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A DECUPLET FOR g_T MESONS

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Assuming that g_T mesons belong to a meson decuplet, we predict the masses of its nonisosinglet members. We discuss how the accuracy of the prediction depends on the experimental errors of the masses of g_T mesons. We note, that the present errors are such that the prediction would be drastically improved, if they were slightly reduced. We calculate also the octet contents of g_T mesons as functions of one parameter and conclude that none of them is excluded as glueball candidate, if only g_T masses are known.

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1. *Introduction*

The tensor mesons g_T ($J^{PC} = 2^{++}$) have been discovered [1] in the reaction

$$\pi^- p \Rightarrow g_T n \Rightarrow \phi\phi n. \tag{1}$$

Actually three g_T mesons are known. We denote them as g_1, g_2, g_3 whenever we want to distinguish a particular g_T. Their masses are quoted in the Table I following Ref. [2]. The authors of the experiment claim that these mesons (or at least one of them) are glueballs [3, 4]. However, the original experiment [1] still remains the only one in which the g_T mesons were observed; the signals seen in other experiments [5–8] cannot be unambiguously assigned to any of these mesons.

The statement that the observation of g_T mesons in reaction (1) constitutes a discover of the glueball(s) raised discussion which was centered mainly around two questions:y
1) are all g_T signals genuine resonances or, possibly, some of them (especially g_T^1) reflect kinematical effects [9],
2) is it justified to interprete g_T mesons as glueballs [10] and, in particular, to expect that all g_T mesons are glueballs [11, 4]; in opinion of several authors at least one of them could

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be the state of different structure: (*i*) 4-quark $qq\bar{q}\bar{q}$ [12, 13], (*ii*) meikton $q\bar{q}g$ [13, 14], (*iii*) excited $q\bar{q}$ [15].

In the present paper all g_T signals are assumed to be genuine resonances. However, they are regarded not as the pure glueballs (i.e. SU(3) singlets), but rather as the mixed states belonging to a reducible SU(3) decuplet ($8\oplus 1\oplus 1$) of mesons [16]. This multiplet would therefore resemble the decuplet of tensor mesons including $f_2(1720)$ and the well known nonet ($a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, $f_2(1525)$) [17]. A decuplet is a good place to look for a glueball [18]. The relation of g_T mesons to glueball will be supported if all these mesons belong to one decuplet. The decuplet itself (defined within the exotic commutator model [19]) is determined by the masses of its members [16]. Unfortunately, the isotriplet and isodublet partners of the decuplet are not known (perhaps apart from a signal of K_2^* with unknown mass [20]). We predict their masses and discuss possible octet-singlet-glueball contents of g_T mesons. However we do not make any assumption concerning the internal structure of the nine non-glueball members of the decuplet; they may be arbitrary states constituing the nonet, like meiktons or somehow excited $q\bar{q}$ states. In particular, we do not assign any definite internal structures to the octet (g_8) and singlet (g_0) states which mix together with glueball (G) (we use only its property of being singlet) to produce the observed g_T states.

2. *The masses of isotriplet and isodublet mesons*

We have at our disposal one mass formula and few inequalities to determine the unknown masses of isotriplet and isodublet mesons. These constraints do not fix a unique solution, but they may strongly restrict the quantities we are looking for.

It is convenient to put the mass formula for decuplet [16] in the form

$$F(a)+2F(b) = 0. \tag{2}$$

Here a is the mass squared of the isotriplet meson, b is defined [19] as

$$b = 2K-a, \tag{3}$$

$F(x)$ is characteristic polynomial of the mass matrix,

$$F(x) = (x-g_1)(x-g_2)(x-g_3), \tag{4}$$

and the particle symbols stand for their mass squared. The unknown quantities in (2) are a and b.

