SEPARATION METHOD AND Λ-PARTICLE BINDING IN NUCLEAR MATTER

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(Received March 7, 1970; Revised paper received September 24, 1970)

The binding energy of a Λ particle in nuclear matter, $B_\Lambda(\infty)$, is calculated with the help of the modified Moszkowski-Scott separation method. This procedure permits one to systematically expand the effective Λ-nucleon interaction in terms of G_s, the reaction operator for free particles caused by the short range part of the potential alone. The form of the series is discussed. The contribution of all first and second-order terms to the binding energy has been calculated numerically with two central ΛN potentials. The rearrangement energy is taken into account. The calculated $B_\Lambda(\infty)$ are compared with the empirical value of $B_\Lambda(\infty)$. Possible ways of reducing the calculated value of $B_\Lambda(\infty)$ are discussed.

1. *Introduction*

Calculations of the binding energy of a Λ particle in nuclear matter, B_Λ, are of great importance and interest for the following reasons. A confrontation of the calculated binding energy with the phenomenological well depth may give information about the Λ-nucleon interactions, particularly about the interaction in higher angular momentum states. Further, because of the simplifications occuring for an infinite system, one may hope to do accurate theoretical calculations of B_Λ and to test indirectly the validity of various approximations also for finite hypernuclei. Finally it can be instructive to consider the similarities and differences with the case of pure nuclear matter.

The methods available for the calculation of B_Λ are in essence the same as for nuclear matter, for potentials with a strong and short-range repulsion: (I) The Brueckner K-matrix approach and (II) variational methods which use correlated, Jastrow-type wave functions. The K-matrix approach was first used for the calculation of B_Λ in the form of the so called "independent pair approximation" by many authors [1-4]. More recent and powerful K-matrix techniques for ΛN potentials are used by Dąbrowski and Hassan [5] — they use the Brueckner-Gammel integral equation — and by Bodmer and Rote [6], who use an integro-differential equation. The variational approach was first used by Downs and Grypeos

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[7] and has recently been the subject of a thorough and careful investigation by Mueller and Clark [8].

In the present paper B_Λ is calculated using the K-matrix technique and the separation method introduced by Moszkowski and Scott [9]. According to the present state of the Brueckner theory [5], B_Λ can be expressed by

$$-B_\Lambda = V_\Lambda(k_\Lambda) = \sum_{k_N} (\boldsymbol{k}_N \boldsymbol{k}_\Lambda | G | \boldsymbol{k}_N \boldsymbol{k}_\Lambda), \tag{1.1}$$

where $\boldsymbol{k}_i = \vec{k}_i s_i t_i$ ($\vec{k}_i$ = momentum below the Fermi sea, s_i = spin and t_i = izospin). The G matrix which represents the effective ΛN interaction is defined by the equation

$$G = v + v \frac{Q}{e} G, \tag{1.2}$$

where v is the Λ-nucleon potential,

$$e = e_N(k_N) + e_\Lambda(k_\Lambda) - \varepsilon(k'_N) - \varepsilon(k'_\Lambda) \tag{1.3}$$

and the Pauli principle operator

$$Q = \sum_{k'_N k'_\Lambda > k_F} |\vec{k}'_N \vec{k}'_\Lambda)\,(\vec{k}'_N \vec{k}'_\Lambda|. \tag{1.4}$$

By ε_N, ε_Λ we denote the kinetic energies and by e_N, e_Λ — single particle energies of the occupied states

$$\begin{aligned} e_N(k_N) &= \varepsilon_N(k_N) + V_N(k_N). \\ e_\Lambda(k_\Lambda) &= V_\Lambda(k_\Lambda) \end{aligned} \tag{1.5}$$

because in the ground state the Λ particle occupies the state with zero momentum.

