THE EFFECTIVE POTENTIAL FOR THE SU(5)×U(1) MODEL I*

By R. Mańka, J. Sładkowski and A. Kornakiewicz

Institute of Physics, Silesian University, Katowice**

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The effective potential in the minimal flipped SU(5)×U(1) model is calculated and the proton life-time in this model is discussed. Results are compared with those from the orthodox SU(5) GUT model.

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1. Introduction

Despite the over short proton life time and the existence of monopoles the minimal SU(5) model is the most effective grand unification model constructed up to now. Recently, superstring theories raised our hopes of constructing a unified theory of the known fundamental interactions [1]. The superstring inspired models are based on either the $E_8 \times E_8$ of the SO(32) gauge group. Such a "big" gauge group is then broken (to E_6, SO(10), $SU(3)^3$ ...). The breaking mechanism is unique. We hope to succeed in constructing a theory with a "smaller" gauge group. The most recent candidate is the SU(5)×U(1) model (the flipped SU(5) model) [2–4]. This is a supersymmetric model with (1, 2) world sheet supersymmetry. The aim of this paper is to construct effective potential, firstly in the minimal nonsupersymmetric version (Part 1) and next in the full supersymmetric version (Part 2), when the supersymmetry is softly broken.

The paper is organized as follows. In Section 2 we describe the orthodox SU(5) model in order to make the paper self-contained. The flipped SU(5) model is described in Section 3. In Section 4 the effective potential of the flipped SU(5) model is calculated and the proton life time is discussed.

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** Address: Instytut Fizyki, Uniwersytet Śląski, Uniwersytecka 4, 40-007 Katowice, Poland.

2. *The orthodox SU(5) model*

The minimal SU(5) model [5] consists of the SU(5) gauge field A_μ^a ($a = 1, \ldots, 24$), two scalar Higgs fields H (adjoint representation) and h (fundamental representation), and two fermion fields for each family ψ_i and $\psi_{[ij]}$ transforming as 5 and 10, respectively. The Lagrange function has the form

$$L = L_b + L_f, \tag{2.1}$$

$$L_b = -1/4 F_{\mu\nu}^a F^{a\mu\nu} + 1/2 (D_\mu H)^a (D^\mu H)^a$$
$$+ D_\mu h^+ D^\mu h - U(H, h), \tag{2.1b}$$

$$L_f = i\bar{\psi}_{Rj}\gamma^\mu (D_\mu \psi_R)_j + i\bar{\psi}_{L[jk]}\gamma^\mu (D_\mu \psi_L)_{[jk]} \tag{2.1c}$$

with

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c, \tag{2.1d}$$

$$(D_\mu H)^a = \partial_\mu H^a + g f_{abc} A_\mu^b H_c, \tag{2.1e}$$

and covariant derivatives defined similarly as for the other fields. The SU(5) gauge field is described in the following basis

$$A_\mu = \sum_\alpha T^a A_\mu^a = 1/2 \sum_\alpha A_\mu^a \Lambda^a, \tag{2.2a}$$

$$[\Lambda^a, \Lambda^b] = 2i f_{abc} \Lambda^c,$$
$$\mathrm{Tr}\,(\Lambda^a \Lambda^b) = 2\delta_{ab}. \tag{2.2b}$$

As we are interested in high-energy breaking of the symmetry group from SU(5) to SU(3) $\times$ SU(2) $\times$ U(1) only the H-dependent part of the potential

$$U(H, h) = U(H) + U_{\mathrm{int}}(H, h) + U(h) \tag{2.3a}$$

$$U(H) = \lambda_1 (\mathrm{Tr}\, H^2)^2 + \lambda_2 (\mathrm{Tr}\, H^4) \tag{2.3b}$$

will be required. Fermions are grouped in the following way:

$$\psi_{R_i} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^c \\ \nu_e \end{pmatrix} \quad \text{and} \quad \psi_{L[i,j]} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}. \tag{2.4}$$

