SPIN STRUCTURE OF THE ABSORPTION CORRECTIONS TO THE STODOLSKY-SAKURAI MODEL

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The absorption correction to the Stodolsky-Sakurai model is decomposed into two parts. The first part differs from the pole term only by a spin independent factor. Consequently, it does not affect the spin density matrix. The corrections which change the density matrix are small.

1. *Introduction*

The Stodolsky-Sakurai model [1] gives very satisfactory predictions for the spin density matrix of isobars produced in the reactions

$$PB \to P'B^*. \tag{1}$$

Here, P, P' denote pseudoscalar mesons, B is a $\frac{1}{2}^+$ baryon and B^* is a $\frac{3}{2}^+$ isobar. The model, however, is a pole model, based on the assumption that the exchanged particle has dipole coupling in the $B\,B^*$ vertex. It is well known that the predictions of pole models are usually strongly modified by the contributions of cuts, *i.e.* by absorption corrections.

In this paper the effects of absorption on the predictions of the Stodolsky-Sakurai model are analysed. It is found that the dominant part of the absorption term has the same spin dependence as the original pole amplitude. This explains why the absorption corrections to the spin-density matrix are small, even if absorption is strong and has dramatic effects on the differential cross-section. Important corrections are expected only when the main term vanishes or nearly vanishes, *i.e.* in the vicinity of dips in the differential cross-section.

The general formulae are given in the next section. The pole term is assumed to have the spin dependence predicted by the Stodolsky-Sakurai model and absorption is assumed to be diagonal in the s-channel helicity. Since no further assumptions are necessary, the

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results apply to a wide variety of pole inputs and absorption models. Section 3 contains a discussion of the formulae. The argument is illustrated by numericalcalcul ations for the processes $\pi^+p \to \pi^0\Delta^{++}$ and $\pi^+p \to \eta\Delta^{++}$ at incident momentum 8 GeV/c. The calculations are done in the framework of the Gottfried and Jackson absorption model [2] using a pole model presented in a previous paper [3]. Our conclusions are summarized in Section 4.

2. *Formulae*

According to the Stodolsky-Sakurai model the amplitude for process (1) can be written in the form

$$T^{\text{POLE}} = f\Sigma_0, \tag{2}$$

where f is a scalar function of the energy and momentum transfer, while

$$\Sigma_0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3}$$

contains the spin dependence. The rows of Σ_0 are labelled by the spin projection of the isobar and the columns are labelled by the spin projections of the initial baryon. The spin states are defined in the Jackson transversity frames. For each of the particles it is a spin reference frame defined in the particle rest frame with the z-axis normal to the reaction plane. The y-axis for the isobar is antiparallel to the momentum of the incident baryon and the y-axis for the baryon is parallel to the momentum of the isobar. The x-axes are chosen so as to make the frames right-handed. Thus (2) means that the spin projection on the normal does not change in the scattering process. This would be just like for πN scattering, but (2) implies in addition that the amplitude does not depend on the spin projection.

Absorption is generally believed to conserve s-channel helicity. Therefore, in order to introduce absorption we transform (2) into the helicity frame. The result is

$$^{H}T^{\text{POLE}} = \frac{i}{\sqrt{2}} f \cos\frac{\chi}{2}\; (\Sigma_+ + \Sigma_-) - \frac{i}{2} f \sin\frac{\chi}{2} \left[\sqrt{\frac{3}{2}}\,(\tau_{++} + \tau_{--}) + \tau_0\right]. \tag{4}$$

Here,

$$\Sigma_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \Sigma_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \sqrt{3} \end{pmatrix},$$

$$\tau_0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}, \qquad \tau_{++} = -\sqrt{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \tau_{--} = \sqrt{2} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{5}$$

with the rows and columns labelled by the helicities of the particles. The angle χ is the sum of the angles χ_B and χ_{B^*} shown on a velocity diagram[1] in Fig. 1. Rotations of the initial and final Jackson transversity frames by $-\chi_B$ and χ_{B^*} around the normal to the reaction plane would convert them into ordinary transversity frames. The angle χ is the sum and not the difference of the angles χ_B and χ_{B^*}, because we deal with the states $|B\rangle$ and $\langle B^*|$.

