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EQUILIBRIUM DEFORMATIONS OF THE GROUND AND EXCITED STATES IN THE RARE-EARTH NUCLEI

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The equilibrium deformations of the ground and excited states in nuclei in the Rare-Earth region are calculated using the Nilsson model. The residual interactions are taken into account in the form of the pairing forces. Comparison is made with previous calculations of similar type together with a suggestion of applying the idea of different deformations in ground and excited states of the nucleus in analogue state physics.

Nuclei in the Rare-Earth region are known to possess a nonspherical distribution of nucleons in their ground states. The aim of this paper is to investigate the possibility that the nucleus when excited, changes its deformation. That is, we look for the equilibrium deformation of an excited state — ε_{ex} — and compare it with the equilibrium deformation of the ground state — ε_0 — for the given nucleus.

In the case of odd-A nuclei the ground state as well as a few lowest excited (non-collective) states are the one-particle states of the last unpaired nucleon. The quantum numbers of these states are known from experiments and they are in good agreement with the predictions of the Nilsson model — Nilsson 1955. Equilibrium deformation of the nucleus in a given state can be found from the minimum of the total energy of the nucleus — E — when treated as a function of the deformation.

Sixteen odd-A Rare-Earth nuclei were investigated, namely the isotopes of Ho, Er, Tm, Yb, Lu, Hf, Ta, and W. Their proton and neutron numbers were respectively:

$$67 \leqslant Z \leqslant 74 \text{ and } 96 \leqslant N \leqslant 109.$$

Their corresponding masses were: $163 \leqslant A \leqslant 183$.

It was assumed that the interactions in the nucleus can be described by the independent particle model with the average deformed potential (Nilsson *et al.* 1966 and Nilsson 1967). Furthermore it was assumed that the nucleons interact between themselves by the residual short-range pairing forces. In this model which can be solved in the independent quasi-

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-particle approximation, the ground and low excited states of an odd-A nucleus are one-quasi-particle states and the total energy is given by:

$$E = E^P_{\mathrm{BCS}} + E^N_{\mathrm{BCS}} + E_{\mathrm{Coul}} \tag{1}$$

where

$$E_{\mathrm{BCS}} = \sum_\nu 2v_\nu^2\,\varepsilon_\nu - \Delta^2/G$$

$$v_\nu^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_\nu-\lambda}{\sqrt{(\varepsilon_2-\lambda)^2+\Delta^2}}\right) \tag{2}$$

$$E_{\mathrm{Coul}} = 0.6\, Ze^2 \cdot g(\varepsilon)/R$$

$$g(\varepsilon) = \left(1-\frac{2}{3}\varepsilon\right)^{1/3}\left(1+\frac{1}{3}\varepsilon\right)^{1/3}\begin{cases} \ln\left(\dfrac{1-\frac{2}{3}\varepsilon}{1+\frac{1}{3}\varepsilon-\sqrt{2\varepsilon-\frac{1}{3}\varepsilon^2}}\right)\dfrac{1}{\sqrt{2\varepsilon-\frac{1}{3}\varepsilon^2}} & \varepsilon>0 \\ \operatorname{arctg}\left(\dfrac{\sqrt{\frac{1}{3}\varepsilon^2-2\varepsilon}}{1+\frac{1}{3}\varepsilon}\right)\dfrac{1}{\sqrt{\frac{1}{3}\varepsilon^2-2\varepsilon}} & \varepsilon<0 \end{cases} \tag{3}$$

Numerical calculations for the nucleus with given Z and N consisted in finding sets of Nilsson levels ε_ν, for protons and neutrons separately, in several deformation points ε. In the next step the BCS equations (Bardeen, Cooper and Schrieffer 1957) were solved so as to find Δ and λ:

$$\frac{2}{G} = \sum_\nu \frac{1}{\sqrt{(\varepsilon_\nu-\lambda)^2+\Delta^2}} \tag{4}$$

$$n = \sum_\nu 2v_\nu^2 \equiv \sum_\nu \left(1 - \frac{\varepsilon_\nu-\lambda}{\sqrt{(\varepsilon_\nu-\lambda)^2+\Delta^2}}\right)$$

where n is an even number of neutrons (N) or protons (Z) in the actual nucleus. The number of terms included in summation (4) was taken to be equal to the number of particles in question (*i. e.* N or Z) and the pairing forces strength was $G_p = 20.8$ MeV/A and $G_n = 15.6$ MeV/A for protons and neutrons respectively (*cf.* Nilsson 1967).

