A DYNAMICAL QUARK MODEL OF MESONS

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A dynamical quark model of mesons based on the principle of a temperate interaction enabling the application of the Tamm-Dancoff method combined with a suitable cut-off is presented. The main result is that it is not at all necessary to assume an unreasonably high value of the coupling constant in order to obtain a very large mass defect of the bound quark-antiquark system.

1. *Introduction*

Our dynamical quark model of mesons is based on the Principle of Temperate Action. It is assumed that the Lagrangian of interaction is a function

$$L' = L'(a) \tag{1}$$

of the argument

$$a = ig\bar{\psi}\gamma_5\psi\varphi \tag{2}$$

which is either bounded or, at least, does not increase too rapidly if its argument a tends to infinity, *e. g.*

$$L' = a\,(1+\lambda a^2)^{-\alpha};\, \alpha > \frac{1}{6}. \tag{3}$$

The quantum field theory with such (or similar) temperate interaction is free of the usual convergence difficulties [1].

The method of computations with such complicated Lagragians consists in the following: it is possible to expand the Lagrangian into a power series in terms of the parameter λ and to curtail the series after a few terms if the field is sufficiently weak. This, however, is a condition upon the state vector. The state represents a weak field if it is a superposition of eigenstates of a small number of particles with comparatively small momenta (in the centre of mass system). Thus in the first order of approximation we may consider only the first term of the expansion of (3)

$$L' = ig\bar{\psi}\gamma_5\psi\varphi \tag{3'}$$

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but, at the same time, we have to consider the smallest possible number of particles and a suitable cut-off dependent upon λ. Thus, the method of approximations is a combination of the Tamm-Dancoff method [2] (further quoted as T. D.) with a suitable cut-off which compensates for the neglection of higher non-linear terms in (3).

Let the field ψ denote the quark field quantity and φ the pseudoscalar meson field quantity. In our case it is fundamentally impossible to distinguish the dressed pseudoscalar meson from the lowest bound state of a quark-antiquark pair. In order to satisfy this

Fig. 1

requirement we assume that the meson bare mass is twice as large as the bare mass of the quark. The dressed mass of the meson is described in the first order approximation by the graph presented in Fig. 1 which is indistinguishable from the graph (Fig. 2) representing

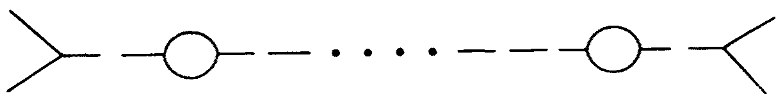

Fig. 2

the energy of the bound state of the pair inasmuch as these graphs represent infinite chains without a beginning and an end (and could be supplemented by dotts to both sides from the right and left). The lowest state represents pseudoscalar mesons whereas

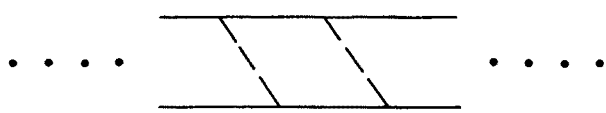

Fig. 3

other mesons are identified with excited bound states of the quark-antiquark system describable by more complicated graphs (*e. g.* Fig. 3).

It is assumed that the coupling constant G should be of the same order of magnitude as the ordinary nuclear forces coupling constant (*i. e.* not much different from 4). Thus, we are left with two free parameters: the quark mass m and the cut-off K, or rather their ratio. Assuming a reasonable ratio $K/m \sim 1$ (see *e. g.* [3]) the self energy of the quark is shown to be quite a small fraction of its bare mass so that we do not need to distinguish between the bare and dressed mass of a free quark. On the other hand, it is shown that (even with the assumed comparatively small value of the coupling constant) quark-antiquark pairs form strongly bound systems with an arbitrarily large mass defect.

The SU (3) symmetry is broken by assuming the λ-quark to be 1.21 times heavier than the remaining two quark (*cf.* [4]) which yields a correct mass splitting within the pseudoscalar meson octet.

2. *Self mass of the quark*

In the lowest T. D. approximation the self energy of a quark is represented as an infinite chain of the graphs (Fig. 4).

