta^2/G - G(\sum_\nu v_\nu^4 - \sum_\nu{}' 1)\} - \{E(g) + \langle E_{\text{pair}}\rangle\} \tag{7a}$$

with X standing for Z (protons) or N (neutrons). In Eq. (7a) e_ν are the single-particle energies, G is the pairing forces strength, Δ is the energy gap and the term $G\Sigma_\nu' 1$ is the subtracted diagonal pairing energy corresponding to a sharp Fermi surface.

The quantity $E(g)$ represents the sum of the single-particle energies when these energies are smeared out to give a continuous density of the levels $g(e)$. The corresponding formula is

$$E(g) = \int_{-\infty}^{e_F} 2eg(e)de \tag{8}$$

with the Fermi energy e_F given by

$$X \equiv Z(\text{or } N) = \int_{-\infty}^{e_F} 2g(e)de. \tag{8a}$$

The level density function $g(e)$ obtained by smearing out of the single-particle levels with the help of the Gauss function is

$$g(e) = \frac{1}{\gamma\sqrt{\pi}} \sum_{\nu} f_{\mathrm{corr}}(u_\nu) e^{-u} \tag{9}$$

with $u_\nu = (e - e_\nu)/\gamma$. The correction function f_{corr} (u_ν) is constructed in such a way as to smooth out the short range fluctuations in the level density with the range l (of the order of the shell spacing $\hbar\omega_0$) and to retain the long range fluctuations with the range L (of the order of the Fermi energy). Thus the smeared level width γ should be: $l < \gamma < L$. Up to the sixth order in u_ν the f_{corr} function is [19, 11]

$$f_{\mathrm{corr}}(u_\nu) = 1 + \left(\frac{1}{2} - u_\nu^2\right) + \left(\frac{3}{8} - \frac{3}{2} u_\nu^2 + \frac{1}{2} u_\nu^4\right) + \left(\frac{5}{16} - \frac{15}{8} u_\nu^2 + \frac{5}{4} u_\nu^4 - \frac{1}{6} u_\nu^6\right). \tag{10}$$

The term $\langle E_{\mathrm{pair}} \rangle$ in Eq. (7a) denotes the average value of the pairing energy of a nucleus at its equilibrium deformation. We adopt here after Ref. [11] the value $\langle E_{\mathrm{pair}} \rangle \approx -2.3$ MeV.

2.2. Details of the calculations

We investigate the total energy of a nucleus as a function of the quadrupole ε and hexadecapole ε_4 deformations. To obtain the single-particle energies of a nucleus as functions of these deformations we use the Nilsson single-particle Hamiltonian [5]

$$H_{\mathrm{sp}} = T + V, \tag{11}$$

where the kinetic energy is

$$T = \frac{1}{2} \hbar\omega_0(\varepsilon, \varepsilon_4) \left[-\Delta\varrho + \frac{2}{3} \varepsilon \, \frac{1}{2} \left(2\frac{\partial^2}{\partial\zeta^2} - \frac{\partial^2}{\partial\xi^2} - \frac{\partial^2}{\partial\eta^2} \right) \right]$$

and the potential energy is

$$V = \frac{1}{2} \hbar\omega_0(\varepsilon, \varepsilon_4) \varrho^2 \left[1 - \frac{2}{3} \varepsilon P_2(\cos\theta_t) + 2\varepsilon_4 P_4(\cos\theta_t) \right] - -\varkappa \hbar \mathring{\omega}_0 \left[2 l_t \cdot s + \mu (l^2 - \langle l^2 \rangle_N) \right].$$

Here, ξ, η, ζ are the stretched coordinates and the subscript t at a quantity denotes that it is expressed in these coordinates, $\varrho^2 = \xi^2 + \eta^2 + \zeta^2$. P_2, P_4 denote the Legendre polynomials of corresponding degrees.

The parameters $\varkappa$ an μ are usually fitted to the experimental single-particle levels and then, for the analysis of nuclei in the superheavy region, they are extrapolated to that region.

