NATURAL PARITY EXCHANGE AND GRIBOV-MORRISON PARITY RULE IN DIFFRACTIVE PRODUCTION OF MESONIC SYSTEMS

By A. Białas

Institute of Physics, Jagellonian University* and Institute of Nuclear Physics, Cracow

J. Dąbkowski

Department of Physics, School of Agriculture, Cracow

Z. Gołąb-Meyer

Institute of Physics, Jagellonian University, Cracow

(*Received February 4, 1971*)

The diffraction dissociation of pions and kaons into three particles is discussed under the following assumptions:

a) dominance of the natural parity exchange

b) Gribov-Morrison parity rule.

The measurable consequences of these assumptions are derived.

In this paper we discuss some consequence of

a) dominance of the natural parity exchange,

b) Gribov-Morrison parity rule [1],

for the spin dependence of the diffraction dissociation of pions and kaons into three particles.

We consider the reactions

$$\pi + X \to (3\pi) + X \tag{1}$$

and

$$K + X \to (K\pi\pi) + X \tag{1a}$$

where X may be any particle or nucleus.

The consequences of the assumptions a) and b) are presented for the case where no measurements of polarization of particle X are performed[1].

* Address: Instytut Fizyki, Uniwersytet Jagielloński, Kraków, Reymonta 4, Poland.

[1] The discussion of Gribov-Morrison rule for scattering on polarized target was given recently by Meggs and Van Hove [2]. We also refer the reader to this paper for a detailed explanation of Gribov-Morrison rule.

If the parity exchanged in the t channel of the reactions (1) and (1a) is natural, then the s channel amplitudes of these reactions satisfy the relations [3]:

$$M_{\mu\beta;\lambda 0} = \eta(-1)^{\beta} M_{\mu-\beta;\lambda 0} \tag{2}$$

where $\eta = P(-1)^{J+1}$.

In the above equation, λ, μ are the initial and final helicities of particle X, β is the helicity of 3π or $K\pi\pi$ system and J^P — its spin parity state, $\eta = +1$ for unnatural and $\eta = -1$ for natural parity of the produced 3π and $K\pi\pi$ systems. Formula (2) implies restrictions on the behaviour of the spin density matrices of 3π ($K\pi\pi$) system.

The immediate consequence of (2) is

$$\varrho_{\beta\beta'} = \eta(-1)^{\beta}\varrho_{-\beta\beta'}. \tag{3}$$

As we have seen, this formula follows just from the assumption that natural parity exchange dominates in diffraction dissociation. It is valid for any spin projection axis provided it is in the reaction plane. If the spin projection axis is chosen along the normal to the reaction plane, the formula (3) can be equivalently written in the form

$$\varrho_{\beta\beta'} = \eta(-1)^{J+\beta}\varrho_{\beta\beta'}. \tag{3a}$$

The physical meaning of (3) and (3a) depends on the value of the spin of the "decaying" 3π ($K\pi\pi$) system.

This may be seen by expressing these equations in terms of statistical tensors. They read

$$\sum_j (-1)^j [T^j_m \langle J-\beta; J\beta'|jm\rangle - \eta(-1)^{\beta} T^j_{m+2\beta}\langle J\beta; J\beta'|jm+2\beta\rangle] = 0$$

$$\beta, \beta' = 0, \pm 1, \pm 2, \dots, \pm J \tag{4}$$

$$\sum_j (-1)^j \langle J-\beta; J\beta'|jm\rangle T^j_m[(-1)^{J+\beta} - \eta] = 0$$

$$\beta, \beta' = 0, \pm 1, \pm 2, \dots, \pm J. \tag{4a}$$

Formula (4) refers to the spin projection axis chosen in the scattering plane; in (4a) the spin projection axis is chosen along the normal to the scattering plane.