With the help of the separation method we can develop the reaction matrix for the Λ-nucleon interaction in the series with finite elements. We divide the ΛN potential into two parts $v = v_s + v_l$ where the separation distance, d, is chosen in such a way that v_s produces zero scattering for two free particles, *i. e.* the attraction in v_s just counter-balances the repulsion in v_s. This means the equality at $r = d$ of the logarithmic derivatives of the functions: $u_l^0(r) = r\Phi_l(r)$ — where $\Phi_l(r)$ is the radial wave function of two free noninteracting particles with angular momentum l — and $u_l(r) = r\Psi_l(r)$ — where $\Psi_l(r)$ is the corresponding wave function to two free (*i. e.* isolated) particles, Λ and nucleon, interacting *via* the potential $v_{\Lambda N}$. Thus we have

$$\frac{1}{u_l^0(r)} \frac{du_l^0}{dr}\bigg|_{r=d} = \frac{1}{u_l(r)} \frac{du_l(r)}{dr}\bigg|_{r=d}, \tag{1.6}$$

where r is the relative distance $r = |\vec{r}_\Lambda - \vec{r}_N|$. In the expansion for the G matrix we shall use the method proposed by Köhler [10] which is a modified Moszkowski-Scott method. We get the series

$$\begin{aligned} G = G_s^F + v_l + v_l \frac{Q}{e} v_l + (\Omega_s^F - 1)(e_0 - e)(\Omega_s^F - 1) + \\ + (\Omega_s^F - 1) e (Q - 1)\,(\Omega_s^F - 1) + 2 v_l Q (\Omega_s^F - 1), \end{aligned} \tag{1.7}$$

where G_s^F, Ω_s^F are the reaction matrix and the wave operator which correspond to v_s and to e_0 — free, noninteracting particles propagator. The validity of the expansion (1.7) for ΛN potential is discussed in [11].

2. *Detailed calculations*

Let us introduce the relative momenta

$$\vec{k} = \frac{\mu}{M_N}\vec{k}_N - \frac{\mu}{M_\Lambda}\vec{k}_\Lambda = \frac{\mu}{M_N}\vec{k}_N$$

and

$$\vec{k}' = \frac{\mu}{M_N}\vec{k}'_N - \frac{\mu}{M_\Lambda}\vec{k}'_\Lambda$$

where μ is the reduced mass of the ΛN system. The center-of-mass momentum is:

$$\vec{P} = \vec{k}'_N + \vec{k}'_\Lambda = \vec{k}_N.$$

Let us introduce also the magnitude k_{av}, the average value of the relative momentum averaging over $\vec{k}_N$

$$k_{\text{av}} = \frac{\int d\vec{k}_N k^2}{\int d\vec{k}_N} \approx 0.5\ \text{fm}^{-1} \quad \text{for} \quad k_F = 1.366\ \text{fm}^{-1}.$$

The k_{av} is useful in the numerical calculations.

2.1. Calculation of d and $\Psi_s^F(r)$

The G matrix expansion imposes conditions for the form of the wave function. We have to replace the ΛN wave function in nuclear matter — Ψ^N, by the wave function describing Λ—N interaction but with no effect of any other nucleons — $\Psi_s^F(\vec{r})$, and by the free wave function $\Phi(\vec{r})$. The condition (1.6) for finding the separation distance is simultaneously a condition to identify the wave functions $\Phi(\vec{r})$ and $\Psi_s^F(\vec{r})$. The separation distance must be calculated for particular values of the angular momentum. To this end we introduce the partial waves expansion of the $\Psi_s^F(\vec{r})$ and we solve the radial part of the Schrödinger equation:

$$\frac{d^2u_l}{dr^2} + \left\{\frac{2\mu}{\hbar^2}[E - v(r)] - \frac{l(l+1)}{r^2}\right\} u_l = 0 \tag{2.1}$$

with given potential $v(r)$ and compare step by step $u_l(r)$, $u_l^0(r)$ and their derivatives, until we satisfy the condition (1.6). Because d is a function of k, it was necessary to find the solution of Eq. (2.1) for all relative momenta in the following region $0 \leqslant k < 1\ \text{fm}^{-1}$. The $d(k)$ dependence is rather essential for angular momentum $l = 0$ and less for $l = 1, 2$. In this last case we assume $d(k_{\text{av}})$ in the numerical calculations.

