COSMIC STRINGS IN EXTRA-U(1) MODEL*

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In this work a cosmic string arising as a result of spontaneous breaking of the $SU(2)_L \times U(1)_Y \times U(1)_E$ symmetry is investigated.

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1. Introduction

Basing on successful applications of the Standard Model, we can assume that all theories which claim to describe elementary particles at low energies ought to contain gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. One can arrive at such theories as low-energy limits of the $E_8 \times E_8'$ superstring theory [1]. After compactification, one of the E_8's gives an E_6 group, whereas the other E_8 describes a „hidden sector" of the theory. The E_6 group can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E \times U(1)_{E'}$ symmetry [2] and, below an intermediate mass scale ($10^8 - 10^{12}$ GeV), to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ [3]. A Spontaneous Symmetry Breaking (SSB) of the electroweak interactions ($SU(2)_L \times U(1)_Y \times U(1)_E \to U(1)_{EM}$) is investigated in this paper. As a result of this SSB not only a homogeneous vacuum, but also a non-trivial vacuum configuration, i.e. cosmic string, can occur. A cosmic string can also arise earlier, near the intermediate mass scale, i.e. during the breaking of the $U(1)_{E'}$ symmetry. Such a string will be more massive than the one arising near the electroweak scale (~ 1 TeV), and can form its core.

The symmetry of the vacuum is broken by Higgs fields: two doublets

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad \bar{H} = \begin{pmatrix} \bar{H}^0 \\ \bar{H}^- \end{pmatrix}$$

and a singlet N. With the extra U(1) group a E_μ gauge field is connected. While mixing with W_μ^i and B_μ fields existing in the Standard Model, this gauge field creates a new boson Z. The effects caused by this boson are analyzed by many authors in the $e^+e^- \to ff$ [4], $e^+e^- \to \nu\bar{\nu}\gamma$ [5], $e^+e^- \to W^+W^-$ [6] and $e^+e^- \to ZH^0$ [7] processes.

$SU(2)_L \times U(1)_Y \times U(1)_E$ quantum numbers of the Higgs fields

Field	T_3	Y	Y_E
H^+	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{2}{3}$
H^0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{2}{3}$
$\bar{H}^0$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{6}$
$\bar{H}^-$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{6}$
N	0	0	$\frac{5}{6}$

Table I shows quantum numbers of the Higgs bosons relevant to $SU(2)_L$, $U(1)_Y$ and $U(1)_E$ groups.

Covariant derivatives are given by:

$$D_\mu = \partial_\mu - ig\vec{T} \cdot \vec{W}_\mu - iYg'B_\mu - iY_E g''E_\mu, \tag{1}$$

where g, g', g'' are coupling constants of $SU(2)_L$, $U(1)_Y$ and $U(1)_E$ groups respectively.

2. The gauge bosons

As in the Weinberg-Salam model, the SSB causes the appearance of gauge fields masses. The SSB is connected with the non-zero Vacuum Expected Value (VEV) of the Higgs fields:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \bar{H} \rangle = \begin{pmatrix} \bar{v} \\ 0 \end{pmatrix}, \quad \langle N \rangle = x. \tag{2}$$

Mass terms of the gauge fields are enclosed in that part of Lagrangian which describes kinetic energy of Higgs particles:

$$\mathcal{L}_k = (D_\mu H)^\dagger D^\mu H + (D_\mu \bar{H})^\dagger D^\mu \bar{H} + (D_\mu N)^\dagger D^\mu N. \tag{3}$$

Writing H, $\bar{H}$, N as the sum of its VEVs and quantum fluctuations around the minimum of potential

$$H = \langle H \rangle + \tilde{H}, \quad \bar{H} = \langle \bar{H} \rangle + \tilde{\bar{H}}, \quad N = \langle N \rangle + \tilde{N}, \tag{4}$$

the Lagrangian (3) takes the form:

$$\mathcal{L}_k = (D_\mu \langle H \rangle)^\dagger D^\mu \langle H \rangle + (D_\mu \langle \bar{H} \rangle)^\dagger D^\mu \langle \bar{H} \rangle + (D_\mu \langle N \rangle)^\dagger D^\mu \langle N \rangle$$

$$+ (D_\mu \tilde{H})^\dagger D^\mu \tilde{H} + (D_\mu \tilde{\bar{H}})^\dagger D^\mu \tilde{\bar{H}} + (D_\mu \tilde{N})^\dagger D^\mu \tilde{N}. \tag{5}$$