In the case of odd number of particles, the blocking method was used in calculation of Δ, λ and E_{BCS}, *i. e.* the level occupied by the last particle ν_0 was excluded from the sums in Eqs (2), (4):

$$E'_{\mathrm{BCS}} = \sum_{\nu\neq\nu_0} 2v_\nu^2\varepsilon_\nu - \frac{\Delta^2}{G} + \varepsilon_{\nu_0} \tag{5}$$

with

$$\frac{2}{G} = \sum_{\nu\neq\nu_0} \frac{1}{\sqrt{(\varepsilon_\nu-\lambda)^2+\Delta^2}}, \qquad n' = \sum_{\nu\neq\nu_0} 2v_\nu^2 + 1 \tag{6}$$

where n' is the odd number of nucleons.

For each nucleus the total energy E was calculated as a function of deformation. In the vicinity of the minimum of $E(\varepsilon)$ the deformation parameter ε was changed with the step 0.005.

Equilibrium deformations of the ground and first excited states are given in Table I. For each nucleus the Nilsson quantum numbers of the ground and first excited states

TABLE I

Nucleus	Ground state	I excited state	ε_0	ε_{ex}	$\Delta\varepsilon$
Ho^{163}_{67}	7/2[523]	1/2[411]	0.260	0.261	0.001
Ho^{165}_{67}	7/2[523]	3/2[411]	0.261	0.267	0.006
Tm^{169}_{69}	1/2[411]	7/2[404]	0.262	0.260	−0.002
Tm^{171}_{69}	1/2[411]	7/2[523]	0.260	0.257	−0.003
Lu^{175}_{71}	7/2[404]	5/2[402]	0.242	0.236	−0.006
Lu^{177}_{71}	7/2[404]	9/2[514]	0.235	0.230	−0.005
Ta^{179}_{73}	7/2[404]	9/2[514]	0.223	0.218	−0.005
Ta^{181}_{73}	7/2[404]	9/2[514]	0.210	0.208	−0.002
Er^{165}_{68}	5/2[523]	1/2[521]	0.263	0.265	0.002
Er^{167}_{68}	7/2[633]	1/2[521]	0.266	0.267	0.001
Yb^{171}_{70}	1/2[521]	7/2[633]	0.258	0.257	−0.001
Yb^{173}_{70}	5/2[512]	7/2[633]	0.256	0.249	−0.007
Hf^{177}_{72}	7/2[514]	9/2[624]	0.235	0.233	−0.002
Hf^{179}_{72}	9/2[624]	7/2[514]	0.222	0.224	0.002
W^{181}_{74}	9/2[624]	5/2[512]	0.211	0.207	−0.004
W^{183}_{74}	1/2[510]	3/2[512]	0.195	0.195	0.000

(Mottelson and Nilsson 1959) are given together with their respective deformations (ε_0 and ε_{ex}). The last column gives the change in deformation $\Delta\varepsilon$ when going from the ground to the excited state: $\Delta\varepsilon = \varepsilon_{x} - \varepsilon_0$. For investigated nuclei one gets a relative change in deformation of the order 0.4 0.9% (for $|\Delta\varepsilon| \leqslant 0.03$) and 1.2 — 3% (for $|\Delta\varepsilon| > 0.003$).

The energy $E(\varepsilon)$ in the vicinity of equilibrium deformation of the ground and excited states of Lu^{175}_{71} is given in Fig. 1.

The same problem of different equilibrium deformations in the ground and excited states was inspected by Soloviev in 1966. He used the old version of the Nilsson model and performed a similar type of calculation for some special excited states in Tm^{169}, Lu^{175}, Ta^{181}, Er^{167}, Yb^{173} Hf^{175} and Hf^{177}. The states he used were characterised by a very strong dependence of the energy on deformation. Soloviev gave rules for the changes in deformatoni as depending on the character of the excited one-quasiparticle state:

$\varepsilon_{ex} < \varepsilon_0$ if the quasiparticle is either in a hole level of the average field, the energy of which decreases rapidly with increasing ε or in a particle level with energy strongly increasing with ε.