Simultaneously, with d we can find the shape of the vave function. This can be done by solving numerically Eq. (2.1) with the v_s potential. Since for the calculation of d, we need

$u_l(r)$ with an accuracy to a constant, renormalization of our results is necessary. Normalization constant can be found comparing the value const $\times u_l(d)$ with $u_l^0(d)$. For a check, the function $\Psi_s^F(r)$ was also calculated by the other method. Eq. (2.1) was solved step by step from $r = d$ to $r = r_c$ with the initial condition (1.6).

2.2. The $Q(\vec{P}, \vec{k})$ operator

The $Q(\vec{P}, \vec{k})$ operator which excludes intermediate states from the Fermi sea, is different from the comparable purely nuclear case. This is connected with the fact that in the Λ-nucleon interaction only one particle is subject to the Pauli principle. This condition can be written in the form

$$|\vec{k}'_N| > k_F \quad \text{or} \quad \left|\vec{k}' + \frac{M_N}{M_\Lambda}\vec{k}\right| > k_F \tag{2.2}$$

Because of the dependence of Q on the angle θ between $\vec{k}$ and $\vec{k}'$, one approximates Q by its angle average $Q(k', k)$, which depends only on the magnitudes $\vec{k}$ and $\vec{k}'$. The detailed discussion of the validity of this approximation for N–N interaction, was gives by Brown, Schappert and Wong (12). We get

$$\bar{Q}(P, k') = \bar{Q}(k, k') = \frac{1}{4\pi}\int d\hat{p} Q(\vec{k}, \vec{k}')$$

Here $\hat{P}$ is a unit vector in the direction of $\vec{P}$ and $\int d\hat{P}$ means integration over angles. With the help of (2.2) we can get the condition for the angle between $\vec{k}$ and $\vec{k}'$ vectors

$$\cos(\hat{kk}') = \cos\theta > \frac{k_F^2 - k'^2 - \left(\frac{M_N}{M_\Lambda}\right)^2 k^2}{2\frac{M_N}{M_\Lambda}kk'}$$

and integrating with this condition we get finally

$$\bar{Q}(k, k') = \begin{cases} 0 & \text{for } k < k_F - \frac{\mu}{M_N}k_N \\ 1 & \text{for } k > k_F + \frac{\mu}{M_N}k_N \\ \dfrac{\left(k' + \frac{M_N}{M_\Lambda}k\right)^2 - k_F^2}{4\frac{M_N}{M_\Lambda}kk'} & \text{otherwise} \end{cases} \tag{2.3}$$

3. Single particle energies

Generally, the energy denominator $e(k, k')$ is given by (1.3). The problem is to find the form of $V_\Lambda(k_\Lambda)$ and $V_N(k_N)$ and its dependence on momenta. $V_\Lambda(k_\Lambda)$ and $V_N(k_N)$ can be defined in terms of the reaction matrix and one can calculate $V_\Lambda(k_\Lambda)$ and $V_N(k_N)$ by iteration

only. In practice one iterates until self-constancy is achieved, modifing the single particle energies with each cycle until the output energies are the same as the input ones.

For the single-nucleon potential we assume, following Weisskopf [13], that the theory and the bare N–N interaction are both correct in the sense that they should give correct results, and then we can use the observed binding energy per nucleon in nuclear matter to obtain $V_N(k_N)$. Therefore we can write

$$-B_N(\infty) = \bar{\varepsilon}_N + \tfrac{1}{2}\,\overline{V}_N$$

Inserting $B_N(\infty) = 15.5$ MeV, $\bar{\varepsilon}_N = \frac{3}{10}\hbar^2 k_F^2\, M_N^{-1} = 23.2$ MeV we have

$$\overline{V}_N = -77.4 \text{ MeV}.$$

This average value is taken as the single particle potential below the Fermi sea $V_N(k_N)$.

