MODELLING OF NARROW AIR SHOWERS

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Calculations are made for the distributions of central densities (of electromagnetic particles) by integrating over radii $R \approx 0.4$ m and $R \approx 0.8$ m, and for the spectrum over the total number of particles in a shower, as well as for the distributions of the fractions of the total number of particles in a detector exhibiting maximum count rate setting the thresholds to be $N_c \geqslant 170$ and $N_c \geqslant 625$ particles. Comparison with the experimental data is performed.

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1. Introduction

Experimental results have been published [1] recently on the structure of an Extensive Air Shower (EAS) within a narrow central region about its axis. In this connection the evaluation of three-dimensional nuclear-electromagnetic showers with account of the characteristics of the experimental installation becomes of interest for the analysis of the experimental data. At present numerous models are available for the evaluation of nuclear and electromagnetic air showers [2]; these models provide satisfactory descriptions and interpretations of the experimental data on the hadron spectrum, the muon spectrum, and so on.

In this work a Monte-Carlo program [3] for three-dimensional simulation of nuclear-electromagnetic air showers is applied for evaluating the characteristics of electromagnetic showers; the model is based on the extrapolation of accelerator data to the region of super-high energies.

Calculations are made for the distributions of central densities (of electromagnetic particles) by integrating over radii $R \approx 0.4$ m and $R \approx 0.8$ m, and for the spectrum over the total number of particles in a shower, as well as for the distributions of the fractions of the total number of particles in detector exhibiting maximum count rate setting the threshold to be $N_c \geqslant 170$ and $N_c \geqslant 625$ particles. Comparison with the experimental data is performed.

2. *The main features of the model*

A primary proton enters the standard atmosphere [4] where it undergoes interaction in accordance with the usual exponential law. A logarithmic dependence was assumed for the mean charged particle multiplicity on energy [5], while the Koba–Nielsen formula [6] was taken as the multiplicity distribution. Each interaction act was required to satisfy energy and momentum conservation. Particle production was considered in two regions: of fragmentation and of pionization. The mean multiplicities in these regions were taken from Refs [5] and [7]. The energy spectrum of secondary particles was chosen in accordance with Ref. [8], and the distribution of transverse momenta was considered to be exponential for $P_\perp \leqslant 1$ GeV/c, while for $P_\perp > 1$ GeV/c the enhancement of the transverse momentum was taken into account [9]. The integral energy spectrum of the primary protons was taken to be of the form

$$I(E_p) = 0.856\, E_p^{-1.7} (\mathrm{cm}^{-2}\mathrm{s}^{-1}\,\mathrm{str}^{-1}). \tag{1}$$

In our calculations the difference between the interaction of a proton passing through the atmosphere with the nucleus of an atom making up the air and its interaction with a nucleon was not taken into account, since the main part of the pions initiating electromagnetic cascades lies within the region of $x \approx 0.1 \div 0.4$, owing to the slope of the primary proton spectrum (1) being steep. In this region of x the ratio [5]

$$\frac{1}{\sigma_{\mathrm{air}}^{\mathrm{in}}} \frac{d\sigma_{\mathrm{air}}(x)}{dx} \Big/ \frac{1}{\sigma_p^{\mathrm{in}}} \frac{d\sigma_p(x)}{dx}$$

differs little from unity.

3. *The procedure for shower simulation*

A proton interacting with the nucleus of an atom of the air produces pions (only the production of pions is assumed). In accordance with isotopic invariance, one-third of the pions is considered to be π^0-s, which initiate electromagnetic cascades. The spatial distribution of particles in these cascades is described, in the Greisen approximation, by the Nishimura–Kamata function (NKG) [10]:

$$f(R, R_0, s) = c(s)\,(R/R_0)^{s-2}(1+R/R_0)^{s-4.5}. \tag{2}$$

$R_0 = 95$ m for the observation level of 840 g/cm^2 (altitude of 1700 m),

$$\int f(R, R_0, s) R\,dR = 1.$$

The shower age s, is determined by the formula [10]:

$$s = \frac{3t}{t + 2\ln(E/\beta)}, \tag{3}$$

where $t = T/37.1$ and $\beta = 0.081$ GeV. The number of particles in an electromagnetic shower is given by the expression:

$$N = \frac{0.31}{\sqrt{\ln (E/\beta)}} \exp (t(1-3/2 \ln s)),$$

$$F(N, R, R_0, s) = Nf(R, R_0, s). \qquad (4)$$

The transition effect due to the concrete roof over the experimental installation and the walls of the vessel, containing the scintillator, having a certain thickness [11] was taken into account (R in meters):

for $0.4 \leqslant s \leqslant 1.35$

$$\begin{aligned} F(R) &= 5.1-1.4\,R, && 0 \leqslant R \leqslant 1.5,\\ F(R) &= 3.9-0.6\,R, && 1.5 < R \leqslant 2.5,\\ F(R) &= 3.525-0.45\,R, && 2.5 < R \leqslant 3.5,\\ F(R) &= 3.55-0.4\,R, && 3.5 < R \leqslant 4.5,\\ F(R) &= 2.45-0.2\,R, && 4.5 < R \leqslant 5.5,\\ F(R) &= 2.25-0.15\,R, && 5.5 < R \leqslant 8.5,\\ F(R) &= 0.95; && \end{aligned}$$

for $s < 0.4$ and $s > 1.35$

$$f(R, s) = \frac{5.12 R^{(0.178s-0.51)}}{4.18^s}, \quad 0 \leqslant R \leqslant 4.5,$$

$$f(R, s) = F(R), \quad R > 4.5. \qquad (5)$$

The experimental installation represented a continuous "carpet" of 400 detectors, (0.7×0.7) m^2 each [12]. The readout of each detector is generated in the form of integer numbers N from 0 to 36 (above this value saturation occurs), which are then transformed into the numbers of particles by the formula (M is the number of particles hitting the detector)

$$M = 9 \cdot (1.25)^{N-1}. \qquad (6)$$

For the dispersion of each individual detector an expression was applied that was obtained in studies of real showers [13]:

$$\left(\frac{\sigma}{M}\right)^2 = \frac{1}{M} + 0.0169 + (N-1)^2 (4.10^{-3})^2 + 1/3 \left[\frac{(2-s)0.35}{R}\right]^2, \qquad (7)$$

where the first term takes into account the Poisson density fluctuations, and the other summands are due to dispersion of the thresholds and of the slopes of the characteristics of the logarithmic transformer for each individual detector, as well as to the steps of the transformer being discrete and to the dimensions of the detector itself.

The true number of particles M' hitting the detector (with account of fluctuation (7)) is: $M' = M + \sigma A_1$, where A_1 is a random number generates by the Gaussian distribution.

4. *Results of the simulation*

Simulation of nuclear-electromagnetic cascades in the atmosphere was perfoımed applying the procedure described in Section 3. The distribution of central densities was determined for different integration radii, coıresponding to adding up densities for different numbers of detectors. In Figs 1 and 2 we present the density distributions N_1 (a single detector, integration radius $R \approx 0.4$ m), N_4 (the sum of densities in four detectors exhibiting maximum count rates, $R \approx 0.8$ m) and N_c (spectrum for the total number of particles in the shower). Showers were selected on the basis of a large release of energy within the area with $R \approx 0.4$ m (this corresponds to the selection of a single detector with a high