The first three parts of this expression give mass terms of the gauge fields, and the last three parts describe interactions between the Higgs and the gauge fields. Assuming the

SSB to be homogeneous in space, the mass terms can be written as follows:

$$\mathscr{L}_{\text{mass}} = -\sum_{ij} M_{ij}^2 \Phi_i^\dagger \Phi_j, \tag{6}$$

where $\Phi_i = (W_\mu^1, W_\mu^2, W_\mu^3, B_\mu E_\mu)$ and the matrix M^2 has the block structure:

$$M^2 = \begin{pmatrix} M_W^2 & 0 \\ 0 & M_Z^2 \end{pmatrix}. \tag{7}$$

The matrices M_W^2 and M_Z^2 are given by[1]

$$M_W^2 = \frac{g^2(v^2+\bar{v}^2)}{4}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \tag{8}$$

$$M_Z^2 = \tfrac{1}{4}\begin{pmatrix} g^2(v^2+\bar{v}^2) & -gg'(v^2+\bar{v}^2) & gg'(\tfrac{4}{3}v^2-\tfrac{1}{3}\bar{v}^2) \\ -gg'(v^2+\bar{v}^2) & g'^2(v^2+\bar{v}^2) & g'^2(-\tfrac{4}{3}v^2+\tfrac{1}{3}\bar{v}^2) \\ gg'(\tfrac{4}{3}v^2-\tfrac{1}{3}\bar{v}^2) & g'^2(-\tfrac{4}{3}v^2+\tfrac{1}{3}\bar{v}^2) & g'^2(\tfrac{16}{9}v^2+\tfrac{1}{9}\bar{v}^2+\tfrac{25}{9}x^2) \end{pmatrix}. \tag{9}$$

Diagonalization of M_W^2 gives, as in the Standard Model, the charged bosons $W^\pm$

$$W^\pm = \frac{W^1 \mp iW^2}{\sqrt{2}} \tag{10}$$

with mass

$$m_W^2 = \frac{g^2(v^2+\bar{v}^2)}{2}, \tag{11}$$

while diagonalization of M_Z^2 will give a massless photon

$$A_\mu = \sin\vartheta_W W_\mu^3 + \cos\vartheta_W B_\mu \tag{12}$$

and two neutral bosons Z

$$Z_\mu^1 = \cos\vartheta_E \cos\vartheta_W W_\mu^3 - \cos\vartheta_E \sin\vartheta_W B_\mu - \sin\vartheta_E E_\mu, \tag{13}$$

$$Z_\mu^2 = \sin\vartheta_E \cos\vartheta_W W_\mu^3 - \sin\vartheta_E \sin\vartheta_W B_\mu + \cos{}_E E_\mu. \tag{14}$$

ϑ_W is the Weinberg angle, $\tg\vartheta_W = g'/g$, whereas ϑ_E is defined by

$$\tg 2\vartheta_E = \frac{2a}{b-1}, \tag{15}$$

where

$$a = \tfrac{1}{3}\frac{g'}{\sqrt{g^2+g'^2}}\frac{4v^2-\bar{v}^2}{v^2+\bar{v}^2},$$

$$b = \tfrac{1}{9}\frac{g'^2}{g^2+g'^2}\frac{16v^2+\bar{v}^2+25x^2}{v^2+\bar{v}^2}.$$

[1] Assuming the coupling constants being equal for both groups U(1): $g' = g''$ [8].

The mass squares of bosons Z^1 and Z^2, which are non-zero eigenvalues of M_Z^2, are:

$$m_{z_1}^2 = \tfrac{1}{2} m_{z_0}^2 [b+1- \sqrt{(b-1)^2+4a^2}],$$

$$m_{z_2}^2 = \tfrac{1}{2} m_{z_0}^2 [b+1+ \sqrt{(b-1)^2+4a^2}], \tag{16}$$

where $m_Z^2 = \tfrac{1}{2}(g^2+g'^2)(v^2+\bar{v}^2)$; a and b are the same as in Eq. (15).