$\varepsilon_{ex} > \varepsilon_0$ if the quasiparticle is either in a hole level the energy of which increases strongly with ε or in a particle level the energy of which decreases rapidly with deformation.

This rule was verified by Soloviev's numerical results for specific nuclei and their various states.

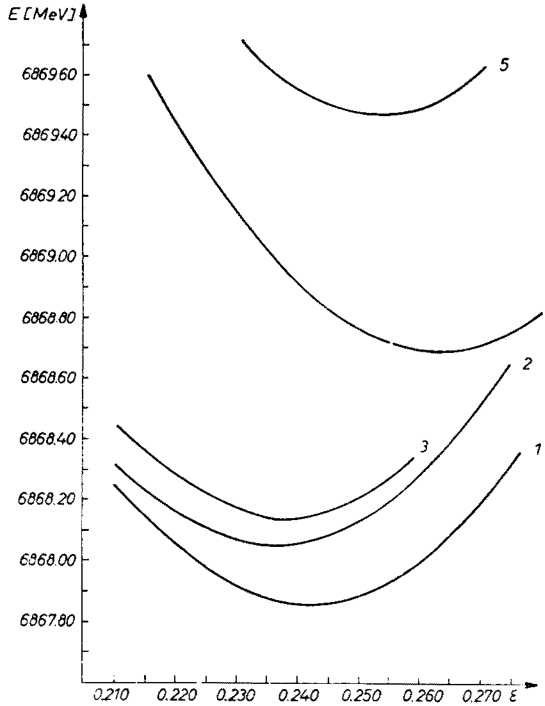


Fig. 1. Total energy E of Lu^{175}_{71} nucleus *versus* deformation ε as calculated for different states. Curve labelled *1)* gives $E(\varepsilon)$ for the ground state, *2)* — I excited state, *3)* — II excited state, *4)* — "Soloviev's" state 1/2[541] and *5)* — 3/2[651] state

In order to compare our present results with these of Soloviev, equilibrium deformations for the higher excited states in Tm^{169}, Yb^{173}, Lu^{175} and Ta^{181} were calculated. The "Soloviev's" states were taken into account as well as the second and sometimes third excited states for each of these nuclei and the numerical results are presented in Table II.

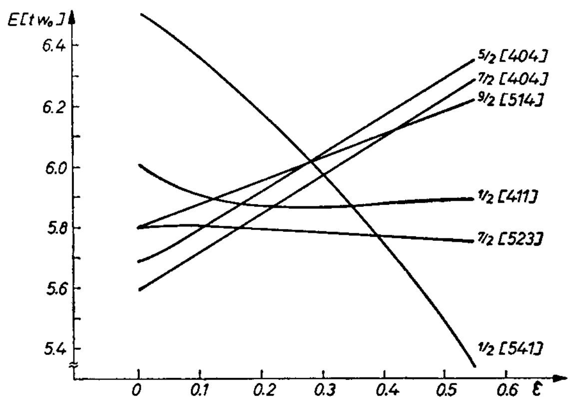
If we compare the deformation dependence of various states used in our calculation (Fig. 2) we can see that the results of this paper confirm predictions of the Soloviev rule. Moreover, numerical results when repeated in the cases chosen by Soloviev, are in good quantitative agreement.

Equilibrium deformations of the excited states, if calculated for the states with energy rapidly changing with deformation, can be changed even by 10% as compared with the ground states deformations.

This type of calculation can be applied for example in dealing with the analogue states. One expects that the energy shift ΔE between a given (ground) state in the nucleus (N, Z) and its analogue state in the nucleus $(N-1, Z+1)$ depends on the Coulomb energy difference. On the other hand one can look for analogue states of the excited states in the same nucleus (N, Z). Once more the energy shift $\Delta E'$ is affected by the Coulomb displacement energy but it is possible to get $\Delta E' \neq \Delta E$ just because of the mentioned before dependence of Coulomb energy on deformation (Eqs (3)). If ε_1 is the ground state deformation