The electric charge is $Q_{\mathrm{el}} = T^3 + Y$, where $Y = 1/2\sqrt{5/4}\,\Lambda^{24}$. The spontaneous symmetry breaking to the group SU(3) $\times$ SU(2) $\times$ U(1) occurs due to Higgs boson condensation along the $a = 24$ axis so that

$$H = -T^{24}\varrho, \tag{2.5a}$$

$$\varrho = \bar{\varrho} + \sigma, \tag{2.5b}$$

where

$$\Lambda^{24} = 2/\sqrt{15}\begin{bmatrix} -1 & & & & \\ & -1 & & & 0 \\ & & -1 & & \\ 0 & & & 3/2 & \\ & & & & 3/2 \end{bmatrix}. \tag{2.5c}$$

This is the unitary gauge representation in which the unphysical Goldston fields are removed at the beginning. The shift in (2.5) describes the condensation, which reveals it self as changing of the vacuum state. Due to the condensation the H-Higgs field could be described by the following Lagrange function

$$L = 1/2\partial_\mu\bar{\varrho}\partial^\mu\bar{\varrho} - 1/2m^2\bar{\varrho}^2 - U(\bar{\varrho}), \tag{2.6a}$$

where

$$m^2 = 3\Lambda\sigma^2 \tag{2.6b}$$

$$U(\bar{\varrho}) = 1/4\lambda\sigma^2 + \lambda\bar{\varrho}\sigma^3 + 1/4\lambda\bar{\varrho}^4 \tag{2.6c}$$

$$\lambda = \lambda_1 + 7/30\lambda_2. \tag{2.6d}$$

Some of the SU(5) gauge fields gain masses

$$L_m = 1/2\sum M_{ab}^2 A_\mu^a A^{a\mu}, \tag{2.7a}$$

$$M_{ab}^2 = -1/2g^2\,\mathrm{Tr}\,([\Lambda^a, V][\Lambda^b, V]), \tag{2.7b}$$

$$V = -T^{24}\sigma.$$

When the condensation occurs along the 24-axis (as we have chosen) the SU(5) group is broken to the SU(3) × SU(2) × U(1) group and the remaining twelve gauge fields (usually denoted by X and Y) gain the mass

$$M_{X,Y}^2 = 5/12h^2\sigma^2. \tag{2.8}$$

The effective potential may now be calculated in a way analogous to that for scalar electrodynamics. The bosons $\bar{\sigma}$, X, and Y will be the source of this potential. Each of them will have 3 degrees of freedom. We can calculate the effective potential as in [6, 7]. The SU(3) × SU(2) × U(1) gauge symmetry will also be broken if the second scalar field (transforming as 5) condensates i.e.

$$h = (\bar{X} + V)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{2.9}$$

As a result of this the fields $W_\mu^\pm$ and Z_μ gain their masses and $\sin^2\vartheta_W$ takes the value 3/8.

3. *The flipped SU(5) model*

The minimal SU(5)×U(1) model [2–4] consists of the 24 SU(5) gauge fields A^a_μ the U(1) gauge field A^{25}_μ, the scalar higgs field $H_{[i,j]}$ (transforming as (10, 1), the scalar field h (transforming as (5, −2)) and the two (for each family) fermion fields ψ_i and $\psi_{[i,j]}$ (transforming as (5, −3) and (10, 1), respectively). The Lagrange function takes the form:

$$L = -1/4 \sum_{a=1}^{25} F^a_{\mu\nu}F^{a\mu\nu} + (D_\mu H^+)_{[ij]}(D^\mu H)_{[ij]} + (D_\mu h)^+(D^\mu h)$$

$$- U(H, h) + i\bar{\psi}_{Rk}\gamma^\mu(D_\mu \Phi_R)_i + i\bar{\psi}_{L[jk]}\gamma^\mu(D_\mu \psi_L)_{[jk]}, \tag{3.1a}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc}A^b_\mu A^c_\nu \quad (a, b, c = 1, \dots 24), \tag{3.1b}$$