We solve the equation

$$H|> = E_0|> \tag{4}$$

with

$$H = H^0 + H' \tag{4'}$$

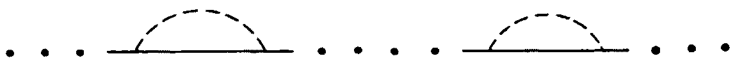

Fig. 4

where

$$H' = ig\colon \int \bar{\psi}\gamma_5\psi\varphi d^3x\colon \tag{4''}$$

where the ket $|>$ is a superposition of one-quark and one-quark plus one-meson states

$$|> = \sum_{\boldsymbol{k}_1} a(\boldsymbol{k}_1)|1_{\boldsymbol{k}_1}> + \sum_{\boldsymbol{k}_2,\boldsymbol{p}} b(\boldsymbol{k}_2, \boldsymbol{p})|1_{\boldsymbol{k}_2}, 1_{\boldsymbol{p}}> \tag{5}$$

where $|1_{\boldsymbol{k}_1}>$ is the amplitude of the one-quark state with momentum $\boldsymbol{k}_1$ and $|1_{\boldsymbol{k}_2}, 1_{\boldsymbol{p}}>$ is the amplitude of one-quark plus one-meson state with momenta $\boldsymbol{k}_2, \boldsymbol{p}$ respectively (the spin indices have been suppressed). From (4) and (5) one gets

$$[E_0 - \omega(\boldsymbol{k}_1)]\, a(\boldsymbol{k}_1) = \sum_{\boldsymbol{k}_2,\boldsymbol{p}} b(\boldsymbol{k}_2, \boldsymbol{p})\, \langle 1_{\boldsymbol{k}_1}|H'|1_{\boldsymbol{k}_2}, 1_{\boldsymbol{p}}\rangle \tag{6}$$

and

$$[E_0 - \omega(\boldsymbol{k}_2) - \omega'(\boldsymbol{p})]\, b(\boldsymbol{k}_2, \boldsymbol{p}) = \sum_{\boldsymbol{k}_1} a(\boldsymbol{k}_1)\, \langle 1_{\boldsymbol{p}}, 1_{\boldsymbol{k}_2}|H'|1_{\boldsymbol{k}_1}\rangle \tag{6'}$$

whence in the system of rest one gets the following non-linear formula for E_0

$$E_0 - m = \frac{1}{2}\left(\sum_{s^+} - \sum_{s^-}\right) \times$$

$$\times \int d^3k\, \frac{\langle 1_{-\boldsymbol{k}}, 1_{\boldsymbol{k}}|H'|1_{\boldsymbol{o}}\rangle\, \langle 1_{\boldsymbol{o}}|H'|1_{\boldsymbol{k}}\cdot 1_{-\boldsymbol{k}}\rangle}{E_0 - \omega\,(\boldsymbol{k}) - \omega'\,(-\boldsymbol{k})} \tag{7}$$

where it was averaged over the initial spins and summed over the final spins.

A straighforward computation yields

$$E_0 - m = \frac{g^2}{4(2\pi)^2} \int_0^K k^2 dk \int_0^\pi \sin\Theta d\Theta k_x^2 \times$$

$$\times\, [(E_0 - (k^2+\mu^2)^{\frac{1}{2}} - (k^2+m^2)^{\frac{1}{2}}\,(k^2+m^2)\,(k^2+\mu^2)^{\frac{1}{2}}]^{-1} \tag{8}$$

Assuming the meson mass to be twice the quark mass we compute (8) as a function of $s = K/m$ under the assumption

$$(i)\ E_0 \ll m$$

$$(ii)\ E_0 \approx m$$

It appears that (i) is inconsistent with the assumption $s \sim 1$ and $G < 10$ whereas from the assumption (ii) we get for $s = 0.91$ and $g^2 = 136$ $(G^2 = g^2/4\pi = 10.8)$ a consistent result $E_0 = 0.972$ m. This means that the dressed mass of the quark differs very little from its bare mass and we do not need to distinguish between them.