The masses of the decuplet mesons are constrained by inequalities [16]:

$$g_1 < a < g_2 < b < g_3. \tag{5}$$

Consequently, we find from Table I (ignoring experimental errors)

$$(2.297\ \mathrm{GeV}/c^2)^2 < b < (2.339\ \mathrm{GeV}/c^2)^2. \tag{6}$$

TABLE I

g_T mesons and their masses [2]

Particle	g_1	g_2	g_3
Mass (GeV/c^2)	$2.011^{+0.062}_{-0.076}$	$2.297^{+0.028}_{-0.028}$	$2.339^{+0.055}_{-0.055}$

If b is restricted by this condition, then the mass formula (2) drastically reduces the interval (5) of allowed values of a. This can be easily seen in Fig. 1 representing (2) graphically. There are two regions, A and B, of possible values of a consistent with (5). These two regions determine two solutions.

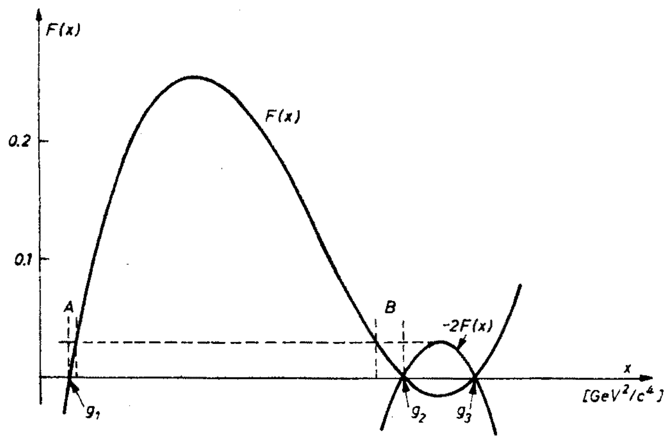

Fig. 1. The characteristic polynomial of the mass matrix $F(x)$. Two regions, A and B, of the values of a (the mass squared of the isotriplet meson) correspond to the allowed values of b (see text).

Solution A

$$g_2+g_1 < b+a < g_3+g_1,$$
$$g_2-g_1 < b-a < g_3-g_1. \tag{7}$$

Hence, taking into account (3),

$$(2.011\ \mathrm{GeV}/c^2)^2 < a < (2.015\ \mathrm{GeV}/c^2)^2,$$
$$(2.159\ \mathrm{GeV}/c^2)^2 < K < (2.183\ \mathrm{GeV}/c^2)^2,$$
$$0.616\,(\mathrm{GeV}/c^2)^2 < K-a < 0.713\,(\mathrm{GeV}/c^2)^2. \tag{8}$$

Solution B

$$2g_2 \lesssim b+a < g_3+g_2,$$
$$0 < b-a < g_3-g_2 \tag{9}$$

and

$$(2.279\ \mathrm{GeV}/c^2)^2 < a < (2.297\ \mathrm{GeV}/c^2)^2,$$
$$(2.297\ \mathrm{GeV}/c^2)^2 \lesssim K < (2.318\ \mathrm{GeV}/c^2)^2,$$
$$0 < K - a < 0.098\ (\mathrm{GeV}/c^2)^2. \tag{10}$$

The solution A predicts a large mase splitting between "pion" and "kaon", while the solution B predicts a small one. The values are well separated and, in each solution, strongly restricted. Also a and K are strongly restricted in both solutions. These properties follow from the narrowness of the mass gap (6).

3. *The octet contents of g_T mesons*

The octet contents of g_T mesons [16] can be expressed in the form:

$$\lambda_1^2 = \frac{1}{3}\frac{(a-g_2)(a-g_3)+2(b-g_2)(b-g_3)}{(g_1-g_2)(g_1-g_3)},$$
$$\lambda_2^2 = \frac{1}{3}\frac{(a-g_1)(a-g_3)+2(b-g_1)(b-g_3)}{(g_2-g_1)(g_2-g_3)},$$
$$\lambda_3^2 = \frac{1}{3}\frac{(a-g_1)(a-g_2)+2(b-g_1)(b-g_2)}{(g_3-g_1)(g_3-g_2)}, \tag{11}$$

where $\sum \lambda_j^2 = 1$.
The quantities λ_j^2 are determined uniquely, if all masses of the decuplet mesons are known. In the present case a is a function of b (via(2)) within regions A and B, and therefore also λ_j^2 are functions of b. These functions are plotted in the Figs. 2 and 3, for the solutions A and B, respectively. Due to narrowness of the interval (6) the magnitude of λ_1^2 is almost totally independent of b, and in each solution, the octet content of g_1 meson has definite value. In contrast, λ_2^2 and λ_3^2 are not determined by the masses of g_T mesons and are rapidly changing functions of b.