Not all of these formulae (4) and (4a) can be easily checked, because for higher spin states the statistical tensors cannot be measured from angular distributions without a detailed knowledge of the decay dynamics. For any particular spin state one can, however, derive the relations between moments of the angular distributions. We give here such relations for 1^+ and 2^- states which are probably the most important in the A_1 and Q regions.

Let $\bar{\Theta}$ and $\bar{\Phi}$ be the polar and azimuthal angles of the normal to the decay plane of 3π ($K\pi\pi$) system in a frame XYZ which is the cm frame of this system with the Z axis normal to the reaction plane.

Furthermore let

$$N^j_m = \int d\bar{\Omega}\, Y^j_m(\bar{\Theta}, \bar{\Phi})\, W(\bar{\Theta}, \bar{\Phi}) \tag{5}$$

where $W(\bar{\Theta}, \bar{\Phi})$ is the normalized distribution of $\bar{\Theta}$ and $\bar{\Phi}$. Then the formula (4a) implies

a) for the 1^+ state

$$N_0^2 = \frac{1}{\sqrt{80\pi}} \tag{6}$$

b) for the 2^- state

$$N_0^1 = 4\sqrt{\frac{7}{3}}\, N_0^3. \tag{7}$$

To derive the above formulae it is necessary to use the relations between the angular distribution of the decay and the statistical tensors, as given *e.g.* in [4]. Formula (7) is trivially fulfilled for the case of charged 3π system. In this case $N_0^1 = N_0^3 = 0$ because two particles in the decaying system are identical to each other (see *e.g.* Ref. [4]).

The formulae (6) and (7) provide the test of the natural parity exchange dominance, but only in the case when one spin-parity state of the 3π ($K\pi\pi$) system dominates in the interesting region.

In the case when different spin parity states interfere we have worked out some tests under the simplifying assumption that the main decay modes of A_1 and Q bumps are $\varrho\pi$ and $K^*\pi$. In this we follow the approach proposed in [6] and derive the relations between the double averages over the production and decay angles of ϱ (K^*).

Let us introduce the following notation:

$$Z_{Mm}^{J\,j} = \int d\Omega d\omega \,\mathrm{Re}\,[Y_M^J(\vartheta, \varphi)] D_{mM}^{*j}(\Phi, \Theta, 0)\, W(\Theta, \Phi; \vartheta, \varphi) \tag{8}$$

where Θ and Φ are the angles determined by the ϱ (K^*) momentum in XYZ frame described before, ϑ, φ are the polar and azimuthal angles of the final $\pi(K)$ momentum from the $\varrho(K^*)$ decay in the rest frame of $\varrho(K^*)$. The frame xyz in which ϑ, φ are defined has its z axis along the $\varrho(K^*)$ direction in the rest frame of A_1 (Q) and the x axis in the plane defined by Z and z axes. $W(\Theta, \Phi; \vartheta, \varphi)$ is the experimental distribution of angles Θ, Φ, φ.

If only 0^-, 1^+ and 2^- states interfere then from (2) we get

$$\frac{2}{5} Z_{02}^{02} - Z_{02}^{22} + Z_{22}^{22} - \frac{18}{5\sqrt{3}} Z_{02}^{04} + 3\sqrt{\frac{3}{5}} Z_{02}^{24} = 0$$

$$Z_{00}^{00} - \frac{\sqrt{5}}{2} Z_{00}^{20} + \frac{15}{4}\sqrt{5}\, Z_{20}^{22} - \frac{5}{2} Z_{00}^{02} + \frac{5}{4}\sqrt{5}\, Z_{00}^{22} +$$

$$+ 9 Z_{00}^{04} - \frac{9}{2}\sqrt{5}\, Z_{00}^{24} = 0; \qquad Z_{M\,m}^{J\,j} = 0 \text{ for } j \geqslant 5. \tag{9}$$

The above relations follow from natural parity exchange dominance, Gribov-Morrison rule, and the assumption that the highest interfering spin does not exceed 2. Thus they may be used in order to test these assumptions.