3. *Energy of the bound quark-antiquark system*

Assuming the same Hamiltonian (4′) and the state vector

$$|\rangle = \sum_{p} a(\boldsymbol{p})|1_{\boldsymbol{p}}\rangle + \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2} b(\boldsymbol{k}_1, \boldsymbol{k}_2)|1_{\boldsymbol{k}_1}, \bar{1}_{\boldsymbol{k}_2}\rangle \tag{9}$$

where $\boldsymbol{k}_2$ denotes the momentum of the antiquark, the problem reduces to the evaluation of the terms corresponding to the graphs represented by Fig. 1 or Fig. 2. This yields the following equation for the energy E_0

$$E_0 - 2m = \sum_{s,r} \int d^3k \langle 1_o | H' | 1_{\boldsymbol{k},s}; \bar{1}_{-\boldsymbol{k},r}\rangle \times$$

$$\times \langle \bar{1}_{-\boldsymbol{k},r}; 1_{\boldsymbol{k},s} | H' | 1_0\rangle (E_0 - 2\omega(\boldsymbol{k}))^{-1} \tag{10}$$

which may be developed in powers of E_0/m for small E_0. This equation has to be generalized so as to accomodate the unitary spin [5].

$$\mathcal{H}' = ig\lambda_j \gamma_5 \psi 0_j \tag{11}$$

where

$$\begin{aligned} 0_1 &= \pi^- + \pi^+ & 0_2 &= -i(\pi^+ - \pi^-) \\ 0_3 &= \sqrt{2}\pi^0 & 0_4 &= K^+ + K^- \\ 0_5 &= -i(K^+ - K^-) & 0_6 &= K^0 + \bar{K}^0 \\ 0_7 &= -i(\bar{K}^0 - K^0) & 0_8 &= \sqrt{2}\,\eta \end{aligned} \tag{12}$$

or, in units of the mass m of the non-strange quark

$$\alpha_1 = \frac{E_0^{p\bar{p}}}{m} = 2 - \frac{g^2}{\pi^2}\left[\ln\left(s + \sqrt{1+s^2}\right) - \frac{s}{\sqrt{1+s^2}}\right] \tag{13}$$

where $s = K/m$.

Denoting the mass of he strange quark by M we get similarly

$$\alpha_2 = \frac{E_0^{\lambda\bar{\lambda}}}{m} = 2 - \frac{g^2\varrho^2}{\pi^2}\left[\ln\left(\frac{s}{\varrho} + \sqrt{1 + \frac{s^2}{\varrho^2}}\right) - \frac{s}{\sqrt{s^2+\varrho^2}}\right] \tag{14}$$

where $\varrho = M/m$.

Similarly for bound states of non-strange quark with a strange antiquark one gets

$$\alpha_3 = \frac{E^{p\bar{\lambda}}}{m} = 2 - \frac{g^2}{4\pi^2(\varrho^2-1)}\Big\{s\,[\sqrt{s^2+\varrho^2}\,(2s^2-\varrho^2-2\varrho) -$$

$$- \sqrt{1+s^2}\,(2s^2-1-2\varrho)] + \varrho^3(\varrho+2)\ln\left(\frac{s}{\varrho}+\sqrt{1+\frac{s^2}{\varrho^2}}\right) - (1+2\varrho)\ln\left(s+\sqrt{1+s^2}\right)\Big\} \tag{15}$$

and the same for the energy of a system composed of a strange quark with a non-strange antiquark.

In the above formulae there appear three parameters g, ϱ and s. The coupling constant g may be eliminated by considering the ratios

$$k_1 = \frac{2-\alpha_1}{2-\alpha_2} \quad \text{and} \quad k_2 = \frac{2-\alpha_1}{2-\alpha_2} \tag{16}$$

In order to get agreement with experimental evidence the quantities k_1 and k_2 must be $k_1 > k_2 > 1$ and $s < \varrho$. In particular, assuming for example $\varrho = 1.21$, $s = 0.91$ we get $k_1 = 1.032$, $k_2 = 1.019$ whence

$$\alpha_1 = 0.0154,\ \alpha_2 = 0.075,\ \alpha_3 = 0.053 \tag{17}$$

Introducing the above values into (13) we get for the coupling constant the numerical result

$$g^2 = 136 \qquad G^2 = g^2/4\pi = 10.8 \tag{18}$$

of the same order of magnitude as is the ordinary nuclear forces coupling constant.