The $g_8 - g_0 - G$ compositions of g_T states are determined by the elements of mixing matrix. This matrix can be parametrized by Euler angles $\theta_j (j = 1, 2, 3)$ [16], where θ is determined by λ_1, θ_3 is determined by λ_2 (and λ_3) (if θ_1 is already known). Therefore θ_1 has definite value in the solutions A and B, but θ_3 remains unknown. The θ_2 is also unknown, since it is not expressible by masses [16]. Thus, the compositions of g_T mesons (and, in particular, the amplitudes of their glueball components) depend on two unknown parameters: θ_2 and θ_3.

A general feature of a decouplet is that g_2 (or g_3) meson is mostly in octet state, if b is close to its mass squared (see Figs. 2 and 3). In contrast, g_1 meson is always mostly in singlet state ($0 < \lambda_1^2 < 1/3$) [18]. The solution A predicts $\lambda_1^2 \approx 1/3$, i.e. it attributes to g_1 the octet content of the less massive isoscalar of ideally mixed quark nonet. The solution B predicts $\lambda_1^2 \simeq 0$ (< 0.0006), assigning g_1 as a singlet (in general, a mixed state

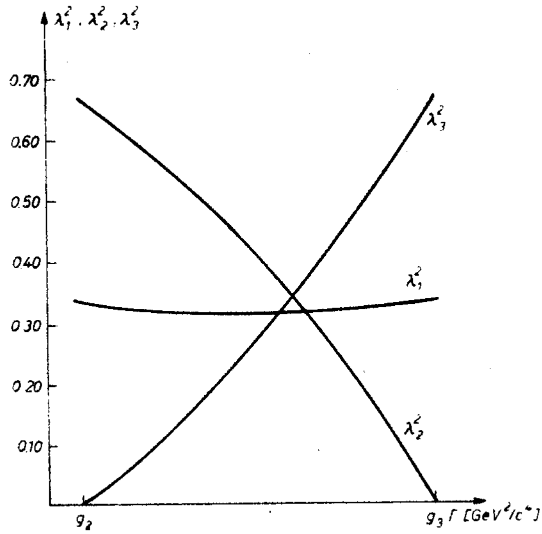

Fig. 2. The octet contents λ_j^2 of g_j states as the functions of b. Solution A

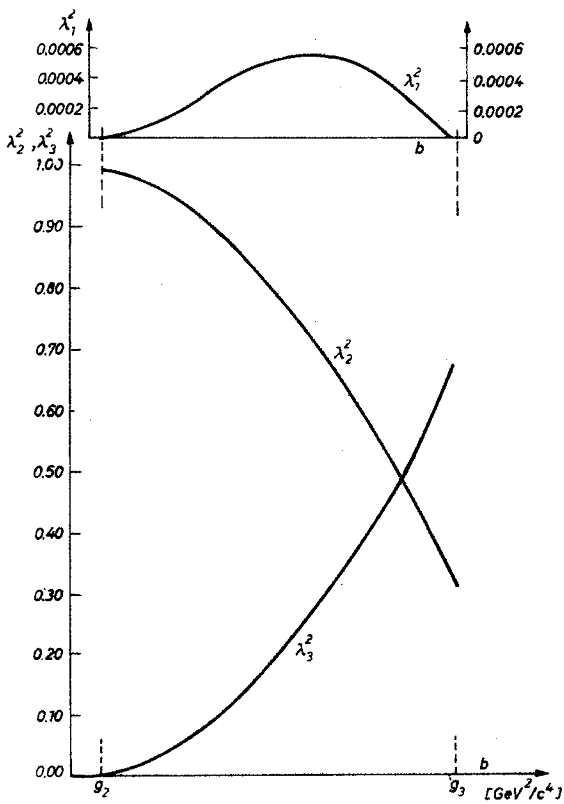

Fig. 3. The octet contents λ_j^2 of g_j states as the functions of b. Solution B

of g_0 and G). Although λ_2^2 and λ_3^2 are not determined, a choice of definite value of λ_1^2 restricts them to some extent. As it is seen from Figs. 2 and 3, the ranges of values of λ_2^2 in the solutions A and B are different and smaller than it is generally allowed ($0 < \lambda_2^2 < 1$) [18]. Therefore these solutions predict different properties of g_2 and g_3 mesons. Let us note some posibilities.

Solution A

— the decouplet may split up into a "pseudoideal nonet" [18] and a singlet $g_2(g_3)$ (at $b = g_3(g_2)$); in particular, the singlet may be the glueball,

— g_1, g_2, g_3 may contain g_8 at almost equal rate.

Solution B

— g_2 and g_3 may belong (at $b = g_3$) to a "pseudoideal nonet"

— g_2 may be a pure octet state ("full degeneracy" [18]: $b = g_2 = a = K$); then g_3 is a singlet and may be the glueball,

— g_2 and g_3 may have hidden flavour reversed (for $q\bar{q}$ mesons this would mean that the $s\bar{s}$ state is the less massive isoscalar of the nonet [18]); such a state may arise, if $\lambda_2^2 \simeq 2/3$, $\lambda_3^2 \simeq 1/3$. We also notice that the state g_1 — in the case A, and g_2 — in the case B cannot be pure (or almost pure) glueball. We can thus conclude:

1° the information on masses of g_T mesons is not sufficient to exclude (such information cannot confirm [18]) any of them as possible candidate for pure glueball (the more, as the mixed state),

2° the choice of one solution (A or B) excludes some options for g_T states; on the other hand, the additional information about the properties of g_T states could help in some cases in making choice between the solutions.

4. *The effect of experimental errors*

The solutions A and B ((8) and (10)) correspond to central values of experimentally determined masses of g_T mesons (Table I). The regions (8) and (10) of predicted masses of a and K mesons are so small that the prediction is practically of the same value as the exact one. Let us now discuss how these regions change, if the experimental errors of g_T masses shown in Table I are taken into account.

As it is seen from Fig. 1, the constraints imposed on a by the mass formula (2) become weaker, when the maxima of the right and left parts of the curve approach each other. This happens, when the ratio

$$k = (g_3 - g_2)/(g_2 - g_1) \tag{12}$$

increases. Also λ_1^2 becoming b-dependent like λ_2^2, and λ_3^2 is less restricted (whereas the values of all three λ_j^2 at $b = g_2$ and $b = g_3$ remain unchanged). If the maxima of the left and right parts of the curve are equal, then the regions A and B join together, the mass formula (2) does not work any more and the masses are restricted only by (5).

The quantities a and b should be interchanged in their roles, if the maximum of the left part of the curve is lower than the right one. When it becomes much lower, then for all a obeying (5) the possible values of b are confined to two narrow regions (like A and B) and λ_3^2 becomes approximately constant for both solutions (with $\lambda_3^2 \simeq 2/3$ and $\lambda_3^2 \simeq 0$).

Within the limits of experimental errors of the masses of g_T mesons (see Table I) we can equate the maxima of left and right parts of the curve plotted on Fig. 1 (more precisely, we can even make the right maximum slightly higher). This is achieved for these marginal experimental values of masses, which maximize $k(\simeq 2/3)$ defined in (12), i.e. for

$$g_1 = (2.011+0.062)^2(\text{GeV}/c^2)^2,$$
$$g_2 = (2.297-0.028)^2(\text{GeV}/c^2)^2, \tag{13}$$
$$g_3 = (2.339+0.055)^2(\text{GeV}/c^2)^2.$$