Fig. 1 shows the dependence of m_{Z_1} and m_{Z_2} on the ratio x/v for $v = \bar{v}$. With the increasing ratio x/v, m_{Z_1} approaches the mass of the Standard Model boson Z^0, whereas m_{Z_2} tends to infinity. Thus for large enough x/v, only one neutral boson Z, with the mass approximate to the mass of Z^0, ought to be found in experiments.
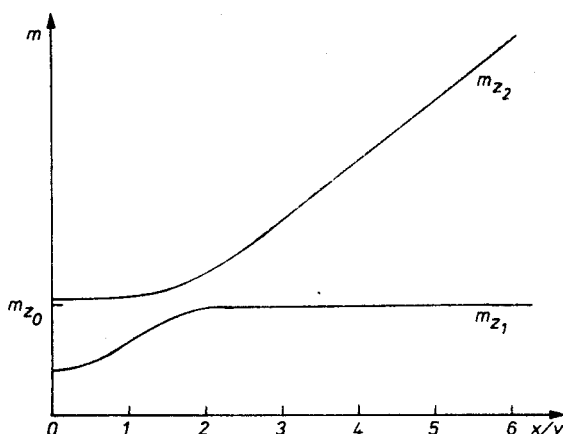

Fig. 1. Dependence of the masses of bosons Z^1 and Z^2 on the ratio x/v

3. Potential

The SSB occurs due to the specific shape of the potential, the one with its minimum for non-zero VEVs of the Higgs fields. The part of the potential generating the $SU(2)_L \times U(1)_Y \times U(1)_F$ symmetry breaking contains the $H, \bar{H}, N$ fields only and is given by [8]

$$V(H, \bar{H}, N) = m_H^2|H|^2 + m_{\bar{H}}^2|\bar{H}|^2 + m_N^2|N|^2 + \lambda^2|H\bar{H}|^2$$

$$+ \lambda^2|N|^2(|H|^2+|\bar{H}|^2) + \lambda a(NH\bar{H}+\text{h.c.}). \tag{17}$$

Parameters $m_H^2, m_{\bar{H}}^2, m_N^2, \lambda, a$ are to be fitted so that the potential should have its minimum for the fields $H, \bar{H}$ and N defined by (2). The most general pattern of the Higgs fields condensation is

$$\langle H \rangle = \begin{pmatrix} v^+ \\ v \end{pmatrix}, \quad \langle \bar{H} \rangle = \begin{pmatrix} \bar{v} \\ \bar{v}^- \end{pmatrix}, \quad \langle N \rangle = x. \tag{18}$$

Assuming for simplification $m_H = m_{\bar{H}}$, we find $v = \hat{v}$ and $v^+ = \bar{v}^-$. Hence (18) takes the form

$$\langle H \rangle = \begin{pmatrix} v_c \\ v \end{pmatrix}, \quad \langle \bar{H} \rangle = \begin{pmatrix} v \\ v_c \end{pmatrix}, \quad \langle N \rangle = x, \qquad (19)$$

where $v_c = v^+ = \bar{v}^-$ describes the condensation of charged higgs. So the potential (17) can be rewritten as

$$V(v, v_c, x) = 2m_H^2 v^2 + 2m_H^2 v_c^2 + m_N^2 x^2 + \lambda^2 v^4 - 2\lambda^2 v^2 v_c^2 + \lambda^2 v_c^4$$
$$+ 2\lambda^2 x^2 v^2 + 2\lambda^2 x^2 v_c^2 + 2\lambda a x v^2 - 2\lambda a x v_c^2. \qquad (20)$$

To get the minimum of this potential at the point defined by (2), i.e. at point $(v, v_c = 0, x)$, the parameters of the potential must satisfy the conditions:

$$\lambda = \left(\frac{m_H^2 v^2 - m_N^2 x^2}{x^2 v^2 - v^4} \right)^{\frac{1}{2}}.$$

$$a = -\frac{1}{\lambda x} (m_H^2 + \lambda^2 v^2 + \lambda^2 x^2),$$

$$\frac{m_H}{m_N} > \left(\frac{x}{v} \right)^2 > 1. \qquad (21)$$

The value of x/v ought to be sufficiently large. This condition is based on the dependence of the masses of bosons Z^1 and Z^2 on the ratio x/v (see Eq. (16), Fig. 1). For our numerical analysis we took $x/v = 5$. For the mass of boson W — calculated by formula (11) — to be in good agreement with experimental data, we must assume $v = 126$ GeV and $x = 634$ GeV. Hence $m_{z_1} = 94.7$ GeV and $m_{z_2} = 265.6$ GeV.