For the single Λ-particle potential we assume, on the ground of the experimental data, the initial value $V_\Lambda(k_\Lambda) = -28$ MeV. Finally we get the starting value $e(k, k') = e_0(k, k') - 105.4$ MeV. Of course we must perform the self-consistency procedure. The $B_\Lambda(\infty)$ is to some extent dependent on the initial $V_\Lambda(k_\Lambda)$ and calculations soon lead to self-consistency. Notice that we use pure kinetic energies in the intermediate states (both the nucleon and Λ particle). This is justified from the point of view of the present state of the theory of nuclear matter [14]. The particle potential energy $V_N(k'_N)$ is very small, whereas the "hole" potential energy is approximately equal to the averege nucleon potential energy in nuclear matter (this was the starting point of our approximation). Brandow [16] calculated the average value of $V_N(k'_N)$ and received the value of 1 MeV which contributes only about 0.1 MeV to the total binding energy. We assume that $V_\Lambda(k'_\Lambda)$ gives similar contribution.

4. *The G matrix elements*

Eq. (1.1) gives the formula for the binding energy of the Λ-particle in nuclear matter (without the rearrangement energy). The G matrix expansion is given by (1.7). Let us proceed to the calculation of the particular terms of this series. Inserting (1.7) into (1.1) we get

$$V_\Lambda = \frac{1}{(2\pi)^3}\int_0^{k_F} d\vec{k}_N \sum_{s,m_s} (\vec{k}_N, s, m_s | G_s^F + v_l + v_l \frac{Q}{e}\, v_l +$$

$$+ (\Omega_s^F - 1)\,(e_0 - e)\,(\Omega_s^F - 1) + (\Omega_s^F - 1)\, e\,(Q-1)\,(\Omega_s^F - 1) +$$

$$+ 2 v_l Q (\Omega_s^F - 1) | \vec{k}_N, s, m_s). \tag{2.4}$$

We consider only the central Λ-nucleon potential, with the same spin dependence for all angular momentum states

$$v_{\Lambda N}(r) = v^s(r) \Lambda^s + v^t \Lambda^t$$

where Λ^s and Λ^t are projection operators on the singlet and the triplet state, respectively.

4.1. The long range part V_A^L

The first term of the G matrix expansion gives no contribution to V_A, because the condition of our choice of the seperation distance is simultaneously the condition that the diagonal elements $(G_s^F)_{kk}$ are equal zero.

For calculating of $V_A^L = (v_l)_{kk}$ we expand the plan wave into the partial waves and after summation over spins and angular integration we get finally

$$V_A^L = \frac{2}{\pi}\left(\frac{M_N}{\mu}\right)^3 \sum_l (2l+1) \int_0^{\frac{\mu}{M_N}k_F} k^2 dk \int_d^\infty j_l^2(kr)(v^s+3v^t)r^2 dr. \tag{2.5}$$

4.2. Born approximation for v_l, V_A^B

This correction is connected with the scattering of particles to the intermediate momenta states above the Fermi sea. Inserting summation over the intermediate states we get

$$V_A^B = \frac{1}{(2\pi)^6}\left(\frac{M_N}{\mu}\right)^3 \int_0^{\frac{\mu}{M_N}k_F} d\vec{k} \int_0^{} d\vec{k'}\, \frac{Q(k,k')}{e(k,k')}\, |v_{kk'}|^2, \tag{2.6}$$

where

$$v_{kk'} = \int_d^\infty \sum_{\substack{ll' \\ mm'}} 16\pi^2(-i)^{l+l'} Y_{lm}(\hat{k}')\, Y_{lm}^*(\hat{r}')\, Y_{l'm'}^*(\hat{k})\, Y_{l'm'}(\hat{r}) \times$$
$$\times\, j_l(k'r) j_{l'}(kr) v(r) r^2 dr d\Omega_r.$$

Because after summation over the spin states we get the following expression

$$\sum_{sm_s} (\vec{k}sm_s|v^sA^s+v^tA^t|\vec{k}'s'm_s')\,(\vec{k}'s'm_s'|v^sA^s+v^tA^t|\vec{k}sm_s)$$
$$= |(\vec{k}|v^s|\vec{k}')|^2+3|(\vec{k}|v^t|\vec{k}')|^2, \tag{2.7}$$

in order to simplify the formulae we shall use in all the following expressions $v(r)$ which equals either $v^s(r)$ or $v^t(r)$. We must multiply this last term by 3.