$$F^{25}_{\mu\nu} = \partial_\mu A^{25}_\nu - \partial_\nu A^{25}_\mu, \tag{3.1c}$$

$$(D_\mu H)_{[ij]} = \partial_\mu H_{[ij]} - ig \sum_{a=1}^{24} A^a_\mu (T^a H)_{[ij]} - g_E Y_E A^{25}_\mu H_{[ij]}, \tag{3.1d}$$

$$(T^a H_{[ij]}) = \sum_k (T^a_{ik} H_{[k,j]} + T^a_{jk} H_{[i,k]}). \tag{3.1e}$$

The structure of this model resembles that of the GSW model. Similarly, we have two coupling constans g (SU(5)) and g_E(U(1)) (but we estimate that g_E is big [2, 3]).

Analogously, we should expect mixing between the field A^{24} and A^{25}_μ and the existence of a Weinberg-like angle ν_E. The electric charge operator takes the form

$$Q = T_3 + (Y + Y_E)1/5. \tag{3.2}$$

Due to the "flipping" the electric charge operator differs from the simple sum $Y_E + Y + T_3$. This has the effect of changing the Weinberg angle formula.

The $Y_E = -1$ scalar field $H_{[i,j]}$ can be represented in the form

$$H = \begin{pmatrix} 0 & \tilde{d}^c & -\tilde{d}^c & \tilde{d} & \tilde{u} \\ -\tilde{d}^c & 0 & \tilde{d} & \tilde{d} & \tilde{u} \\ d^c & -\tilde{d}^c & 0 & \tilde{d} & \tilde{u} \\ -\tilde{d} & -\tilde{d} & \tilde{d} & 0 & \tilde{v} \\ -\tilde{u} & -\tilde{u} & -\tilde{u} & -\tilde{v} & 0 \end{pmatrix}, \tag{3.3}$$

where the scalar fields $\tilde{d}$, $\tilde{u}$, $\tilde{v}$ have charges −5/3, −1/3, −4/3, 0, respectively. The scalar field h_i has $Y_E = -2$ (transforms as 5 with respect to SU(5))

$$h_i = \begin{pmatrix} \tilde{d} \\ \tilde{d} \\ \tilde{d} \\ \tilde{e}^c \\ v \end{pmatrix}. \tag{3.4}$$

If only the v and $\tilde{v}$ field condensate then the vacuum state is chargeless and the fields $\tilde{d}$, u, $\tilde{e}$ become unphysical (Goldstone particles). The $\tilde{v}$ field condensation breaks the SU(5) × U(1) symmetry group to the SU(3) × SU(2) × U(1) group. Breaking of the latter symmetry to the SU(3) × U(1) group is achieved due to the $\tilde{v}'$ field condensation. We describe the $\tilde{v}$ field condensation by the following parametrization:

$$H_{[ij]} = 1/\sqrt{2}\, C_{ij}\varrho, \tag{3.5a}$$

where

$$\varrho = \bar{\varrho}+\sigma, \tag{3.5b}$$

$$C_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \tag{3.5c}$$

As previously, the $\bar{\varrho}$ field takes the Higgs field role. The Higgs sector can now be described by the Lagrange function (2.6) but the coupling constant λ will be different. As in the orthodox SU(5) model, some of the gauge fields gain masses:

$$L_M = 1/2 \sum_{ab=1}^{25} M_{ab}^2 A_\mu^c A^{b\mu}, \tag{3.6a}$$

where

$$M_{ab}^2 = 1/2 g^2 \sum_{k,m,n,l,v} (V^{[k,n]}T_{km}^a + V^{[ml]}T_{ln}^a)(T_{mv}^b V^{vn} + T_{nv}^b V^{mv}) \tag{3.6b}$$

$$V_{[k,n]} = 1/\sqrt{2}\, C_{kn}\sigma, \tag{3.6d}$$

$$T^a = \{1/2\Lambda^a,\ a = 1, \ldots, 24,\ I\}, \tag{3.6e}$$

$$A_\mu^a = (A_\mu^a,\ a = 1, \ldots, 24,\ A_\mu^{25}). \tag{3.6f}$$

The mass matrix has a block structure. The SU(3) × U(2) × U(1) fields are still massless. The X, Y gauge bosons mass matrix has the form:

$$M_{X,Y}^2 = 1/4\, g^2\sigma^2 \begin{pmatrix} 2 & -2i \\ 2i & 2 \end{pmatrix}. \tag{3.7}$$