Describing the case of homogeneous vacuum we must take into consideration the filds which represent the quantum fluctuations around the minimum of the potential, i.e. the fields $\tilde{H}, \tilde{\bar{H}}, \tilde{N}$ from formula (4). The fields $\tilde{H}$ and $\tilde{\bar{H}}$ can be written as

$$\tilde{H} = \begin{pmatrix} \tilde{H}^+ \\ \tilde{H}^0 \end{pmatrix}, \quad \tilde{\bar{H}} = \begin{pmatrix} \tilde{\bar{H}}^0 \\ \tilde{\bar{H}}^- \end{pmatrix},$$

so, in agreement with (2) and (4), we get

$$\tilde{H}^0 = H^0 - v, \quad \tilde{\bar{H}}^0 = \bar{H}^0 - v, \quad \tilde{N} = N - x, \quad \tilde{H}^+ = H^+, \quad \tilde{\bar{H}}^- = \bar{H}^-.$$

After introducing the notation $\Phi_i = (\tilde{H}^0, \tilde{\bar{H}}^0, \tilde{N}, \tilde{H}^+, \tilde{\bar{H}}^-)$, the expansion of the potential (17) around the minimum will take the form

$$V(\Phi_i) = V(\Phi_i = 0) + \sum_i \left(\frac{\partial V}{\partial \Phi_i} \Phi_i + \frac{\partial V}{\partial \Phi_i^\dagger} \Phi_i^\dagger \right) + \sum_{ij} \frac{\partial^2 V}{\partial \Phi_i^\dagger \partial \Phi_j} \Phi_i^\dagger \Phi_j. \qquad (22)$$

Since the first two terms vanish, the expansion (22) can be rewritten as

$$V(\Phi_i) = \sum_{ij} \overline{M}_{ij}^2 \Phi_i^+ \Phi_j, \qquad (23)$$

where

$$\overline{M}_{ij}^2 = \left. \frac{\partial^2 V}{\partial \Phi_i^+ \partial \Phi_j} \right|_{\Phi_k = 0}.$$

The mass matrix $\overline{M}^2$ has the structure

$$\overline{M}^2 = \begin{pmatrix} M^2 & 0 & 0 \\ 0 & m_H^2 + \lambda^2 x^2 & 0 \\ 0 & 0 & m_H^2 + \lambda^2 x^2 \end{pmatrix}, \qquad (24)$$

where M^2 is given by

$$M^2 = \begin{pmatrix} -\lambda ax & 4\lambda^2 v^2 + 2\lambda ax & 4\lambda^2 vx + 2\lambda av \\ 4\lambda^2 v^2 + 2\lambda ax & -\lambda ax & 4\lambda^2 vx + 2\lambda av \\ 4\lambda^2 vx + 2\lambda av & 4\lambda^2 vx + 2\lambda av & -\lambda a \dfrac{v^2}{x} \end{pmatrix}. \qquad (25)$$

The diagonalization of M^2 gives neutral, physical fields Ω_1, Ω_2, Ω_3 as a linear combination of the fields $\tilde{H}^0$, $\tilde{\tilde{H}}^0$, $\tilde{N}$. The masses of these fields (their squares are eigenvalues of M^2) are:

$$m_{\Omega_1} = 10.0 \text{ TeV}, \quad m_{\Omega_2} = 1.3 \text{ TeV}, \quad m_{\Omega_3} = 1.5 \text{ TeV}.$$

The masses of the two charged fields are equal 7.1 TeV each.

4. Non-homogeneous vacuum: a cosmic string

Till now it has been assumed that the condensation of the Higgs fields is defined by the minimum of potential only. But SSB can also generate such a field configuration in which a minimalization of potential in the whole space will be impossible. In that case the regions of high energy density (domain walls, strings, monopoles) appear. The SSB leads to non-zero VEV of Higgs field ϕ, which can be written as $\langle\phi\rangle = \sigma \exp(i\vartheta)$. σ is fixed by the minimum of potential, while the phase ϑ remains arbitrary. When ϑ changes by $2\pi n$ around a closed loop, where n is a non-zero integer, a cosmic string, i.e. a linear topological defect with $\langle\phi\rangle = 0$, must exist. n is called a ,,winding number". The cosmic string, which arises during SSB in a model with gauge symmetry U(1), has been described by Nielsen and Olesen [9].

The aim of this chapter is to present a cosmic string arising during SSB in an extra-U(1) model:

$$SU(2)_L \times U(1)_Y \times U(1)_E \rightarrow U(1)_{EM}.$$

Like in the U(1) model, we can obtain a profile of the string from Euler-Lagrange equations for string forming fields. To get a neutral vacuum only the neutral Higgs fields can condensate, i.e. only the fields H^0, $\bar{H}^0$, N can form the string. Also the gauge field E_μ can get a non-zero VEV. The other gauge fields cannot condensate, because that would violate the conservation of the electric charge.

