EFFECTS OF NUCLEUS SHAPE ON K^- MESON NUCLEAR CAPTURE

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The influence of the nuclear deformation on the K^- meson nuclear absorption is discussed for the states $7i$, $8k$, $9l$ in dysprosium. It is found that the absorption rate, Γ^{capt}, increases by a factor of 1.5 to 1.7 due to the nuclear deformation. The value of Γ^{capt} has also been calculated for the same nucleus on the assumption that a tail of 3 fm has been added to the Fermi distribution of nuclear matter. It is found that Γ^{capt} increases by a factor of 2 as compared with the Fermi distribution without a tail.

1. *Introduction*

In the past few years the yields, energies and widths of K-mesoatomic X ray lines have been measured in some laboratories [1]. The results of these experiments have been analyzed by several authors [2]. Ericson and Scheck [2] have calculated the yields of the kaonic X ray lines and have compared them with the experimental values obtained by Wiegand [1]. They have defined the experimental yield as the ratio of an X ray line intensity $I_{n+1\to n}$ to the intensities of the transitions feeding the level $n+1$:

$$Y_{\text{exp}}^{n+1} = \frac{I_{n+1\to n}}{\sum\limits_{ni>n+1} I_{ni\to n+1}}. \tag{1}$$

This yield has been compared with the theoretical value

$$Y_{\text{theor}}^{n+1\to n} = \frac{\Gamma_{n+1,n}^{\text{rad}}}{\Gamma_{n+1,n}^{\text{rad}}+\Gamma_{n+1,n}^{\text{capt}}}, \tag{2}$$

where Γ_{nl}^{rad} is the radiative dipole transition rate and $\Gamma_{nl}^{\text{capt}}$ is the absorption rate. The latter is given by Ericson and Scheck in the form:

$$\Gamma_{nl}^{\text{capt}} = \frac{4\pi}{m_K}\left[1+\frac{m_K}{m_N}\right]\cdot \text{Im}\,\bar{A}\cdot \int |\Phi_{nl}|^2 \varrho(\vec{r})d\vec{r}, \tag{3}$$

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where $\overline{m}_K$ is the reduced mass of the kaon and the nucleus, Im $\bar{A} = 0.7$ fm is the isospin average of the $K^-\hat{N}$ scattering length, Φ_{nl} is the normalized meson wave function and ϱ is the density of nuclear matter.

In Fig. 1 of the paper of Ericson and Scheck [2] the experimental yields and the theoretical curves are shown. It is seen that the experimental yields of the X ray lines in heavy elements are slightly lower than the predicted ones, on the average by about 30%.

In the present work we look for an explanation of the discrepancy between the experimental and theoretical X ray yields in kaonic atoms. We are concerned with the dependence of the absorption rate and, consequently, the yields of the kaonic X rays on the nuclear deformation. We also examine the influence on the absorption rate of nuclear tail added to the Fermi distribution of nuclear matter [3].

2. *Nuclear deformation*

In order to evaluate the effect of the nuclear deformation we have performed the calculations for dysprosium which is one of the most deformed nuclei.

There are several possible ways of taking into account the deformation of the nuclear matter density distribution [4]. We have used the form with a uniform skin thickness introduced by the Columbia group [5], which gives a good agreement with the Coulomb Excitation Data:

$$\varrho(r) = \frac{\varrho_0}{1 + \exp\left(\frac{r - c(1+\beta Y_{20})}{t}\right)} . \tag{4}$$