Since absorption conserves s-channel helicity, the absorption correction to a helicity amplitude ${}^HT_{\lambda\lambda'}$ can be written in the form

$$ {}^HT^{\text{ABS}}_{\lambda\lambda'} = F_{\lambda\lambda'}({}^HT^{\text{POLE}}_{\lambda\lambda'}). \tag{6}$$

In order to illustrate our argument we will use the absorption model of Gottfried and Jackson [2], further called the usual absorption model. In this model absorption depends mainly on the amount of helicity flip

$$ \nu = |\lambda - \lambda'|. \tag{7}$$

The dependence on individual helicities enters only through so-called spherical corrections, which are small at reasonably high energies [5]. Therefore, in order to simplify our formulae we put

$$ {}^HT^{\text{ABS}}_{\lambda\lambda'} = F_\nu({}^HT^{\text{POLE}}_{\lambda\lambda'}). \tag{8}$$

We could handle the general case, but this does not seem worthwhile. In the usual absorption model neglecting the spherical corrections one finds, using the formulae given, *e.g.*, in [5], the explicit form of the functional F_ν

$$ F_\nu(g(z)) = \frac{k^2\sigma_T}{4\pi} \int_{-1}^{+1} dx\, e^{\alpha(xz-1)}\, g(x)\, I_\nu(\alpha \sqrt{1-x^2}\, \sqrt{1-z^2}\,). \tag{9}$$

Here, z is the cosine of the scattering angle, σ_T and k denote the total cross-section and the centre of mass momentum for the elastic processes $PB \to PB$ or $P'B^* \to P'B^*$. Elastic scattering in the initial and final states is here assumed to be similar; therefore, we insert the parameters for the first process. The parameter

$$ \alpha = Ak^2, \tag{10}$$

where A is the slope of $(d\sigma/dt)_{\text{elast}}$ and I_ν denotes the modified Bessel function of order ν.

Using the formulae (8) and (9) we find the absorption correction to the helicity amplitudes,

$$ {}^HT^{\text{ABS}} = \frac{i}{\sqrt{2}} F_1\left(f \cos\frac{\chi}{2}\right)(\Sigma_+ + \Sigma_-) - $$

$$ - \frac{i}{2}\left[\sqrt{\frac{3}{2}}\, F_2\left(f \sin\frac{\chi}{2}\right)(\tau_{++} + \tau_{--}) + F_0\left(f \sin\frac{\chi}{2}\right)\tau_0\right]. \tag{11}$$

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[1] The angles χ_B and χ_{B^*} are equal to the crossing angles for the baryon and the isobar, respectively, defined according to Ref. [4], with the convention that the baryon is the "2-nd particle" and the isobar is the "4-th particle".

The set of corrected helicity amplitudes then takes the form

$$ {}^{H}T = {}^{H}T^{\text{POLE}} - {}^{H}T^{\text{ABS}}. \tag{12} $$

Transforming back to the Jackson transversity frame we find

$$ T = (f-a)\Sigma_0 - b\tau_0 - c(e^{i\bar{\chi}}\tau_{--} + e^{-i\bar{\chi}}\tau_{++}], \tag{13} $$

where

$$ a = F_1\left(f\cos\frac{\chi}{2}\right)\cos\frac{\chi}{2} + \left[\frac{3}{4}F_2\left(f\sin\frac{\chi}{2}\right) + \frac{1}{4}F_0\left(f\sin\frac{\chi}{2}\right)\right]\sin\frac{\chi}{2}, \tag{14} $$

$$ b = -i\,F_1\left(f\cos\frac{\chi}{2}\right)\sin\frac{\chi}{2} + i\left[\frac{3}{4}F_2\left(f\sin\frac{\chi}{2}\right) + \frac{1}{4}F_0\left(f\sin\frac{\chi}{2}\right)\right]\cos\frac{\chi}{2}, \tag{15} $$

$$ c = \frac{i}{2}\sqrt{\frac{3}{8}}\left[F_0\left(f\sin\frac{\chi}{2}\right) - F_2\left(f\sin\frac{\chi}{2}\right)\right], \tag{16} $$

and

$$ \bar{\chi} = \frac{3}{2}\chi_{B^*} - \frac{1}{2}\chi_B, \tag{17} $$

with the angles defined in Fig. 1.