The Euler-Lagrange equations have the form:

$$\partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi_i^\dagger)} = \frac{\partial \mathscr{L}}{\partial \phi_i^\dagger}, \qquad (26)$$

where $\phi_i = H^0, \bar{H}^0, N, E_\mu$,

$$\mathscr{L}(H, \bar{H}, N) = (D_\mu H)^\dagger D^\mu H + (D_\mu \bar{H})^\dagger D^\mu \bar{H} + (D_\mu N)^\dagger D^\mu N$$

$$-\tfrac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \tfrac{1}{4} E_{\mu\nu} E^{\mu\nu} - V(H, \bar{H}, N),$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\varepsilon_{ijk} W_\mu^j W_\nu^k,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad E_{\mu\nu} = \partial_\mu E_\nu - \partial_\nu E_\mu.$$

Under the assumption that the modulus of VEV depends on the distance from the centre of the string only, we can write — in cylindrical coordinates (r, φ, z) —

$$|\langle \phi_i \rangle| = \sigma^i(r), \qquad \phi_i = H^0, \bar{H}^0, N. \qquad (27)$$

According to the last column in Table I, the simplest solutions in the cylindrical coordinates, securing a non-zero winding number are

$$\langle H^0 \rangle = \sigma^1(r)e^{-i4n\varphi}, \qquad \langle \bar{H}^0 \rangle = \sigma^2(r)e^{-in\varphi}, \qquad \langle N \rangle = \sigma^3(r)e^{+i5n\varphi}. \qquad (28)$$

The string is formed for $n \neq 0$ only. We shall take $n = 1$ for the forthcoming analysis.

Let us assume, similarly to Nielsen and Olesen [9], that only the angular component of the gauge field E_μ is non-zero (see Fig. 2) and it depends only on r. We denote this component as $\alpha(r)$.
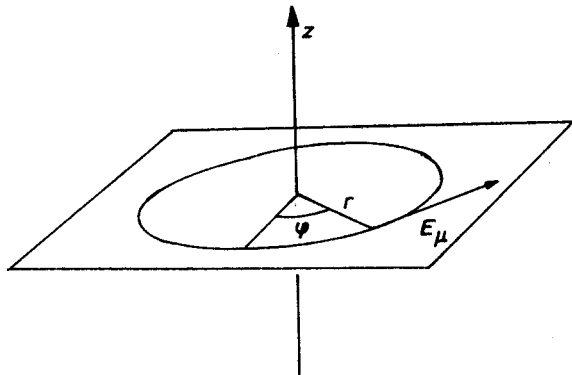

Fig. 2. The Ansatz for the gauge field E_μ in the neighbourhood of a cosmic string. The string is placed along the z axis

Substituting solutions (28) into equations (27) we get:

$$\frac{d^2\sigma^1}{dr^2} = -\frac{1}{r}\frac{d\sigma^1}{dr} + \left\{\left(\frac{4}{r} + \tfrac{1}{3}g''\alpha\right)^2 + m_H^2 + \lambda^2[(\sigma^2)^2 + (\sigma^3)^2]\right\}\sigma^1 + \lambda a\sigma^3\sigma^3,$$

$$\frac{d^2\sigma^2}{dr^2} = -\frac{1}{r}\frac{d\sigma^2}{dr} = \left\{\left(\frac{1}{r} + b\alpha\right)^2 + m_H^2 + \lambda^2[(\sigma^1)^2 + (\sigma^3)^2]\right\}\sigma^2 + \lambda a\sigma^1\sigma^3,$$

$$\frac{d^2\sigma^3}{dr^2} = -\frac{1}{r}\frac{d\sigma^3}{dr} + \left\{\left(\frac{5}{r} - \tfrac{5}{6}g''\alpha\right)^2 + m_N^2 + \lambda^2[(\sigma^1)^2 + (\sigma^2)^2]\right\}\sigma^3 + \lambda a\sigma^1\sigma^2,$$

$$\frac{d^2\alpha}{dr^2} = -\frac{1}{r}\frac{d\alpha}{dr} + \frac{\alpha}{r^2} + \tfrac{1}{3}g''\left(\tfrac{1}{6}g''\alpha - \frac{1}{r}\right)[16(\sigma^1)^2 + (\sigma^2)^2 + 25(\sigma^3)^2], \qquad (29)$$

where $b = g\cos\vartheta_W\cos^2\vartheta_E - g'\sin\vartheta_F + 4/3g''$.