The ratios of the masses are

$$\alpha_1 = \frac{E_0^{p\bar{p}}}{m} = \frac{m_\pi}{m} \qquad \alpha_3 = \frac{E_0^{p\bar{\lambda}}}{m} = \frac{m_K}{m}$$

$$\frac{m_K}{m_\pi} = \frac{\alpha_3}{\alpha_1} = 3.44$$

$$m_\eta^2 = \frac{1}{3}\,[(E_0^{p\bar{p}})^2 + 2(E_0^{\lambda\bar{\lambda}})^2] = \frac{m^2}{3}\,(\alpha_1^2+2\alpha_2^2)$$

whence

$$\frac{m_\eta}{m_\pi} = \left[\frac{1}{3}\left(1+\left[\frac{\alpha_3}{\alpha_1}\right]^2\right)\right]^{\frac{1}{2}} = 4.0.$$

The experimental values are respectively

$$\left(\frac{m_K}{m_\pi}\right)_{\text{exp}} = 3.5 \qquad \left(\frac{m_\eta}{m_\pi}\right)_{\text{exp}} = 3.93.$$

The mass of the non-strange quark is

$$m = \frac{m_\pi}{\alpha_1} = 65\ m_\pi = 9.1\ \text{GeV}.$$

and that of the strange quark is

$$M = 1.21 \text{ m} = 80 m_\pi = 11 \text{ GeV}$$

Thus, it is possible to choose the parameters so as to obtain reasonable values for the coupling constant and the masses of the quarks as well as a correct mass splitting within the pseudo-scalar meson octet.

4. *The meson exchange forces*

Inasmuch as the quark-quark interaction is described by a different type of graphs (Fig. 5) it was not clear whether it is attractive or repulsive. Therefore we estimated also the energy corresponding to this graph and found that it cannot yield bound two-quark systems. It follows that a similar graph will contribute to the binding of the quark-antiquark

Fig. 5

system. However, inasmuch as now 3-particle states come into play, we have to go to a higher T. D. approximation so that this contribution may be regarded to be only a correction to the main term represented by Fig. 1 or Fig. 2. By taking this correction into account the value of g may be still diminished.

5. *Discussion*

The above described model in the lowest T. D. approximation is very similar to the Lee model [6]. Putting the bare meson mass equal to two quark masses we limit ourselves to a case intermediate between stable and unstable states of exchanged particle in this model. This is first reason why the bare mass of meson is so large.

The second reason is that, if we consider the exchange of a light physical particle (for example the pion) and we want to preserve the weak interaction — we must cut the momentum of quark at a lower value (in fact any physical particle is built from moving quarks) than for an exchange of a heavy boson without internal structure. In case of π meson exchange the formula (8) reads

$$E_0 = m \left\{ 1 - \frac{g^2}{48\pi^2} \left[s - \operatorname{arctg} s + \frac{s}{2} \sqrt{1+s^2} + \right.\right.$$
$$\left.\left. + \ln(1+s^2) - \frac{s^2}{2} - \frac{1}{2} \ln\left(s + \sqrt{1+s^2}\right) \right] \right\} \quad (19)$$

which, for cut-off at $s = 0.4$ yields

$$E_0 = 0.968\, m$$

The effect is of the same order of magnitude as for the exchange of a heavy boson (*cf.* chapter 2) with the same coupling constant (in fact the coupling constant for the exchange of a physical particle composed from moving quarks would be less than that for the exchange of a heavy boson without internal structure because of the vacuum polarization).

We note here the recently issued paper by Nitti and Pusterla [7] with the hamiltonian interaction term of a vector form. The authors used the Padé approximation and obtained very similar results to ours, namely

$$g^2 \cong 100, \qquad \mu = 1.6 \div 1.8\, m, \qquad m \cong 10\ \text{GeV}.$$

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