In the present paper we fit the parameters $\varkappa$ and μ to the levels obtained in the Woods-Saxon potential. The details of this fitting are described in Ref. [20]. The resulting values of $\varkappa$ and μ are given in Table I. The corresponding single-particle levels, obtained with the

TABLE I

The Nilsson scheme parameters $\varkappa$ and μ, used in the present calculation, *versus* shell number N

N	$\varkappa_p$	$\varkappa_n$	μ_p	μ_n
0	0.112	0.102	0	0
1	0.098	0.102	0	0
2	0.086	0.102	0.230	0.145
3	0.078	0.076	0.385	0.225
4	0.068	0.066	0.510	0.288
5	0.057	0.059	0.810	0.337
6	0.049	0.053	0.739	0.377
7	0.046	0.047	0.702	0.409
8	0.038	0.043	1.000	0.435
9	0.033	0.039	1.140	0.457
10	0.029	0.035	1.220	0.475
11	0.025	0.032	1.310	0.490
12	0.022	0.030	1.380	0.504

Hamiltonian (11) at zero deformation ($\varepsilon = \varepsilon_4 = 0$), are shown in Fig. 2*a* for protons and Fig. 3*a* for neutrons. The Woods-Saxon levels are shown in (*b*) of the figures and the levels obtained with the extrapolated parameters $\varkappa$ and μ and used in the previous calculations [9–11] are presented in (*c*), for comparison.

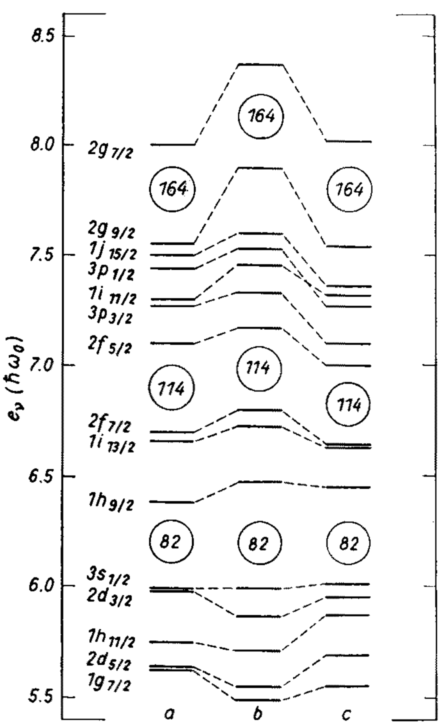

Fig. 2. Single-particle schemes for protons: *a*) Nilsson scheme used in the present paper, *b*) scheme obtained in the Woods-Saxon potential and *c*) extrapolated Nilsson scheme used in Refs [9–11]

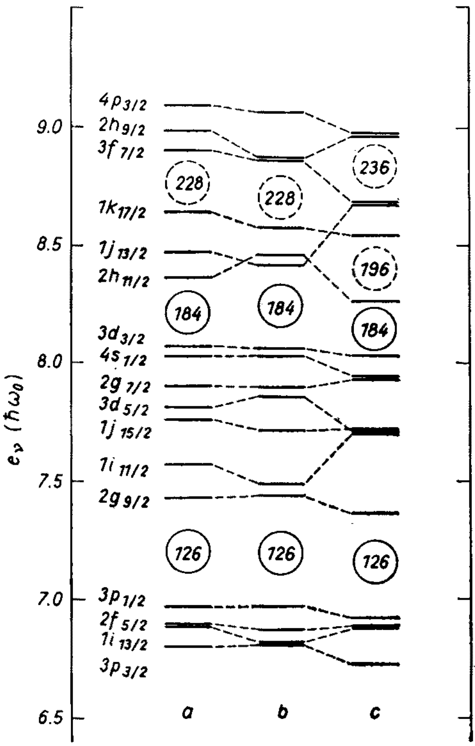

Fig. 3. The same, as in Fig. 2, for neutrons

To solve the pairing equations for Z protons (or N neutrons) we take $m = Z$ (or N) levels, from the lowest level up to the Z-th (or N-th) one, for which the pairing interaction is switched on. The pairing interaction strength $G(m)$ is calculated from the experimental odd-even mass differences in the actinides region. For $m \approx 114$ we obtain for protons $G_p \approx (18.6/A)$ MeV and for $m \approx 184$ we obtain for neutrons $G_n \approx (13.3/A)$ MeV. These are by 5.2% (protons) and 5.0% (neutrons) lower than the corresponding values $G_p = (19.6/A)$ MeV and $G_n = (14.0/A)$ MeV used in the previous calculations [9–11]. This should be kept in mind when comparing the results of the present paper with those of Refs [9–11].

All our calculations of the fission barrier $W(\varepsilon) \equiv E(\varepsilon)$ and the mass parameter $B(\varepsilon)$ are performed with the pairing strength proportional to the surface area of a nucleus: $G \sim S$ (cf. Ref. [21]), although the effect of using $G =$ const instead of $G \sim S$ is discussed. Thus the values $G_p = (18.6/A)$ MeV and $G_n = (13.3/A)$ MeV, given above, correspond only to the equilibrium deformations ($\varepsilon = \varepsilon_4 = 0$) of the investigated nuclei and the values $G(\varepsilon)$ corresponding to other deformations are obtained from the relation $G \sim S$ (see Fig. 10).