With the help of the orthogonality condition for Y_{lm} we can perform the angular integration and we have

$$V_A^B = \sum_l (2l+1)\,\frac{4}{\pi^2}\left(\frac{M_N}{\mu}\right)^3 \int_0^{\frac{\mu}{M_N}k_F} k^2 dk \int_0^\infty dk' k'^2\, \frac{Q(k,k')}{e(k,k')} \times$$
$$\times \left| \int_d^\infty dr\, r^2 j_l(k'r) j_l(kr) v(r) \right|^2. \tag{2.8}$$

4.3. The Pauli correction V_A^P

From the definition of the wave operators and from the definition of the operation of the G_s^F matrix on the $\Phi(\vec{r})$ function we have

$$(\Omega_s^F - 1)|\Phi(\vec{r})) = \frac{1}{e_0} G_s^F |\Phi(\vec{r})) = \Psi_s^F(\vec{r}) - \Phi(\vec{r}).$$

Using this formula we can eliminate in (2.4) the unknown operator Ω_s^F, and V_Λ^P becomes

$$V_\Lambda^P = \sum_{kk'} (\vec{k}|(\Omega_s^F - 1)|\vec{k}')e(k, k')\ (Q(k, k') - 1)\ (\vec{k}'|(\Omega_s^F - 1)|\vec{k})$$

$$= \frac{1}{(2\pi)^6} \left(\frac{M_N}{\mu}\right)^3 \int_0^{\frac{\mu}{M_N} k_F} d\vec{k} \int_0^\infty d\vec{k}'\ e(k, k')\ (Q(kk') - 1) \times$$

$$\times \Big| \int_0^\infty \Phi_{\vec{k}'}^*(\vec{r})\ [\Psi_{s\vec{k}}^F(\vec{r}) - \Phi_{\vec{k}}(\vec{r})] d\vec{r} \Big|^2. \tag{2.9}$$

We expand $\Psi_s^F(r)$ and perform the angular integration. The function $u(r)/r$ is calculated numerically and also given in analytic form, convenient for integrating.

4.4. The dispersion term, V_Λ^D

The important correction term is the dispersion factor due to the presence of other nucleons:

$$V_\Lambda^D = \sum_{kk'} (\vec{k}|\Omega_s^F - 1)|\vec{k}')\ [e_0(k, k') - e(k, k')]\ (\vec{k}'|(\Omega_s^F - 1)|\vec{k}). \tag{2.10}$$

Remembering that $e_0(k, k') - e(k, k') = -[V_N(k_N) + V_\Lambda(k_\Lambda)]$ and calculating analogously as for V_Λ^P we get

$$V_\Lambda^D = -[V_N(k_N) + V_\Lambda(k_\Lambda)]\ \frac{1}{(2\pi)^6} \left(\frac{M_N}{\mu}\right)^3 \int_0^{\frac{\mu}{M_N} k_F} d\vec{k} \int_0^\infty d\vec{k}' \times \tag{2.11}$$

$$\times \Big| \int_0^\infty \Phi_{\vec{k}'}^*(\vec{r})(\Psi_{s\vec{k}}^F(\vec{r}) - \Phi_{\vec{k}}(\vec{r})] d\vec{r} \Big|^2.$$

4.5. The interference term, V_Λ^I

The last term of our expansion is calculated similarly as V^D.

$$V_\Lambda^I = \frac{2}{(2\pi)^6} \left(\frac{M_N}{\mu}\right)^3 \int_0^{\frac{\mu}{M_N} k_F} d\vec{k} \int_0^\infty d\vec{k}' Q(k, k') \times$$

$$\times \Big[\int_0^\infty \Phi_{\vec{k}}^*(\vec{r}') v(r') \Phi_{\vec{k}'}(\vec{r}') d\vec{r}' \int_0^\infty \Phi_{\vec{k}'}^*(\vec{r})(\Psi_{s\vec{k}}^F(\vec{r}) - \Phi_{\vec{k}}(\vec{r})) d\vec{r} \Big]. \tag{2.12}$$