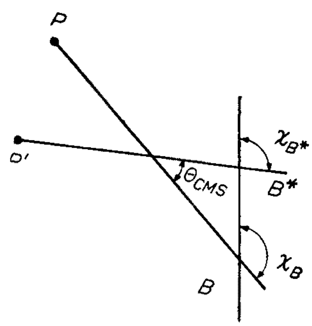

Fig. 1. Velocity diagram for process $PB \to P'B^*$, which defines the angles χ_B and χ_{B^*}

Experimentally, one measures the differential cross-section

$$ \frac{d\sigma}{dt} = 2|f-a|^2 + 2|b|^2 + 4|c|^2 \equiv N, \tag{18} $$

where N is defined by this formula, and the statistical tensor components, which in the Jackson transversity frame read

$$ T_0^2 = \frac{4}{N}|c|^2 - \frac{1}{2}, \tag{19} $$

$$ T_2^2 = \frac{2}{N}e^{i\bar{\chi}}\left[\text{Re}\,(bc^*) + i\,\text{Im}\,((f-a)c^*)\right]. \tag{20} $$

This statistical tensor is related to the more familiar spin-density matrix elements defined in the Jackson frame by the formulae

$$T_0^2 = \frac{1}{4} - (\varrho_{33} + \sqrt{3}\,\mathrm{Re}\,\varrho_{3-1}), \tag{21}$$

$$\mathrm{Re}\,T_2^2 = \frac{\sqrt{6}}{8} - \frac{1}{\sqrt{2}}(\sqrt{3}\,\varrho_{33} - \mathrm{Re}\,\varrho_{3-1}), \tag{22}$$

$$\mathrm{Im}\,T_2^2 = -\sqrt{2}\,\mathrm{Re}\,\varrho_{31}. \tag{23}$$

The predictions of the Stodolsky-Sakurai model are reproduced after putting $a = b = c = 0$. Then $T_0^2 = -\frac{1}{2}$ and $T_2^2 = 0$, or, equivalently, $\varrho_{33} = 3/8$, $\mathrm{Re}\,\varrho_{31} = 0$, $\varrho_{3-1} = \sqrt{3}/8$.

3. Discussion

In this section we discuss the implications of the general formulae given in the preceding section. In order to illustrate the argument, we also present the relevant numerical results for the reactions

$$\pi^+ p \to \pi^0 \Delta^{++}, \tag{24}$$

$$\pi^+ p \to \eta \Delta^{++}, \tag{25}$$

at incident momentum 8 GeV/c. The numerical calculations are performed using approximation (9) with $\sigma_T = 25$ mb, *i.e.*

$$\frac{k^2 \sigma_T}{4\pi} = 18, \tag{26}$$

and for an elastic slope $A = 8\,(\mathrm{GeV/c})^{-2}$.

The amplitudes found in Ref. [3] were used as the pole input. In order to obtain a good fit when absorption is included, one should readjust the parameters used in this amplitude. We have not done this, however, because the numerical results are used in the present paper only as a qualitative illustration.

Our basic remark is that the amplitude a is much larger than the amplitudes b and c (*cf.* Figs 2*a* and 2*b*). There are two reasons for this. Firstly, the angle $\chi/2$ is small (*cf.* Fig. 3). In the limiting case $\chi/2 = 0$ only the amplitude a survives. Secondly, the phase of the functional F_ν defined by formula (9) depends little on the value of the index ν (Figs 4*a* and 4*b*). Consequently, according to the formulae (15) and (16) there are important cancellations in the expressions for the amplitudes b and c.

We propose to discuss the implications of the absorption model for reaction (1), considering the corrections b and c as first order small quantities with respect to the correction a.