As mentioned above, we consider the fission problem as a penetration of the one-dimentional barrier. We obtain this barrier as the line on the two-dimentional total energy surface $E(\varepsilon, \varepsilon_4)$ corresponding to the minimal energy. In other words, for each ε we find $\varepsilon_4(\varepsilon)$, for which the energy is minimal. We have found that, on the average, for all nuclei investigated in the present paper, this minimal energy path is relatively well described by the straight line

$$\varepsilon_4 = 0.1\varepsilon \tag{12}$$

for $\varepsilon = 0-0.7$. Thus all fission barriers in our calculations correspond to the line (12).

The liquid drop model parameters are taken according to the Lysekil paper by Myers and Swiatecki [22].

The sixth order correction function f_{corr}, *i.e.* just the one given by Eq. (10), is taken. The value of the shell-smearing parameter $\gamma = 0.8\,\hbar\omega_0(\varepsilon, \varepsilon_4)$ is used. It was shown in Refs [11, 19] that the shell correction energy ΔE_{SHELL} is a rather flat function of γ, when the correction function of as high order as six is taken.

In the calculation of the spontaneous fission half-life T_{sf} (*cf.* Eq. (1)), the beta vibration energy of a nucleus $\hbar\omega_\beta = 1$ MeV is assumed. This corresponds to the number of assaults of the nucleus on the fission barrier, due to this vibration, $n \approx 10^{20.38}$ sec^{-1}.

In the calculation of the fission probability P (*cf.* Eq. (2)), we assume that the height of the barrier is lowered by the zero-point vibration energy $E - W(\varepsilon_{eq}) = \frac{1}{2}\hbar\omega_\beta = 0.5$ MeV (*cf.* Fig. 1).

3. *Results and discussion of various effects*

3.1. Results of the calculations

Fig. 4 presents the map of the shell correction, *i.e.* the shell correction ΔE_{SHELL} as a function of Z and N, calculated for all investigated nuclei at zero deformation ($\varepsilon = \varepsilon_4 = 0$). We see that 11 MeV, the highest shell correction (in absolute value) is obtained. It is even

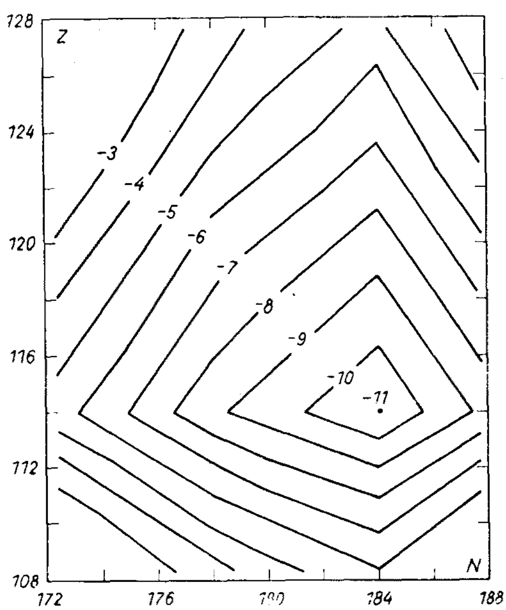

Fig. 4. Shell correction ΔE_{SHELL} in MeV as a function of the proton Z and neutron N numbers at zero deformation

slightly larger than that for the nucleus ^{208}Pb (10 MeV). The map gives an idea for what nuclei to expect the largest spontaneous fission half-lives T_{sf}. As the equilibrium deformations for almost all of the investigated nuclei are zero, the map also gives an idea about the shell correction contributions to the alpha decay energies.

The fission barriers are presented in Figs 5 and 6. As stated above, they are calculated along the deformation path (12). Fig. 5 illustrates how quickly does the barrier of the magic nuclei (isotopes) with $Z = 114$ change when the neutron number N becomes more distant, in both direction, from the magic value $N = 184$. Fig. 6 illustrates the same for magic