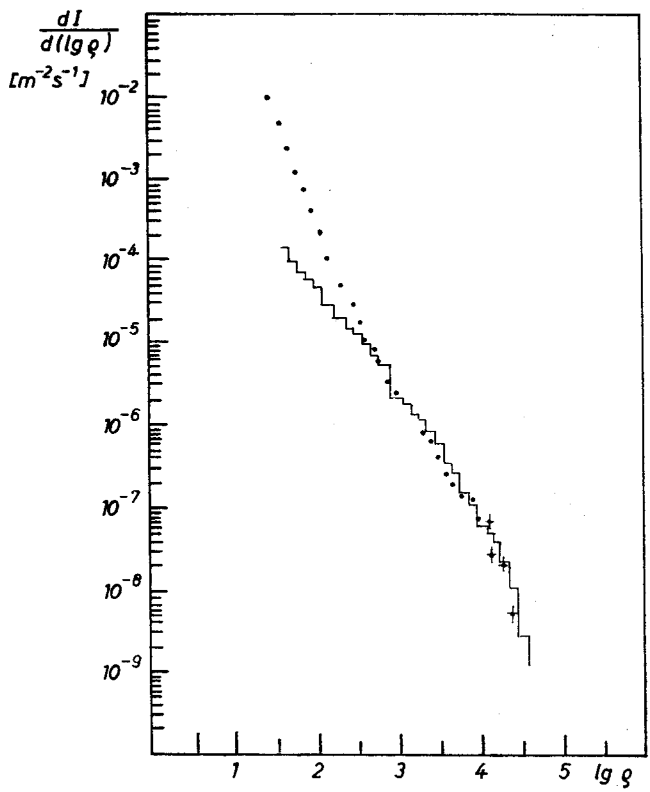

Fig. 1. Differential spectrum of central densities $N_1(R \approx 0.4$ m). Points — data from Ref. [1], histogram — our calculation

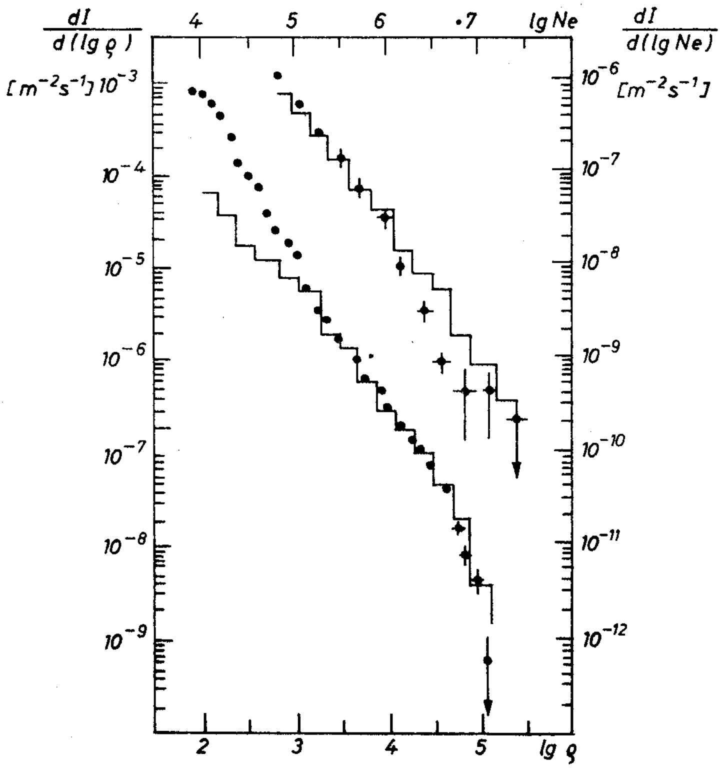

Fig. 2. Differential spectrum of central densities $N_4(R \approx 0.8\text{ m})$ (left scale) and differential spectrum of showers over number of particles (right scale). Points — experimental data from Ref. [1], histogram — our calculation

energy deposit). The thresholds for the energy release were chosen to be the following:

$$6 \leqslant N < 13,$$

$$13 \leqslant N < 17,$$

$$17 \leqslant N < 20,$$

$$20 \leqslant N < 24,$$

$$24 \leqslant N < 29,$$

$$29 \leqslant N < 36,$$

$$N \geqslant 36,$$

where N is determined by formula (6) (where $M \to M'$).