As in the GSW model, mixing between A_μ^{24} and A_μ^{25} takes place. The mixing matrix has the folowing form:

$$M_E^2 = 1/4\,\sigma^2 \begin{pmatrix} 72/15\, g^2 & 24/\sqrt{15}\, g g_E \\ 24/\sqrt{15}\, g g_E & 8 g_E^2 \end{pmatrix}$$

$$= 2\sigma^2 \begin{pmatrix} g'^2 & g' g_E \\ g' g_E & g_E^2 \end{pmatrix}, \tag{3.8}$$

where $g' = \sqrt{3/5}g$ is the weak coupling constant. It has the same value as in orthodox SU(5) theory. Diagonalization of the matrices (3.7) and (3.8) gives masses of the fields X, Y

$$M^2_{X,Y} = 1/4\, g^2\sigma^2, \tag{3.9}$$

and the additional neutral bosons Z'

$$M^2_{Z'} = 1/4\, \sigma^2(72/15\, g^2+8g^2_E) = 2\sigma^2(g^2_E+g'^2). \tag{3.10}$$

$M^2_{Z'} \geqslant M_{XY}$ because g_E is rather large.

The second of the mixed A^{24}_μ and A^μ_{25} fields remains massless. If we define the new fields Z'_μ and B_μ by the relation

$$\begin{pmatrix} A^{24}_\mu \\ A^{25}_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_E & -\sin\vartheta_E \\ \sin\vartheta_E & \cos\vartheta_E \end{pmatrix} \begin{pmatrix} Z'_\mu \\ B_\mu \end{pmatrix}. \tag{3.11}$$

Then the B_μ field is massless. As in the GSW model we have

$$\sin^2\vartheta_E = \frac{g^2_E}{g'^2+g^2_E}. \tag{3.12}$$

The subsequent condensation of the field $\tilde{v}'$ results in breaking of the symmetry to the SU(3)×U(1) group. There is an enormous energy gap between these condensations (10^2 GeV–10^{15} GeV). The subsequent condensation (practically) virtually does not influence the Z'_μ boson mass, so that we may neglect the mixing between the Z'_μ field and A^3_μ and A_μ fields. The later fields gain their masses via the low energy condensation. This may be described by the transformation

$$\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \tag{3.13}$$

where A_μ is the electromagnetic field and ϑ the usual Weinberg angle:

$$\sin^2\vartheta = \frac{\bar{g}^2}{g^2+\bar{g}^2}. \tag{3.14}$$

The "effective" coupling constant $\bar{g}$ is defined by

$$\bar{g} = 5g\,\sin\vartheta_E \tag{3.15a}$$

or

$$\bar{g} = 5g_E\cos\vartheta_E. \tag{3.15b}$$

This means that

$$\bar{g} = \frac{5g'g_E}{\sqrt{g^2_E+g'^2}} \tag{3.16}$$

is proportional to g' when g_E is small and tends to $5g'$ in the limit $g_E \to \infty$. If $g_E > g'/\sqrt{24}$ the effective coupling constant is even greater than in the SU(5) model. This results in increase in the (clasical) value of $\sin^2\vartheta$. To obtain a more realistic value for $\sin^2\vartheta$ we must according to renormalization group analysis, increase the value of M, the renormalization group point. As the result of this the proton lifetime increases! As in the standard model the couplings g and $\bar{g}$ define the electric charge value:

$$e = g \sin\vartheta, \tag{3.17a}$$

$$e = \bar{g} \cos\vartheta. \tag{3.17b}$$

We are now able to present the low energy (electroweąk) covariant derivative

$$D_\mu = \partial_\mu - ieQA_\mu - i(-g\cos\vartheta T_3 + 1/5\bar{g}(Y+Y_E)\sin\vartheta)Z_\mu$$
$$-i(-g'\cos\vartheta_E Y + g_E Y_E \sin\vartheta_E)Z'_\mu. \tag{3.18}$$