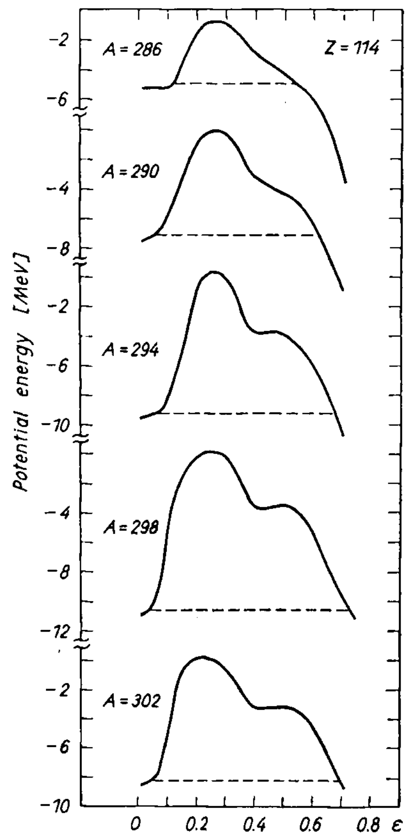

Fig. 5

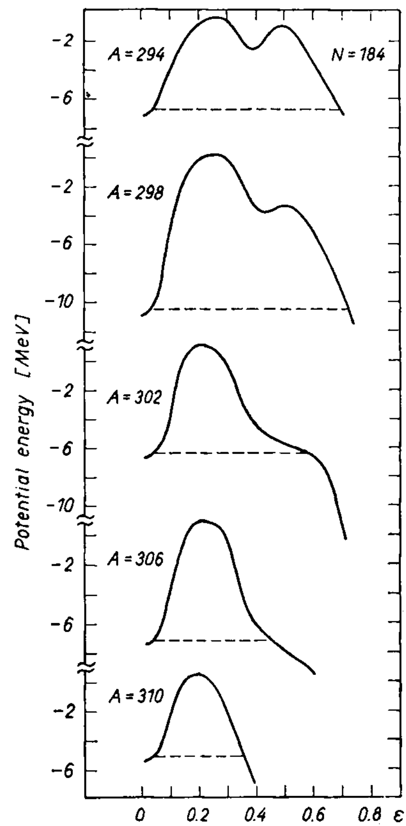

Fig. 6

Fig. 5. Fission barrier for a few isotopes with $Z = 114$. The dashed straight line cutting each barrier corresponds to the zero-point energy equal 0.5 MeV

Fig. 6. The same, as in Fig. 5, for a few isotones with $N = 184$

nuclei (isotones) with $N = 184$ and with the proton number Z becoming more distant from the magic value $Z = 114$. We see that the decrease in the barrier is quite fast when we remove the numbers Z or N from the magic values. It results in the corresponding rapid decrease of the spontaneous fission lifetimes.

The dependence of the mass parameter B on the deformation is illustrated in Fig. 7 for a few isotones with $N = 184$. As in all our calculations, the pairing strength $G \sim S$ is taken. We see that B fluctuates, both as a function of the deformation and the proton number, around some average value (which is approximately $B \approx 1000\,\hbar^2\,\mathrm{MeV}^{-1}$ for the nuclei presented in the figure). It is connected with the fluctuation of the density of the energy levels close to the Fermi level, as was pointed out in Ref. [24].

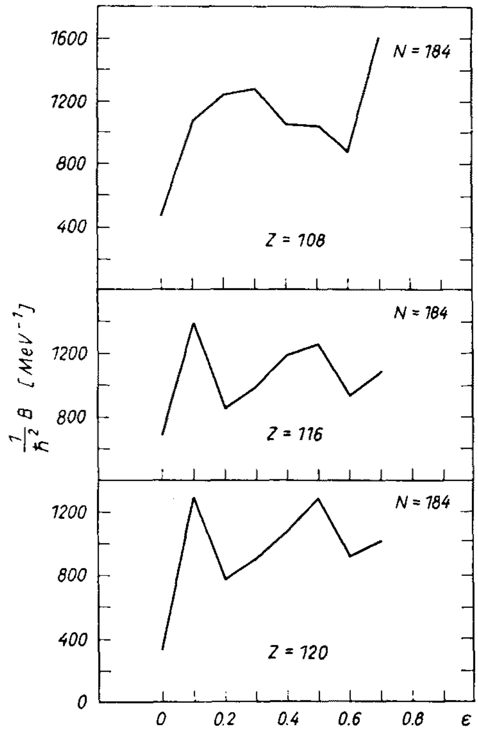

Fig. 7. Mass parameter B as a function of the deformation for three isotones with $N = 184$

At last, the map of the spontaneous fission and alpha decay half-lives is presented in Fig. 8. The T_{sf} half-lives are calculated according to Eqs (1) and (2) with the zero-point vibration energy equal 0.5 MeV. Phenomenological values of the mass parameter $B^{\mathrm{phen}} \approx 0.054\,A^{5/3}\hbar^2$ MeV^{-1} (see Ref. [9]) have been used. We will see later (*cf.* Fig. 12) that these phenomenological values are quite close to the corresponding microscopic values averaged over deformation. The alpha half-lives T_α are calculated according to Eq. (4). The values of T_α calculated according to Eq. (5) are larger for about (0.1–2) orders, as may be seen from Table II.