We have taken the parameters c, t, β from Wu and Wilets [5]:

$$c = 6.02 \text{ fm},$$
$$t = 0.548 \text{ fm},$$
$$\beta = 0.34.$$

The parameter ϱ_0 has been calculated from the condition $\int \varrho d\tau = A$. In our case ϱ_0 is equal to 0.166 fm^{-3}. We have calculated the absorption rates for the states $7i$, $8k$ and $9l$ for $\beta = 0$ (spherical nucleus), and $\beta = 0.34$. The nuclear capture rate, Γ^{capt}, is proportional to the overlap integral of the nuclear and mesonic densities. In an explicit form it reads:

$$\Gamma_{nl}^{\text{capt}}(\beta, \vartheta', \varphi') = \frac{4\pi}{\overline{m}_K}\left[1 + \frac{m_K}{m_N}\right] \cdot \text{Im}\,\bar{A} \cdot \int r^{2l+2} \cdot \exp\left(\frac{-2Zr}{n \cdot a}\right) |Y_{lm}(\vartheta, \varphi)|^2 \times$$
$$\times \frac{\varrho_0 d\tau}{1 + \exp\left(\frac{r - c(1+\beta\, Y_{20}(\vartheta', \varphi'))}{t}\right)} \tag{5}$$

where (r, ϑ, φ) and $(r, \vartheta', \varphi')$ are polar coordinates in mesonic and nuclear quantization frames, Z is the atomic number, $a = \frac{\hbar^2}{\overline{m}_K \cdot e^2}$ is the radius of the $1s$ Bohr orbit in the K-mesonic hydrogen atom.

To obtain the measured value of $\Gamma_{nl}^{\text{capt}}(\beta)$ we must average over all possible positions of the axis of the symmetry of the nucleus system. To do this, we have expressed $Y_{20}(\vartheta', \varphi')$ in the form:

$$Y_{20}(\vartheta',\varphi') = \sum_{\mu} \sqrt{\frac{4\pi}{5}}\, Y_{2\mu}^{*}(\beta_E, \alpha) \cdot Y_{2\mu}(\vartheta, \varphi), \tag{6}$$

where β_E, α are the Euler angles of the axis of symmetry of the nucleus in the mesonic system. Since $Y_{20}(\vartheta', \varphi')$ depends on the difference $\alpha - \varphi$, we have put $\alpha = 0$ and have averaged only over β_E:[1]

$$\Gamma_{nl}^{\text{capt}}(\beta) = \frac{1}{2} \cdot \frac{4\pi}{\bar{m}_K}\left[1 + \frac{m_K}{m_N}\right] \cdot \operatorname{Im} \bar{A} \cdot \int_0^{\pi}\!\!\int r^{2l+2} \cdot \exp\left(\frac{-2Zr}{n \cdot a}\right) |Y_{lm}(\vartheta, \varphi)|^2 \times$$
$$\times \frac{\varrho_0\, d\tau \sin\beta_E\, d\beta_E}{1 + \exp\left(\dfrac{r - c(1 + \beta Y_{20}(\vartheta', \varphi'))}{t}\right)}. \tag{7}$$

The results of our calculations are given in Table I. It is seen that taking into account the nuclear deformation leads to an increase in the value of Γ^{capt} by a factor of 1.5–1.7, depending on the transition. Since Y_{theor}, as given by formula (2), depends on the sum of Γ^{rad} and Γ^{capt}, the above change in the value of Γ^{capt} can mostly influence Y_{theor} when both rates are comparable. This is only the case for the $7i \to 6h$ transition.

TABLE I

Transition $n+1$, $l+1 \to n, l$	Γ_{nl}^{rad} (keV)[1]	$\Gamma_{nl}^{\text{capt}}(\beta=0)$ (keV)	$\dfrac{\Gamma_{nl}^{\text{capt}}(\beta)}{\Gamma_{nl}^{\text{capt}}(\beta=0)}$	$Y_{\text{theor}}(\beta=0)$	$\Gamma_{\text{theor}}(\beta)$	Y_{exp}
$10m \to 9l$	2.46×10^{-3}	1.0×10^{-7}	1.7	1	1	—
$9l \to 8k$	4.50×10^{-3}	3.2×10^{-5}	1.6	1	1	—
$8k \to 7i$	8.96×10^{-3}	7.0×10^{-3}	1.5	0.99	0.99	1 ± 0.7
$7i \to 6h$				0.56	0.46	0

[1] Taken from Ericson and Scheck [2].