Q is given by Eq. (3.2). This describes all electroweak interactions. It is interesting that even fermions, which have $Y_E = 0$, will interact with the Z'_μ particle. This interaction vanishes only in the strong coupling limit $g_E \to \infty$. Interactions mediated by the Z'_μ particle will be difficult to observe in experimenta because its mass is large.

4. *The effective potential*

The effective potential could be calculated by using either the path integral method [9] or canonical formalism [6–8]. In the one-loop approximation this is simply the vacuum energy. Each physical degree of freedom contribution is equal to

$$\mathscr{E}_{0i} = 1/2\,\hbar \int \frac{d^3k}{(2_\pi)^3} \ln\left(1 + \frac{m_i^2}{k_E^2 + M^2}\right), \tag{4.1}$$

where m_i denotes the mass k_E the Wick-rotated four momentum (Euclidean) and M^2 is introduced to remove infrared divergences. Obviously clearly $\mathscr{E}_{0i}$ is divergent and should be regularized [7, 8]:

$$\mathscr{E}_{0i} = m_i^2 \frac{M}{16\pi^2}\left\{-\frac{1}{\varepsilon} + \ln\left(\frac{M^2\pi}{\mu^2}\right) - (1-\gamma)\right\}$$
$$-1/2\frac{m_i^2}{16\pi^2}\left\{\frac{1}{\varepsilon} - \ln\left(\frac{M^2\pi}{\mu^2}\right) - \gamma\right\}$$
$$+\frac{M^2}{32\pi^2}\left\{\left(1+\frac{m_i^2}{M^2}\right)\ln\left(1+\frac{m_i^2}{M^2}\right) - 3/2\left(\frac{m_i^2}{M^2}\right) - \frac{m_i^2}{M^2}\right\}. \tag{4.2}$$

Bearing in mind that we have three degrees of freedom for each of the twelve X, Y bosons and the Higgs field $\bar{\varrho}$ of the SU(5) model, we obtain:

$$U_{\text{eff}}(\sigma) = \frac{\lambda}{4}\sigma^4 + \frac{(3\lambda\sigma^2)^2}{64\pi^2}\left\{-\frac{1}{\varepsilon} + \ln\left(\frac{3\lambda\sigma^2}{\mu^2}\right) + \ln\pi\right.$$
$$\left.+\gamma - \tfrac{3}{2}\right\} + 36\frac{(M_{XY}^2)^2}{64\pi^2}\left\{-\frac{1}{\varepsilon} + \ln\left(\frac{M_{XY}^2}{\mu}\right) + \ln\pi + \gamma - \tfrac{3}{2}\right\} - \tfrac{1}{4}\,\delta\lambda\sigma^4. \tag{4.3}$$

Similarly we may calculate the effective potential of the SU(5) × U(1) model:

$$U_{\text{eff}}(\sigma) = \frac{\lambda}{4}\sigma^4 + \frac{(32\sigma^2)^2}{64\pi^2}\left\{-\frac{1}{\varepsilon} + \ln\left(\frac{32\sigma^2}{\mu^2}\right) + \ln\pi\right.$$

$$\left. + \gamma - \tfrac{3}{2}\right\} + 36\frac{(M_{XY}^2)^2}{64\pi^2}\left\{-\frac{1}{\varepsilon} + \ln\left(\frac{M_{XY}^2}{\mu^2}\right) + \ln\pi\right.$$

$$\left. + \gamma - \tfrac{3}{2}\right\} + 3\frac{(M_{Z'}^2)^2}{64\pi^2}\left\{-\frac{1}{\varepsilon}\ln\left(\frac{M_{Z'}^2}{\mu^2}\right) + \ln\pi + \gamma - \tfrac{3}{2}\right\} - \tfrac{1}{4}\,\delta\lambda\sigma^4. \tag{4.4}$$