The zero order approximation is obtained by putting $b = c = 0$. In this approximation formula (13) yields

$$T = (f - a)\Sigma_0. \tag{27}$$

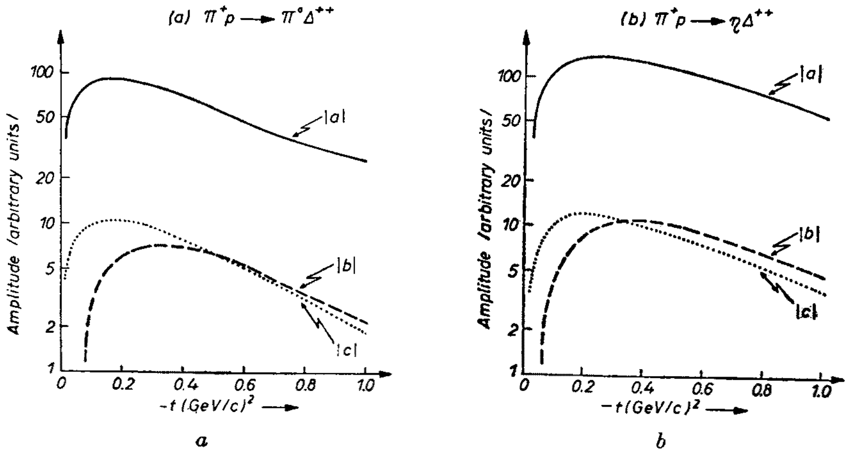

Fig. 2. Moduli of the amplitudes a (solid line), b (dashed line) and c (dotted line) for the reactions $\pi^+p \to \pi^0\Delta^{++}$ and $\pi^+p \to \eta\Delta^{++}$

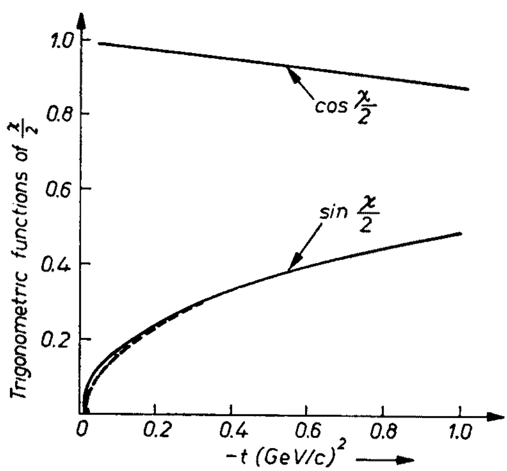

Fig. 3. Dependence of $\cos\frac{\chi}{2}$ and $\sin\frac{\chi}{2}$ on the momentum transfer. The solid curve refers to the reaction $\pi^+p \to \pi^0\Delta^{++}$ and the dashed line to the reaction $\pi^+p \to \eta\Delta^{++}$

This is identical with the pole term (2), except that the scalar amplitude f has been replaced by $f-a$. This changes the differential cross-section (*cf.* Figs 5a and 5b), but leaves the spin dependence, and consequently the spin density matrix, exactly as predicted by the Stodolsky-Sakurai model. We stress that at this stage almost all the absorption corrections are already included. What is left out are very small higher-order terms. Consequently, the complete absorption correction to the Stodolsky-Sakurai model has little effect on the predicted spin density matrix.