It is seen in Fig. 8 that the largest total half-lives are obtained for $Z \approx 108$–110 (and $N = 184$) and they are of the order of 10^{10} years. The values of T_{sf} obtained in the present paper are larger than those of the previous papers [9–11]. It is connected with the larger energy gap for $N = 184$ in our scheme (*cf.* Fig. 3) and thus with the larger (for about 2 MeV)

TABLE II

Logarithms of the spontaneous fission T_{sf} and alpha decay T_{α} half-lives, both in years, and the alpha decay energies Q_{α} in MeV for nuclides specified in the first two columns. The variants $(A)-(D)$ of the calculation of T_{sf} are described in text. The variants (E) and (F) of the calculation of T_{α} correspond to Eq. (4) and (5), respectively

Z	N	log $T_{sf}(y)$				log $T_{\alpha}(y)$		Q_{α}(MeV)
		(A)	(B)	(C)	(D)	(E)	(F)	
108	180	−3.7	2.5	8.0	9.5			
	182	1.9	9.8	14.7	17.0			
	184	8.8	18.6	23.0	28.7			
	186	1.6	9.9	14.8	18.0			
	188	−5.5	1.3	6.6	8.0			
110	178	−3.7	3.5	9.3	10.5	0.1	1.7	7.05
	180	2.7	10.3	16.2	18.0	4.5	6.4	6.19
	182	8.3	16.5	21.3	25.0	7.3	9.3	5.72
	184	15.0	24.8	29.0	36.7	9.5	11.6	5.39
	186	8.5	17.0	21.5	27.0	0.3	2.0	7.00
112	176	−2.5	6.0	11.3	13.0	−4.2	−2.8	8.25
	178	4.5	13.7	18.3	22.5	0.6	2.3	7.11
	180	10.8	21.7	26.9	30.7	3.5	5.4	6.52
	182	16.0	25.6	29.9	36.0	4.3	6.2	6.38
	184	22.3	34.0	37.8	47.3	6.2	8.3	6.04
	186	16.6	27.0	31.0	39.3	−1.9	−0.4	7.68
114	172	−8.0	−1.7	4.7	5.7			
	174	−0.7	9.6	17.5	17.3	−5.9	−4.6	8.92
	176	6.6	21.3	28.3	29.8	−1.8	−0.3	7.81
	178	13.3	32.8	39.0	43.5	1.2	3.0	7.13
	180	19.0	39.3	44.8	49.7	0.9	2.6	7.20
	182	24.0	45.3	50.0	57.5	1.6	3.4	7.05
	184	29.8	55.0	59.3	69.6	3.4	5.2	6.70
	186	24.9	48.6	53.3	62.7	−4.0	−2.6	8.37
	188	20.5	43.0	47.8	55.8	−2.5	−1.0	8.00
116	174	−9.7	−5.4	0.5	1.5	−12.3	−11.5	11.37
	176	−2.0	5.6	10.7	12.5	−11.5	−10.7	11.05
	178	5.5	13.3	17.5	21.8	−10.9	−10.0	10.80
	180	11.3	22.6	28.0	30.3	−11.2	−10.3	10.91
	182	16.6	25.5	29.7	34.8	−10.9	−10.0	10.80
	184	22.5	33.7	37.0	44.8	−10.0	−9.0	10.45
	186	17.0	26.9	30.7	37.3	−14.2	−13.5	12.19
118	178	−3.0	2.0	7.0	9.0	−12.3	−11.5	11.56
	180	3.9	12.0	18.1	17.3	−12.6	−11.9	11.68
	182	9.6	15.5	19.8	22.8	−12.3	−11.5	11.56
	164	15.9	23.8	27.3	32.5	−11.4	−10.6	11.21
	186	9.8	16.0	20.1	24.0	−15.3	−14.8	12.97

Table II (continued)

Z	N	log $T_{sf}(y)$				log $T_\alpha(y)$		Q_α(MeV)
		(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)	
120	180	−4.0	3.0	8.5	5.3	−13.9	−13.4	12.52
	182	1.0	4.6	10.6	11.3	−13.7	−13.2	12.41
	184	8.5.	14.8	18.7		−12.9	−12.3	12.04
	186	0.5	4.0	9.7	11.3	−16.6	−16.3	13.84
122	182	−4.0	−1.0	2.5	0.3	−15.1	−14.8	13.32
	184	0.6	5.0	8.3	7.3	−14.4	−14.0	12.95
	186	−4.2	−1.2	1.6	0.3	−17.9	−17.8	14.80
	188	−7.7	−6.0	−3.2	−4.7	−17.2	−17.1	14.43

shell corrections and consequently higher fission barriers. Also the shape of our contour lines corresponding to T_{sf} = const is different, especially for $N > 184$, from this of Refs [9–11, 13]. This is due to the fact that we do not observe in our scheme the neutron shell at $N = 196$ which is obtained in the scheme used in the papers [9–11, 13] (*cf.* Fig. 3).