The divergences in (4.3) and (4.4) are removed by appropriate counterterms:

$$\delta\lambda = \left[\frac{(3\lambda)^2}{64\pi^2} + \frac{36}{64\pi^2}\left(\tfrac{5}{12}g^2\right)^2\right]\left(-\frac{1}{\varepsilon} + c'\right) \tag{4.5}$$

in the SU(5) case and

$$\delta\lambda = \left[\frac{(3\lambda)^2}{64\pi^2} + \frac{36}{64\pi^2}(\tfrac{1}{4}g^2)^2 + \frac{3}{64\pi^2}(\tfrac{72}{15}g^2 + 8g_E^2)\right]\left(-\frac{1}{\varepsilon} + c'\right) \tag{4.6}$$

in the SU(5) × U(1) case. The counterterms introduce arbitrary constants c' into the theory. They can be chosen so that [7, 8]

$$\tilde{\lambda} = \left.\frac{\partial^4 U_{\text{eff}}}{\partial\sigma^4}\right|_{\sigma=\Lambda} = 6\lambda. \tag{4.7}$$

Similarly as in the classical theory case where $\left.\frac{\partial^4 U_{\text{cl}}}{\vartheta\sigma^4}\right|_{\sigma=\lambda} = 6\lambda$. $(U_{\text{cl}}(\sigma) = \lambda/4\sigma^4)$. The relation $\sigma = \Lambda$ has the following interpretation. The coupling constant $\tilde{\lambda}$ is equal to that in the classical theory. Of course, the point $\sigma = \Lambda$ may not be a minimum of potential U_{eft}. The above renormalization is only fragmentary. In fully renormalized theory all the coupling constants ($g(\Lambda) = g$) will be equal to those of the classical theory. Thus Λ defines the classical point of the renormalization group. This is the point where $g' = \sqrt{3/5}\,g$ in the SU(5) model. In the minimal SU(5) model, the point Λ fixes X, Y bosons scale. The fixing of the constant C' results in Coleman-Weinberg type of potential. In the SU(5) and SU(5) × U(1) models we obtain respectively:

$$U_{\text{eff}}(\sigma) = \frac{\lambda}{4}\sigma^4 + \frac{\sigma^4}{64\pi^2}\left(9\lambda^2 + 36\,(\tfrac{5}{12}g^2)^2\right)\left(\ln\left(\frac{\sigma^2}{\Lambda^2}\right) - \tfrac{25}{6}\right) \tag{4.8}$$

and

$$U_{\text{eff}}(\sigma) = \frac{\lambda}{4}\sigma^4 + \frac{\sigma^4}{64\pi^2}(9\lambda^2 + 36\,(\tfrac{1}{4}g^2)^2 + 3(\tfrac{72}{15}g^2 + 8g_E^2)^2)\left(\ln\left(\frac{\sigma^2}{\Lambda^2}\right) - \tfrac{25}{6}\right). \tag{4.9}$$

When we reject the Higgs boson contribution $9\lambda^2$ in (4.8) we obtain the text-book potential [9]. Let as compare the potential (4.8) and (4.9). Both describe a discontinous phase transition. Introducing dimensionless, coordinates as in [7]

$$\sigma = X\sigma_0, \tag{4.10a}$$

$$\sigma_0 = \Lambda e^{11/b}, \tag{4.10b}$$

we obtain the following expressions

$$U_{eff}(x) = 1/4\lambda' x^4 + 1/4x^4\{\ln(x^2) - 1/2\}, \tag{4.11a}$$

where

$$\lambda' = \frac{16\pi^2\lambda}{9\lambda^2 + 36(5/12g^2)} \text{ (SU(5) case)}. \tag{4.11b}$$

$$\lambda' = \frac{16\pi^2\lambda}{9\lambda^2 + 36(1/4g^2)^2 + (72/15g^2 + 8g_E^2)^2}\cdot \tag{4.11c}$$