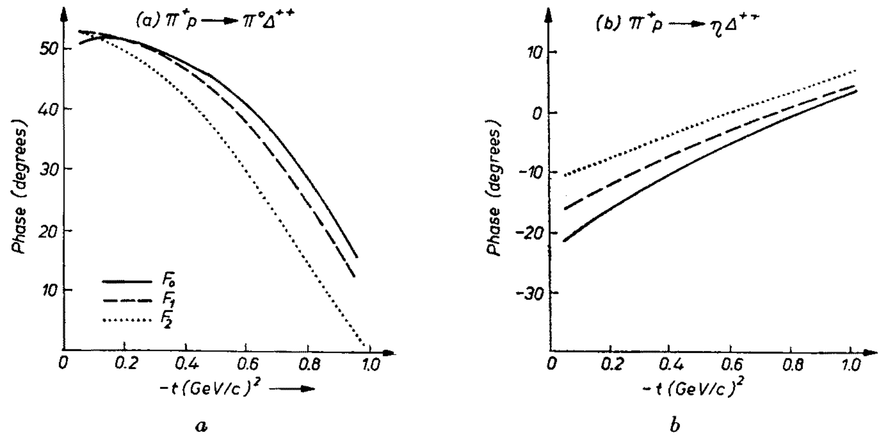

Fig. 4. Phases of the integrals F_0, F_1 and F_2 for the reactions $\pi^+p \rightarrow \pi^0\Delta^{++}$ and $\pi^+p \rightarrow \eta\Delta^{++}$

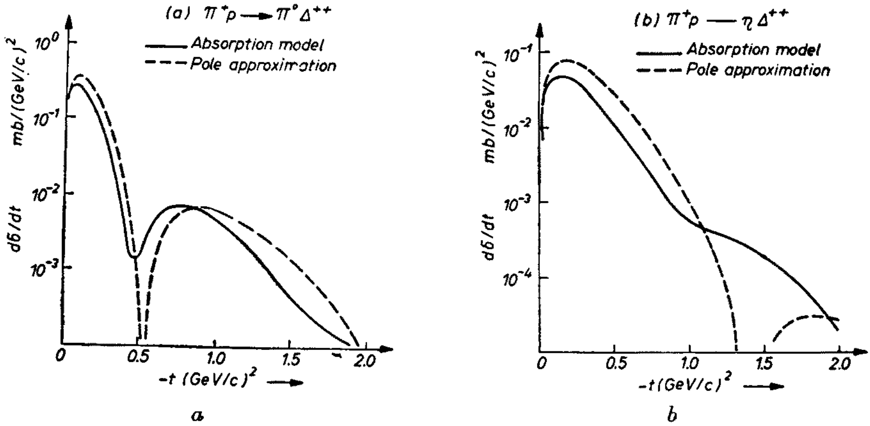

Fig. 5. Differential cross-section for the reactions $\pi^+p \rightarrow \pi^0\Delta^{++}$ and $\pi^+p \rightarrow \eta\Delta^{++}$. The solid curve comes from the absorption model and the dashed curve is a prediction of the pole model

Let us now consider the first order corrections. They contribute nothing to the differential cross-section and nothing to the tensor component T_0^2. One obtains, however, in this approximation

$$T_2^2 = ie^{i\bar{\chi}}\,\mathrm{Im}\,\frac{(f-a)c^*}{|f-a|^2}. \tag{28}$$

According to the formulae (21)–(23), this implies small corrections to all the measurable density matrix elements. The matrix elements predicted for reactions (24) and (25) are shown in Figs 6*a* and 6*b*. Using formula (28), it is possible to explain all the qualitative features of the deviations from the Stodolsky-Sakurai prediction. We omit this discussion. Let us note only that the corrections are relatively large and show structure in the dip region, where the modulus of $f-a$ is small and the phase of $f-a$ varies rapidly. This is clearly

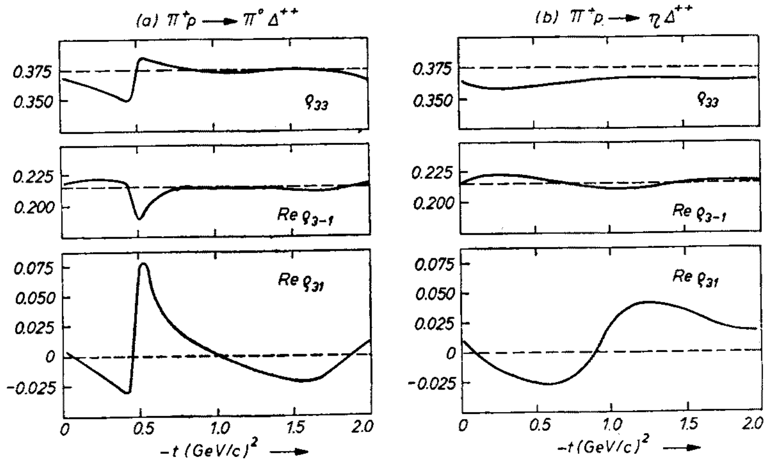

Fig. 6. Spin-density matrix elements in Jackson frame for the reactions $\pi^+p \to \pi^0\Delta^{++}$ and $\pi^+p \to \eta\Delta^{++}$. The horizontal dashed lines correspond to the predictions of the Stodolsky-Sakurai model