We can understand this increase of the K meson nuclear capture in a deformed nucleus by making use of the following simple picture. A deformed nucleus may be likened to a rugby ball wobbling around to produce a smearing-out effect that effectively increases the nuclear radius. This leads to a larger overlap with the mesonic cloud surrounding it which increases the value of Γ^{capt}.

Our results may be compared with the old results of Wilkinson [6] and those of Rook and Aslam published recently [7]. Rook and Aslam have used the nuclear matter distribu-

[1] It may be mentioned that $\Gamma_{nl(m)}^{\text{capt}}(\beta)$ is independent of m.

tion in the form:

$$\varrho = \frac{\varrho_0}{1 + \exp\left(\frac{r(1-\beta Y_{20})-c}{t}\right)}, \tag{8}$$

and for the state $7i$ in gadolinium obtained the ratio

$$\Gamma(\beta = 0.358)/\Gamma^{\text{capt}}(\beta = 0) = 1.729.$$

It is seen that the two methods of making allowance for nucleus deformation give similar results. The corresponding numbers given by Wilkinson are two orders of magnitude higher.

3. *Nuclear tail*

In order to examine the dependence of the absorption rate on the form of the nuclear matter distribution, we have considered the Fermi distribution with a tail described by two parameters, the height, h, and the length, l:

$$\varrho(r) = \begin{cases} \dfrac{\varrho_0}{1+\exp\dfrac{r-c'}{t'}} & r < r_1 \\ h & r_1 \leqslant r < r_1 + l \\ 0 & r \geqslant r_1 + l \end{cases} \tag{9}$$

The value of r_1 is defined by the condition $\varrho(r_1) = h$ and the parameters c', t', while corresponding to the parameters c and t of the formula (4), are not the same. They have been fitted in such a way that the nuclear volume and the r. m. s. radius, $\langle r^2 \rangle^{1/2}$, are the same as for the Fermi distribution without a tail. We have again calculated the capture rate, $\Gamma_{nl}^{\text{capt}}$, for the state $7i$, $8k$ and $9l$ in dysprosium, inserting the density ϱ from the formula (9) into the formula (3). In Table II our results are compared with the values obtained for the standard Fermi distribution without a tail. It is seen that Γ^{capt} for the distribution with a tail is about twice as large as Γ^{capt} for the distribution without a tail. This change in the value of Γ^{capt} reduces the yield of the $7i \to 6h$ transition by about 25%. Using the same parameters of the tail for nuclei with atomic numbers ranging from about 20 to about 70, and the states $4f$, $5g$ and $6h$, Rook [3] has obtained a decrease of Γ^{capt} as compared with the values calculated without a tail.

TABLE II

Transition $n+1$, $l+1 \to n, l$	$\Gamma_{nl}^{\text{capt}}$(Fermi distr. + tail) / $\Gamma_{nl}^{\text{capt}}$(Fermi distr.)	Y_{theor} (Fermi distr.)	Y_{theor} (Fermi distr. + tail)	Y_{exp}
$10m \to 9l$	1.9	1	1	—
$9l \to 8k$	2.0	1	1	—
$8k \to 7i$	1.8	0.99	0.99	1 ± 0.7
$7i \to 6h$		0.56	0.42	0

4. Conclusions

We have evaluated the effect of the nuclear deformation of the K mesic X-ray yields for one of the most deformed nuclei and we have found it less than 20%. We conclude that while the deformation of nuclei should be taken into account, it does not explain the observed discrepancy between the experimental and theoretical values of the X-ray yields in kaonic atoms.

We have also discussed the sensitivity of the kaonic X ray yields to a tail added to the Fermi distribution of the nuclear matter. We have found that even a large tail diminishes the yields by no more than 25%.

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