TABLE II

Nucleus	II excited state		III excited state		Other states	
	quantum numbers	$\Delta\varepsilon$	quantum numbers	$\Delta\varepsilon$	quantum numbers	$\Delta\varepsilon$
Er^{167}_{68}	5/2[512]	−0.006	—	—	7/2[503]	−0.021
Yb^{173}_{70}	—	—	—	—	9/2[505]	−0.02 2
					1/2[651]	0.019
Lu^{175}_{71}	9/2[514]	−0.007	1/2[411]	−0.004	1/2[541]	0.021
					3/2[651]	0.020
Ta^{181}_{73}	5/2[402]	−0.005	1/2[411]	−0.008	1/2[541]	0.022
Tm^{169}_{69}	7/2[523]	−0.004	—	—	1/2[541]	0.018
					1/2[660]	0.015
Hf^{177}_{72}	7/2[633]	−0.010	7/2[503]	−0.015	—	—



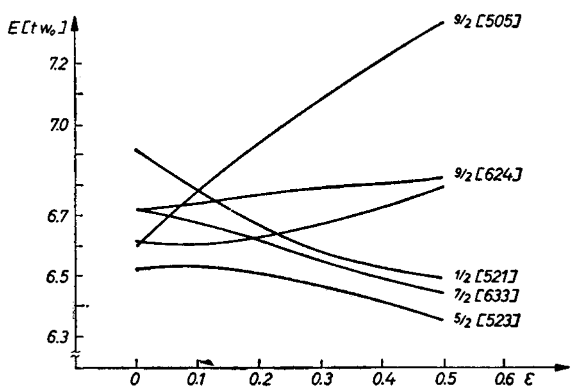


Fig. 2. Deformation dependence of single particle Nilsson levels in case of the few states used in calculation. a — proton states, b — neutron states

of the investigated nucleus and $\Delta\varepsilon_1$ — the change in deformation when going to the excited state, ε_2 and $\Delta\varepsilon_2$ — the deformation and its change of corresponding analogue states, we get for the Coulomb part of ΔE:

$$\Delta E_C = E_{\text{Coul}}(Z, \varepsilon_1) - E_{\text{Coul}}(Z-1, \varepsilon_2) \qquad (7)$$
$$\Delta E'_C = E_{\text{Coul}}(Z, \varepsilon_1 + \Delta\varepsilon_1) - E_{\text{Coul}}(Z-1, \varepsilon_2 + \Delta\varepsilon_2).$$

The Coulomb energy changes $\Delta E_{\text{Coul}} = E_{\text{Coul}}(Z, \varepsilon + \Delta\varepsilon) - E_{\text{Coul}}(Z, \varepsilon)$ for different values of ε and $\Delta\varepsilon$ are given in Table III as average values for different Z in the investigated region.

TABLE III

$\Delta\varepsilon$	$-\Delta E_{\text{Coul}}$ (MeV)					
	$\varepsilon = 0.18$	$\varepsilon = 0.20$	$\varepsilon = 0.22$	$\varepsilon = 0.24$	$\varepsilon = 0.26$	$\varepsilon = 0.28$
0.005	0.118	0.126	0.144	0.160	0.177	0.191
0.010	0.235	0.263	0.293	0.323	0.355	0.386
0.015	0.357	0.401	0.445	0.491	0.540	0.586
0.020	0.484	0.544	0.605	0.662	0.725	0.790
0.025	0.614	0.688	0.761	0.836	0.917	—
0.030	0.747	0.833	0.925	1.017	1.032	—
0.035	0.885	0.987	1.092	1.200	1.311	—
0.040	1.026	1.143	1.266	1.399	1.515	—

The expansion of $\Delta E'_C$ in power series of $\Delta\varepsilon$ gives an approximate formula:

$$\Delta E'_C = \Delta E_C + \Delta\varepsilon_1 \frac{\partial E_{\text{Coul}}(Z, \varepsilon_1)}{\partial\varepsilon} - \Delta\varepsilon_2 \frac{\partial E_{\text{Coul}}(Z-1, \varepsilon_2)}{\partial\varepsilon}. \qquad (8)$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon$ we get simply $\Delta E'_C = \Delta E_C + f(Z, \varepsilon) \cdot (\Delta\varepsilon_1 - \Delta\varepsilon_2)$ and $\Delta E'_C \neq \Delta E_C$ if only $\Delta\varepsilon_1 \neq \Delta\varepsilon_2$.

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