seen in the figures. The absorptive corrections to the Stodolsky-Sakurai results are so small that with present-day experimental accuracy they are hardly observable. Consequently, the deviations from the Stodolsky-Sakurai predictions, which seem to be observed for very small angle scattering (*cf.*, *e.g.*, Ref. [6]), should be explained by changing the pole term and not by absorption. We refer to models which try to reproduce both the differential cross-section and the spin density matrix. Amplitude (2) with absorption can reproduce the observed deviations from the Stodolsky-Sakurai distribution when f is not required to give a reasonable differential cross-section [7]. In particular, using the Born approximation for the pole term one can obtain very good spin density matrices at the cost of having quite unreasonable differential cross-sections. These are the curves which are usually quoted as the absorption model predictions (*cf.*, *e.g.*, first reference of [2]).

In order to recover the full formulae of the absorption model, we must still add the second-order terms. Their effect, however, what is most clearly seen in the deviations of T_0^2 from its Stodolsky-Sakurai value (*cf.* Fig. 7), is very small.

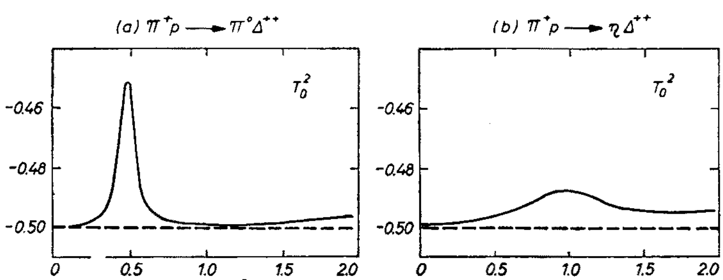

Fig. 7. Tensor element T_0^2 in Jackson transversity frame for the reactions $\pi^+p \to \pi^0\Delta^{++}$ and $\pi^+p \to \eta\Delta^{++}$. The horizontal dashed lines are the predictions of the Stodolsky-Sakurai model

Finally, let us note that the predictions of the Stodolsky-Sakurai model are invariant under rotations round the axis perpendicular to the scattering plane. Therefore, one can choose the amplitudes (2) as the spin amplitudes in any frame with the spin-quantization axis perpendicular to the reaction plane. The different choice of reference frame lead to different absorption corrections to the density matrix. In particular, starting from the ordinary transversity frame one can find that there are no corrections to the Stodolsky-Sakurai decay distribution, because absorption conserves s-channel helicity. The choice of the Jackson frame, however, seems to be natural for the pole models.

4. *Conclusions*

Absorption corrections affect in two ways the predictions of the Stodolsky-Sakurai model.

1. The scalar amplitude, denoted f in formula (2), is changed.

2. The spin dependence of the amplitude is modified. In our notation this is due to the amplitudes denoted b and c in formula (13).

We find that for a wide variety of absorption models the first effect is much stronger than the second one. Consequently, it is a very good approximation to assume that absorption affects only the scalar amplitude f of the Stodolsky-Sakurai model. The correction terms of the second kind are important in the dip regions, where the main term is abnormally small. A high statistics study of these regions could be very interesting, but with present experimental accuracy the effect is probably too small to be visible.

For very small angle scattering ($|t| \leqslant 0.1\ (\mathrm{GeV}/c)^2$) deviations from the Stodolsky-Sakurai predictions for the spin-density matrix are reported [6]. These deviations are in a much wider t range than expected from the Stodolsky-Sakurai model, if the model is required to give a reasonable differential cross-section. We interpret these deviations as evidence of the existence of small additional pole terms with a different spin structure. A quadrupole electric coupling is the most popular candidate for these correction terms [8].

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