It was shown in (4) that some relations of this type may follow from the Gribov-Morrison rule only and can be very useful as the test of the rule. We have also derived such relations, they read:

$$Z^1_0 = Z^2_1 = Z^3_0 = Z^3_2 = Z^4_1 = Z^4_3 = 0$$

$$Z^{21}_{00} = Z^{22}_{01} = Z^{23}_{00} = Z^{23}_{02} = Z^{24}_{01} = Z^{24}_{03} = 0$$

$$Z^{23}_{20} = Z^{23}_{22} = Z^{24}_{21} = Z^{24}_{23} = 0 \tag{10}$$

$$Z^{04}_{04} - \frac{\sqrt{5}}{2} Z^{24}_{04} + \sqrt{3}\, Z^{24}_{24} = 0$$

$$Z^{04}_{02} - \frac{\sqrt{5}}{2} Z^{24}_{02} + \sqrt{3}\, Z^{24}_{22} = 0$$

$$Z^{04}_{00} - \frac{\sqrt{5}}{2} Z^{24}_{00} + \sqrt{3}\, Z^{24}_{20} = 0$$

$$Z^{03}_{03} - \frac{\sqrt{5}}{2} Z^{23}_{03} + \frac{3}{2} Z^{23}_{23} = 0$$

$$Z^{03}_{01} - \frac{\sqrt{5}}{2} Z^{23}_{01} + \frac{3}{2} Z^{23}_{21} = 0 \tag{10a}$$

and

$$Z^{J\,j}_{M\,m} = 0 \text{ for } j \geqslant 5.$$

Here $Z^j_m = Z^{0j}_{0m}$ and $Z^{J\,j}_{M\,m}$ have the same meaning as in (9). It is worthwhile to note that (10) follows immediately from equations (13) and (15) of Ref. [5].

At the end we give the formulae analogous to (10) and (10a) for the case when only 0^- and 1^+ interfere. They read:

$$Z^{J\,j}_{M\,m} = 0 \text{ for } j \geqslant 3$$

$$Z^1_0 = Z^2_1 = 0$$

$$Z^{21}_{00} = Z^{22}_{01} = Z^{21}_{21} = Z^{21}_{01} = 0$$

$$Z^{22}_{22} - Z^{22}_{02} + \frac{2}{\sqrt{5}} Z^{02}_{02} = 0$$

$$\frac{4}{5} Z^{00}_{00} - \frac{2}{\sqrt{5}} Z^{20}_{00} + 3\sqrt{5}\, Z^{22}_{20} - 2Z^{02}_{00} + \sqrt{5}\, Z^{22}_{00} = 0. \tag{11}$$

To conclude, we have derived the experimental consequences of the Gribov-Morrison rule and natural parity exchange dominance for the processes of diffraction dissociation of mesons into three particles. It seems that our relations are fairly easy to check with the existing data. Therefore they may be perhaps useful in the experimental verification of the assumptions used.

The authors thank Dr A. Kotański, Dr K. Zalewski and Mr A. Golemo and Mr P. Gizbert-Studnicki for useful discussions.

REFERENCES

[1] V. N. Gribov, *Yadernaya Fizika* (USSR), **5**, 197 (1967); D. R. O. Morrison, *Phys. Rev.*, **165**, 1699 (1968).
[2] W. J. Meggs, L. Van Hove, CERN preprint, TH. 1256 (1970).
[3] G. Cohen-Tannoudji, Ph. Salin, A. Morel, *Nuovo Cimento*, **55A**, 412 (1968).
[4] A. Kotański, K. Zalewski, Jagellonian University preprint TPJU-31/70, November 1970. See also, Proceedings of The XI Cracow School of Theoretical Physics, Vol. II, p. 23, Cracow 1971
[5] A. Kotański, K. Zalewski, *Nuclear Phys.*, **B22**, 317 (1970).
[6] A. Białas, J. Dąbkowski, L. Van Hove, Jagellonian University preprint TPJU-23/70, September 1970.