This potential has a minimum at $x^0 = \sqrt{e^{-\lambda'}}$, which gives the X, Y bosons mass

$$M_{XY}^2 = 5/12g^2\Lambda^2 e^{11/3}e^{-\lambda'} \text{ (SU(5))}, \tag{4.12}$$

$$M_{XY}^2 = 1/4g^2\Lambda^2 e^{11/3}e^{-\lambda'} \text{ (SU(5)} \times \text{U(1))}. \tag{4.13}$$

The Z'_μ boson mass is equal to:

$$M_{Z'}^2 = 2(g_E^2 + g'^2)\Lambda^2 e^{11/3}e^{-\lambda'}. \tag{4.14}$$

At the point $\sigma = \lambda$, which is not a minimum,

$$M^2 = 5/12\, g^2\Lambda^2 \quad \text{(SU(5))}, \tag{4.15a}$$

$$M^2 = 1/4\, g^2\Lambda^2 \quad \text{(SU(5)} \times \text{U(1))}. \tag{4.15b}$$

These are the values which correspond to the classical point of the renormalization group, $M \sim 4 \cdot 10^{14}$ GeV. On the basis of (4.12)–(4.14) we obtain:

$$M_{XY} = M^2 e^{11/3}e^{-\lambda'} \text{ (SU (5))}, \tag{4.16}$$

and

$$M_{XY} = M^2 e^{11/3}e^{-\lambda'} \text{ (SU (5)} \times U(1)), \tag{4.17a}$$

$$M_{Z'}^2 = 24/15M^2\, \frac{e^{11/3}e^{-\lambda'}}{\cos^2\vartheta_E}\cdot \tag{4.71b}$$

It is obvious that the X, Y boson mass increases when the effective coupling constant λ' decreases. The M_{XY} can be increased maximally $e^{11/6} \approx 6.25$ times. The proton life time is proportional to M_{XY}^4 and hence increases up to 1500 times. This procedure is difficult to perform in the minimal SU(5) model because it involves decreasing λ, that is the Higgs boson mass (2.6b). This is harmless in the SU(5) × U(1) model because the decrease in λ' can be achieved by increasing g_E. The flipped SU(5) model (by assumption) describes strong interaction via the U(1) gauge boson. This is a natural way of increasing proton lifetime. The potential (4.11) leads to inflation [10]. The effective coupling constant λ' is too large in the SU(5) model [10] so that the inflation is of no cosmological significance. It should be considerably smaller. The SU(5) × U(1) model may describe a realistic inflation as in this model λ' is considerably smaller than in the SU(5) model.

5. *Conclusion*

We have compared the effective potential of the SU(5) grand unification model with that of the nonsupersymmetric SU(5) × U(1) model. It is shown that strong U(1) coupling (large g_E) generates a considerable increase in the X, Y boson mass, thus extending the proton lifetime. In the SU(5) × U(1) model monopoles do not exist due to the different structure of the scalar field sector. The presented version of the model is minimal. It is not supersymmetric. The full suppersymmetric version contains richer fermion and scalar field sector. The scalar field $H, \bar{H}, h, \bar{h}, \varphi_m$ transform as 10, $\overline{10}$. 5, $\bar{5}$, and 1 with respect to the SU(5) group. The fermion field F_i, f_i and L transform as 10, 5, 1 with respect to the SU(5) group. As long as the supersymmetry is unbroken, the fermions' and bosons' contributions to the vacuum energy cancel each other and the effective potential is a purely classical one without any radiative corrections. When the supersymmetry is broken the appropriate quantum corrections to the effective potential emerge. This will be investigated in the following paper.

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