Solutions of these equations tend to homogeneous solutions, securing the minimalization of free energy density, when $r \to \infty$:

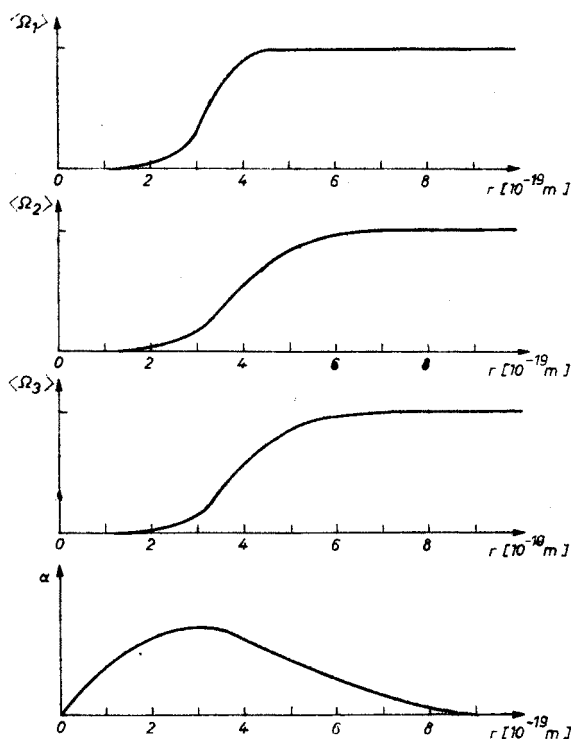$$\sigma^1 \to v, \quad \sigma^2 \to v, \quad \sigma^3 \to x, \quad \alpha \to 0.$$

Fig. 3. The behaviour of the fieds $\Omega_1, \Omega_2, \Omega_3$ and the gauge field E_μ near the centre of the string

Near the centre of the string the terms proportional to r^{-2} and r^{-1} are dominant in Eqs. (29); so we get approximate equations:

$$\frac{d^2\sigma^1}{dr^2} = -\frac{1}{r}\frac{d\sigma^1}{dr} + \frac{16}{r^2}\sigma^1, \quad \frac{d^2\sigma^2}{dr^2} = -\frac{1}{r}\frac{d\sigma^2}{dr} + \frac{1}{r^2}\sigma^2,$$

$$\frac{d^2\sigma^3}{dr^2} = -\frac{1}{r}\frac{d\sigma^3}{dr} + \frac{25}{r^2}\sigma^3, \quad \frac{d^2\alpha}{dr^2} = -\frac{1}{r}\frac{d\alpha}{dr} + \frac{1}{r^2}\alpha, \tag{30}$$

whose solutions are given by

$$\sigma^1 = A_1 r^4, \quad \sigma^2 = A_2 r, \quad \sigma^3 = A_3 r^5, \quad \alpha = A_4 r,$$

where $A_1 \div A_4$ are arbitrary constants.

We obtain the profile of the string, shown in Fig. 3, by a numerical solution of Eqs. (29). The Higgs fields shown in this figure are represented by physical fields $\Omega_1, \Omega_2, \Omega_3$, obtained as a result of diagonalization of mass matrix of the fields $H^0, \overline{H}^0, N$ (25). As the respective figures show, the Higgs fields decrease near the centre of the string and the rate of the decrease depends on the masses of the fields. The most massive field Ω_1 forms an internal layer surrounded by layers formed by less massive fields Ω_2 and Ω_3.

5. Conclusions

We have investigated a vacuum in the $SU(2)_L \times U(1)_Y \times U(1)_E$ model. This vacuum may be both homogeneous and — due to he extra $U(1)$ group — non-homogeneous. In the last Section it was shown that the non-homogeneous vacuum can form cosmic strings. Such strings, arising near the electroweak scale, have the mass per unit length only about 10^{-6} g/cm and they cannot be observed through their gravitational interactions. But because, as it was shown by Witten [10], such a string can be superconducting, there is another possibility of observing cosmic string created during the breaking of the $SU(2)_L \times U(1)_Y \times U(1)_E$ symmetry. While passing through the interstellar plasma the magnetic field wrapped around a superconducting string will create a bow shock and synchrotron emission from relativistic particles in the bow shock will be seen [11]. Oscillating strings will produce low-frequency electromagnetic radiation. They can also be sources of cosmic rays, of γ-ray bursts and of many other astrophysical phenomena [12].

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