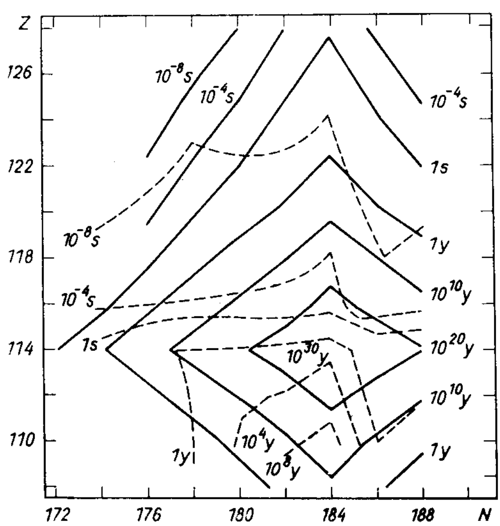

Fig. 8. Map of the spontaneous fission (solid lines) and the alpha decay (dashed lines) half-lives

3.2. Discussion of various effects

In this section we will discuss mainly the sensitivity of the lifetimes on the pairing strength G and the effect of the deformation dependence of the mass parameter on these lifetimes. The effect of a change in the Nilsson scheme parameters on the half-lives was discussed very recently by Gustafson [23].

3.2.1. Sensitivity on the pairing interaction strength G

Fig. 9 presents the fission barrier of three nuclei obtained in two cases: one calculated with $G \sim S$ and the other with $G = G(0) = \text{const}$. It illustrates in what degree is the barrier reduced when we use G increasing with deformation (as $G \sim S$) with respect to the barrier calculated with G independent of deformation. The dependence of the strength $G \sim S$

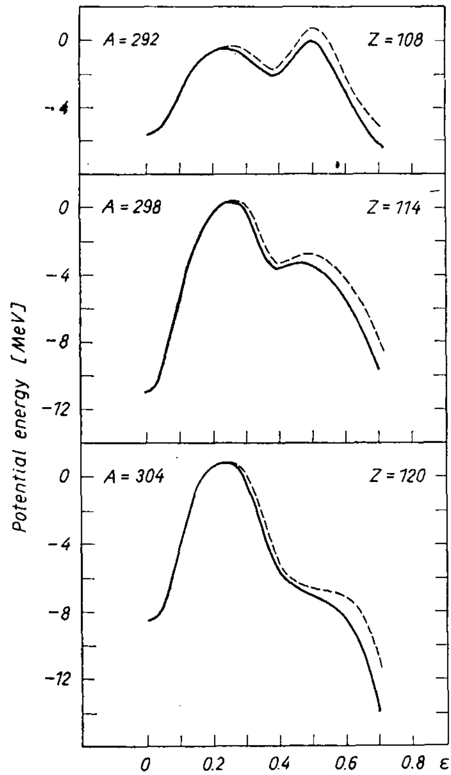

Fig. 9. Fission barriers calculated with $G \sim S$ (solid lines) and $G = \text{const}$ (dashed lines) for three isotones with $N = 184$

on the deformations ε and ε_4, related by Eq. (12), is shown in Fig. 10. We see that for the largest deformations $\varepsilon \approx 0.7$, involved in the problem of the barrier penetration of the nuclei considered, the corresponding increase in G is obout 9%.

The reduction of the mass parameter B (calculated at the point $\varepsilon = 0.2$, $\varepsilon_4 = 0.02$ which is close to the saddle points of the considered nuclei), due to the increase in the pairing strength G, is illustrated in Fig. 11 for a few nuclei. Each curve is drawn through three points corresponding to 0%, 5% and 10% increase in G. We see that this reduction, although different for different nuclei, is quite large. For example, the 5% increase in G reduces B by about (25–45)%. As a result, for most nuclei the reduction in B, due to an increase in G, influencies the spontaneous fission lifetimes T_{sf} stronger than the corresponding reduction in the fission barrier.