In order to check our integral methods and the approximation used for the wave function, V_Λ^I was calculated also in an other way. Consider

$$V_\Lambda^{I'} = 2 \sum_{kk'} (\vec{k}|v_l(r)|\vec{k}')\ (Q(kk') - 1)\ (\vec{k}'|(\Omega_s^F - 1)|\vec{k}).$$

On the other hand

$$V_\Lambda^{I'} = V_\Lambda^I + \sum_{\vec{k}} (\vec{k}|v_l|\Psi_{s\vec{k}}^F - \Phi_{\vec{k}}).$$

This term is equal to zero because $v_l = 0$ for $r \leqslant d$, and $\Psi_s^F = \Phi$ for $r > d$. Hence we get $V_\Lambda^I = V_\Lambda^{I'}$. Numerical results for both methods of calculating the V_Λ^I are in very good agreement. This confirms the method of integrating and the wave function approximation.

5. *Numerical calculations*

All the numerical computations were performed on the GIER computer of the University of Warsaw. Two kinds of calculation were performed:

1) Calculation of the $d(k)$ and $\Psi_s^F(r)$.
2) The numerical calculation of the diagonal matrix elements of the G matrix for the angular momentum $l = 0, 1, 2$.

Calculations were performed with the two various central spin-dependent, Λ-nucleon potentials. The first one is the HTS potential fitted by Herndon, Tang, and Schmid [16] to the binding energy of the s shell hypernuclei. Its intrinsic range is equal to that of a purely attractive two-pion exchange Yukawa potential. The form of this potential is

$$v_{\Lambda N} = \begin{cases} \infty & \text{for } r \leqslant 0.4 \text{ fm} \\ -V_0 \exp(-5.059\,(r-0.4)) & \text{for } r > 0.4 \text{ fm} \end{cases}$$

where $V_0 = 1221.1$ (the singlet state) and 954.1 MeV (the triplet state).

Potential AGK has been fitted by Ali, Grypeos and Kok [17] to Λ-p scattering under the assumption of a common intrinsic range, for both the singlet and triplet interactions. No fit to hyperfragments energies has been attempted. The form of this potential is

$$v_{\Lambda N} = \begin{cases} \infty & \text{for } r \leqslant 0.4 \text{ fm} \\ -V_0 \exp(-2.12\,r)/(2.12\,r) & \text{for } r > 0.4 \text{ fm} \end{cases}$$

where $V_0 = 1118$ MeV (the singlet state) and 929.4 MeV (the triplet state).

The Schrödinger equation was solved by the Runge-Kutt method modified for our problem. Calculations of the matrix elements were performed using the wave functions for the average value of the relative momentum k_{av}. For checking the form of $u(r)$ the diagonal matrix elements G_s was calculated, which with the exact wave function should be equal to zero. We get for HTS potential:

$$(G_s)_{kk} = \begin{cases} -0.004 & \text{for the singlet state} \\ -0.005 & \text{for the triplet state} \end{cases}$$

This shows that $\Psi_s^F(r)$ has the proper form.

6. *Results and conclusion*

The results of our calculations are shown in Tables I and II. All results were obtained for the S, P, D waves. Results show that there is no need to go beyond the D waves. The contribution from the P wave is greater for the potential with greater intrinsic range and

TABLE I

Results for HTS potential (in MeV)

	V_Λ^L	V_Λ^B	V_Λ^P	V_Λ^D	V_Λ^I	V_Λ
$l=0$	−36.9	−2.0	0.4	9.6	−1.6	−30.5
$l=1$	−8.3	−0.9	0	0.2	0	−9.0
$l=2$	−0.3	0	0	0	0	−0.3
Total	−45.5	−2.9	0.4	9.8	−1.6	−39.8

TABLE II

Results for AGK potential (in MeV)

	V_Λ^L	V_Λ^B	V_Λ^P	V_Λ^D	V_Λ^I	V_Λ
$l=0$	−53.2	−1.1	0.6	13.9	−2.2	−42.0
$l=1$	−21.1	−0.9	0	0.3	0	−21.7
$l=2$	−2.3	−0.1	0	0	0	−2.4
Total	−76.6	−2.1	0.6	14.2	−2.2	−66.1

equal to one half of the S wave contribution. This dependence was observed also by other authors [3]. The greatest contribution is connected with the long range part V_Λ^L. The dispersion term is the important correction term.