Considering also the lower limit of g_1, as well as the higher limit of g_2, we obtain the following constraints:

$$(1.935\ \text{GeV}/c^2)^2 < a < (2.325\ \text{GeV}/c^2)^2,$$
$$(2.109\ \text{GeV}/c^2)^2 < K < (2.360\ \text{GeV}/c^2)^2, \tag{14}$$
$$0 < K-a < 0.994\ (\text{GeV}/c^2)^2.$$

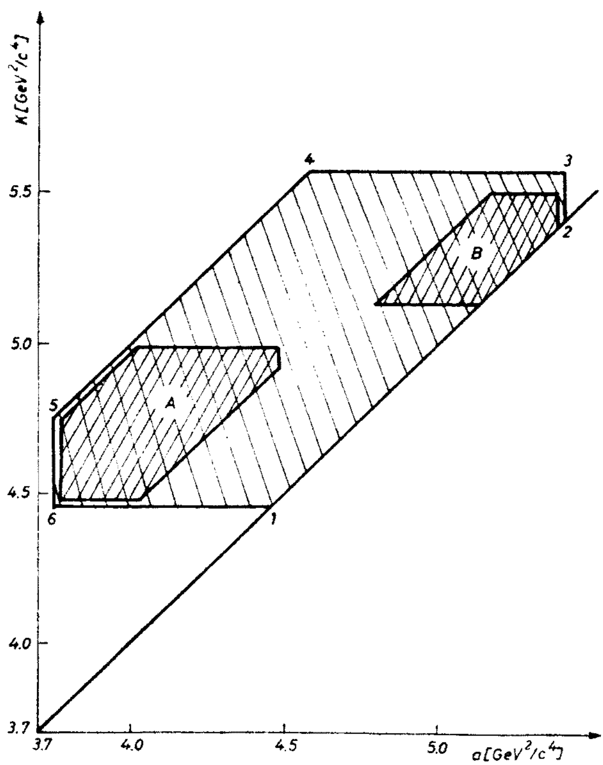

Fig. 4. The predicted values of a and K calculated from data for g_T masses. The polygon 1–6 corresponds to the data with present experimental errors. The crossed domains A and B correspond to the same data with the errors reduced by 10%

where by assumption $b-a > 0$. The values of a and K obeying the inequalities (14) are indicated in Fig. 4 (the hatched domain). The bounds are not very restrictive: at fixed $a(K)$ the allowed values of the masses of $K(a)$ meson belong to the interval which may be as large as 220(240) MeV/c^2. However, Fig. 1 suggests that even a small decrease of the experimental errors should strongly reduce the region of possible values of a and K and divide this region into two parts. Fig. 4 illustrates these effects under assumption that the errors are reduced by 10% (the crossed domains A and B); the ranges of possible values of a and K are reduced more than twice.

5. *Summary and conclusion*

(*i*) A necessary challenge for g_T mesons is to treate the mas isoscalar particles belonging to a meson decuplet. Other members of this decouplet are not observed as yet. We can predict the masses of the unobserved mesons by applying the exotic commutator model and using the masses of g_T mesons as the input data. We find out then, that the prediction is not unique and the predicted masses are distributed over two regions. However, for central values of g_T masses these regions are very small (of the order of ten MeV/c^2) and we get two quite definite solutions.

(*ii*) The octet contents of g_T states (λ_j^2) can be calculated as the functions of one parameter. If only the masses of these particles are known, then none of g_T mesons can be excluded as a candidate for a glueball. To exclude any of them, an additional information is necessary (e.g. the information about the mass of the a meson would enable us to choose between the solutions and, consequently, to eliminate some possibilities.)

(*iii*) The present experimental errors (see Table I) allow large uncertainties for the values of predicted masses. However, even small decrease of the errors indicated in Table I reduces these uncertainties significantly for each mase.

(*iv*) There may exist a few different tensor multiplets in the mass region of g_T mesons. Therefore, the different g_T mesons may belong, in principle, to different multiplets. The confirmation of the decouplet assignment of g_T mesons would make more reliable the possibility that they contain an admixture of the glueball state. It would be thus very useful to measure the masses of g_T mesons with better accuracy. This will appear quite necessary, if a or/and K mesons are observed.

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