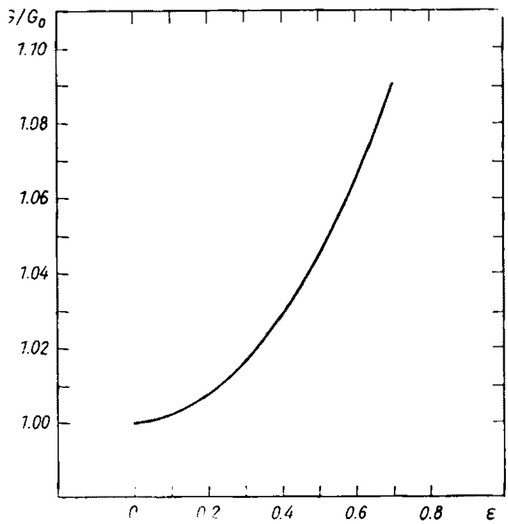

Fig. 10. The dependence of the strength $G \sim S$ on the deformation. The quantity G_0 denotes the value of G at zero deformation

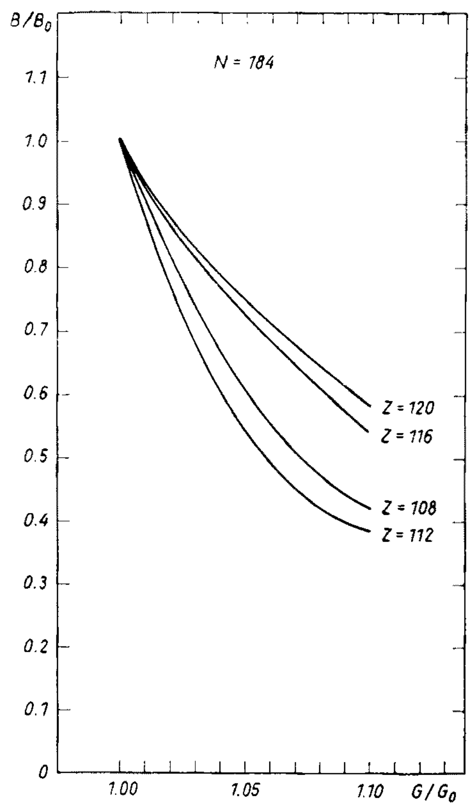

Fig. 11. The dependence of the mass parameter B on the pairing strength G for a few isotones with $N = 184$. The quantity B_0 denotes the value of B calculated at $\varepsilon = 0.2$ and $\varepsilon_4 = 0.02$

3.2.2. Effect of the dependence of the mass parameter on the deformation

The dependence of the mass parameter B on the deformation is shown in Fig. 12 for three isotones with $N = 184$. The parameter B is calculated in three variants:

(a) with $G = G(0) = \text{const}$ } and $Q = Q^{\text{unif}}$
(b) with $G \sim S$ } and $Q = Q^{\text{unif}}$
(c) with $G \sim S$ and $Q = Q^{\text{micr}}$.

The phenomenological value $B^{\text{phen}} = 0.054\, A^{5/3}\hbar^2\, \text{MeV}^{-1}$ is also shown in Fig. 12 (denoted by (d)), for comparison.

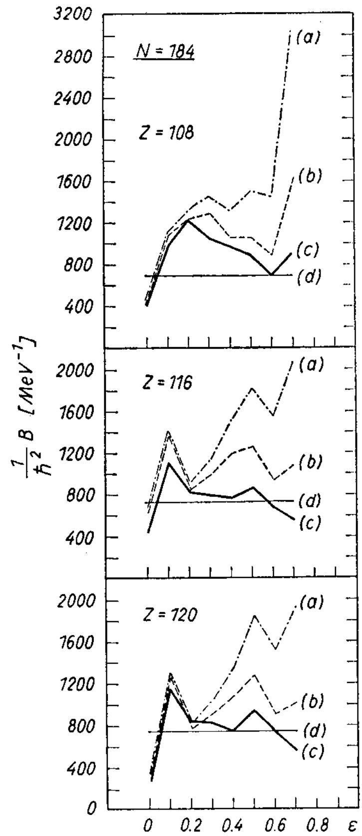

Fig. 12. Mass parameter B as a function of the deformation, calculated in four variants described in text, for three isotones with $N = 184$. Variant (b) coincides with that of Fig. 7

The quantities Q^{unif} and Q^{micr} correspond to different methods of the calculation of the quadrupole moment of a nucleus. The quadrupole moment is needed for calculation of its derivative appearing in the formula (3) for the mass parameter B.