We must take into account the rearrangement energy V_R [4]. This quantity is proportional to V_Λ and to the $N-N$ correlation volume in the nuclear matter. Our present knowledge of the $N-N$ interaction does not allow us to make a precise estimate for V_R. We take $V_R = 0.1\,V_\Lambda$ [5] and by introducing V_R we get $B_\Lambda(\infty) = 35.8$ MeV for HTS potential and $B_\Lambda(\infty) = 59.5$ MeV for AGK potential. Results of [5] agree with our results. The Moszkowski-Scott separation method was used for the Λ particle in the nuclear matter problem by Taharzadeh, Moszkowski and Sood [18]. However, the results obtained in Ref. [18] are outdated because of the assumed form of $v_{\Lambda N}$ and of the single particle energies. The separation of the Λ-nucleon potential into two parts was also discussed by Bodmer and Rote [6]. They calculated only V_Λ^L with the Herndon, and Tang potential [19]. The difference $V_\Lambda - V_\Lambda^L$ is, for potentials with hard core 0.45 fm, very similar to that for AGK potential of this paper.

The experimental value of $B_\Lambda(\infty)$ is to be understood as an extrapolation value measured from hypernuclei with a high but definite A number. At present, however, experiments in this field are still very difficult and the uncertainty of the experimental value of the Λ binding energy in heavy hyperfragments is still large. We assume the experimental value $B_\Lambda(\infty) = 32 \pm 3$ MeV [20] and agreement with the theoretical value is rather bad. One may notice the large difference between both theoretical values — nearly 80%. The value of $B_\Lambda(\infty) = 59.5$ MeV obtained with AGK potential seems to eliminate this potential as a realistic representation of the Λ-nucleon interaction in the nuclear matter. The value $B_\Lambda(\infty) = 35.8$ MeV (HTS potential) is reasonably close to the range of the empirically de-

termined value of $B_\Lambda(\infty)$. However, the HTS potential is fitted only to the binding energy of the s shell hypernuclei and this potential has been outdated by the new potentials of Herndon and Tang [19], determined from both sources of information about ΛN interaction. Calculations with the new potentials gives also rather wrong results [5, 6], only one potential gives the value of B which is in agreement with the empirical estimate. One conclusion to be drawn from this is that components other than the central component in the Λ-nucleon interaction may be important in $B_\Lambda(\infty)$, and one can reduce the theoretical value of $B_\Lambda(\infty)$ in the following way:

(I) One can reduce the $B_\Lambda(\infty)$ by weakening the p-state interaction. This possibility was first pointed out by Walecka [1]. Herndon and Tang [19] suggested 40% p-wave suppresion. This effect corrects our results and we get 33 MeV and 52.7 MeV, respectively.

(II) Inclusion of the tensor force. The tensor part, adding to the Λ-nucleon potential, gives the binding energy smaller by about 2–3 MeV [21].

(III) Effect of the three-body ΛNN forces. No doubt, there are theoretical reasons to expect the existence of an appreciable ΛNN interaction and several authors have investigated its possible effects in hypernuclei [21]. With the present possibilities of deriving theoretically $v_{\Lambda NN}$ and with the present possibilities of solving the hypernuclear problem, the task of determining $v_{\Lambda NN}$ seems to be extremely difficult.

(IV) Isospin suppresion effect, pointed out by Bodmer [23], would lead to the conclusion that the whole idea of treating hypernuclei as systems of nucleons and a Λ particle with the same $v_{\Lambda N}$ potential as in an isolated ΛN system is wrong.

All those effects can give essentially a suppression of the binding energy, but which one is important, remains to be understood.

The author wishes to thank Professor J. Dąbrowski for many helpful discussions throughout the course of this work.

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