The quantity Q^{unif} is the quadrupole moment of an axially symmetric ellipsoid with a uniform distribution of mass. The formula for it is (*cf. e.g.* Ref. [16])

$$Q^{\text{unif}}(\varepsilon) = \frac{2}{5} AR_0^2 \frac{c^2 - a^2}{R_0^2} \equiv 0.8\, AR_0^2 F(\varepsilon) \tag{13}$$

with

$$F(\varepsilon) = 0.5\left(1-\frac{1}{3}\varepsilon^2-\frac{2}{27}\varepsilon^3\right)^{\frac{2}{3}}\left[\frac{1}{\left(1-\frac{2}{3}\varepsilon\right)^2}-\frac{1}{\left(1+\frac{1}{3}\varepsilon\right)^2}\right].$$

In Eq. (13) c is the semi-axis of the ellipsoid measured along the symmetry axis and a the semi-axis along perpendicular direction. The radius of the non-deformed nucleus $R_0 = r_0 A^{1/3}$ with $r_0 = 1.2$ fm has been taken in the calculation.

The formula for the microscopic value of the quadrupole moment Q^{micr} is

$$Q^{\mathrm{micr}} = Q_p^{\mathrm{micr}} + Q_n^{\mathrm{micr}} \tag{14}$$

with

$$Q_{p(n)}^{\mathrm{micr}} = \sum_{\nu_{p(n)}} q_{\nu\nu}\, 2v_\nu^2 .$$

The index p corresponds to protons and n to neutrons.

It is seen in Fig. 12 that the use of $G \sim S$ instead of $G = \text{const}$ decreases B very strongly. For the deformation $\varepsilon = 0.7$, the decrease is about 50% which is in line with the results of Fig. 11. This stresses once more the strong sensitivity of the mass parameter B on the pairing strength G and its dependence on deformation. The consistent use of Q^{micr} in the microscopic calculation of B, instead of Q^{unif} used up to now [9, 11, 16], leads also, as a rule, to a decrease in B. It is seen that the mean value (obtained by averaging over deformations involved in the barrier penetration) of such B^{micr} is quite close to the phenomenological value B^{phen} (*cf.* especially the nuclei with $Z = 116$ and 120 in Fig. 12). This is the reason why we have used B^{phen} in the calculation of T_{sf} presented in Fig. 8 and why we consider these values of T_{sf} the most realistic of the four variants gives in Table II.

Table II presents logarithms of the spontaneous fission T_{sf} and alpha decay T_α half-lives, both in years, and the alpha decay energies Q_α in MeV.

The half-lives T_{sf} are calculated in the four variants:

(*A*) $G \sim S$,	$B = B^{\mathrm{phen}}$,	$E_0 = 0.5$ MeV
(*B*) $G \sim S$,	$B = B(\varepsilon)$ with Q^{unif},	$E_0 = 0.5$ MeV
(*C*) $G \sim S$,	$B = B(\varepsilon)$ with Q^{unif},	$E_0 = 0$
(*D*) $G = G(0) = \text{const}$,	$B = B(\varepsilon)$ with Q^{unif},	$E_0 = 0.5$ MeV.

In other words: in the first three cases we use $G \sim S$ and in the last one $G = \text{const}$. In the last three cases we use B dependent on deformation and calculated with the quadrupole moment Q^{unif}, and in the first case we use $B = B^{\mathrm{phen}} = \text{const}\ (\varepsilon)$. At last, the zero-point energy E_0 is 0.5 MeV in all cases excluding only the third case, where it is zero.

Comparing variants (*B*) and (*D*), we see that the use of $G \sim S$ instead of $G = \text{const}$ decreases T_{sf} very strongly (up to about 14 orders for nuclei with the largest T_{sf}). The rather small differences in the barriers of the two variants (*cf.* Fig. 9) give a relatively small contribution to this effect. The main contribution follows from the large differences in B (*cf.* variants (*a*) and (*b*) in Fig. 12).

The effect of the zero-point energy on T_{sf} is illustrated by comparison of variants (*B*) and (*C*). It is seen that the increase of the zero-point energy for 0.5 MeV decreases T_{sf} by a few orders. This illustrates the importance of the reliable estimation of this energy.

Variant (*A*) of the calculation gives the lowest values of T_{sf}. As already stated above, we consider this variant the most realistic of the four variants presented in Table II. However, contemplating the values T_{sf} of this variant themselves, one should remember that we use the pairing strength *G* (fitted to actinide region) lower by about 5% than that recomended in Ref. [9]. If we took *G* of Ref. [9] the T_{sf} values would be still lower by a few orders.

The large sensitivity of T_{sf} on the strength *G* (mainly *via* the mass parameter *B*) is one of the main points we tried to illustrate in the present paper.

The alpha lifetimes are calculated in two variants: one (*E*) corresponding to Eq. (4) and the other (*F*) to Eq. (5). It is seen that the lifetimes (*E*) are shorter for all nuclei (by up to two orders) than the lifetimes (*F*). They are presented graphically in Fig. 8.

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