The points in Figs 1 and 2 are experimental ones from Ref. [1] (multiplied by the normalizing factor $1/\lg 1.25$, included in transition from the distribution over ϱ to the distribution over $\lg \varrho$), and the histogram is calculated. The results of calculation are in agreement with the experimental data, with the exception of the region of small lg ϱ. This discrepancy is due to the unsatisfactory performance of the NKG-function (2), used in the present study, at low energies; for this reason a more precise evaluation of electromagnetic showers is required in this region.

In Tables I, II and III calculations are presented for N_1, N_4 and N_c with account of the bend in slope of the integral primary spectrum ($\gamma = 1.7$) at energies $E_0 \geqslant 10^6$ GeV [4] ($\gamma = 2.2$). From these Tables it can be seen that taking into account the bend in the primary spectrum leads to a significant change in the slope of the density distribution and

TABLE I (N_1)

γ	lg ϱ	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5
1.7	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	1.0 10^{-7}	6.0 10^{-8}	5.1 10^{-8}	3.9 10^{-8}	1.7 10^{-8}	1.0 10^{-8}	4.4 10^{-9}	2.0 10^{-9}
2.2	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	8.6 10^{-8}	5.3 10^{-8}	3.6 10^{-8}	1.2 10^{-8}	7.0 10^{-9}	3.3 10^{-9}	3.3 10^{-9}	3.0 10^{-9}

γ	lg ϱ	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3
1.7	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	2.4 10^{-9}	2.6 10^{-9}	1.4 10^{-9}	1.6 10^{-9}	9.0 10^{-10}	5.0 10^{-10}	3.3 10^{-10}	1.6 10^{-10}
2.2	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	2.3 10^{-9}	1.2 10^{-9}	6.9 10^{-10}	6.6 10^{-10}	2.9 10^{-10}	1.5 10^{-10}	1.5 10^{-10}	7.3 10^{-11}

TABLE II (N_4)

γ	lg ϱ	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8
1.7	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	7.9 10^{-8}	4.3 10^{-8}	2.3 10^{-8}	5.5 10^{-9}	2.7 10^{-9}	2.6 10^{-9}	1.0 10^{-9}	2.4 10^{-10}
2.2	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	4.6 10^{-8}	2.0 10^{-8}	4.4 10^{-9}	1.4 10^{-9}	7.2 10^{-10}	6.1 10^{-10}	2.5 10^{-10}	1.5 10^{-10}

TABLE III (N_c)

γ	lg ϱ	5.9	6.1	6.3	6.5	6.7	6.9	7.1	7.3
1.7	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	7.5 10^{-8}	4.4 10^{-8}	2.3 10^{-8}	7.3 10^{-9}	5.2 10^{-9}	1.7 10^{-9}	1.4 10^{-9}	4.2 10^{-10}
2.2	$\frac{dI}{d(\lg \varrho)}$ [m^{-2} s^{-1}]	7.6 10^{-8}	3.8 10^{-8}	1.5 10^{-8}	6.1 10^{-9}	2.2 10^{-9}	1.4 10^{-9}	7.8 10^{-10}	1.1 10^{-10}

in the total number of particles (if the bend is taken to be in the region of $E_0 \geq 5 \cdot 10^6$ GeV, then calculations for $\gamma = 1.7$ and for $\gamma = 2.2$ coincide within the errors for the given intervals of $\lg \varrho$).

In integration over the solid angle in expression (1) the effective solid angle, at which observation was performed with the experimental installation, was taken to be 1.7 str.

In Figs 3 and 4 we present the distributions (normalized to unity) of the fraction of the total number of particles detected by one detector ($k = \varrho_{max}/\varrho_{tot}$) for the thresholds $N_c \geq 170$ and $N_c \geq 625$ (the points are experimental data from Ref. [1], the errors are

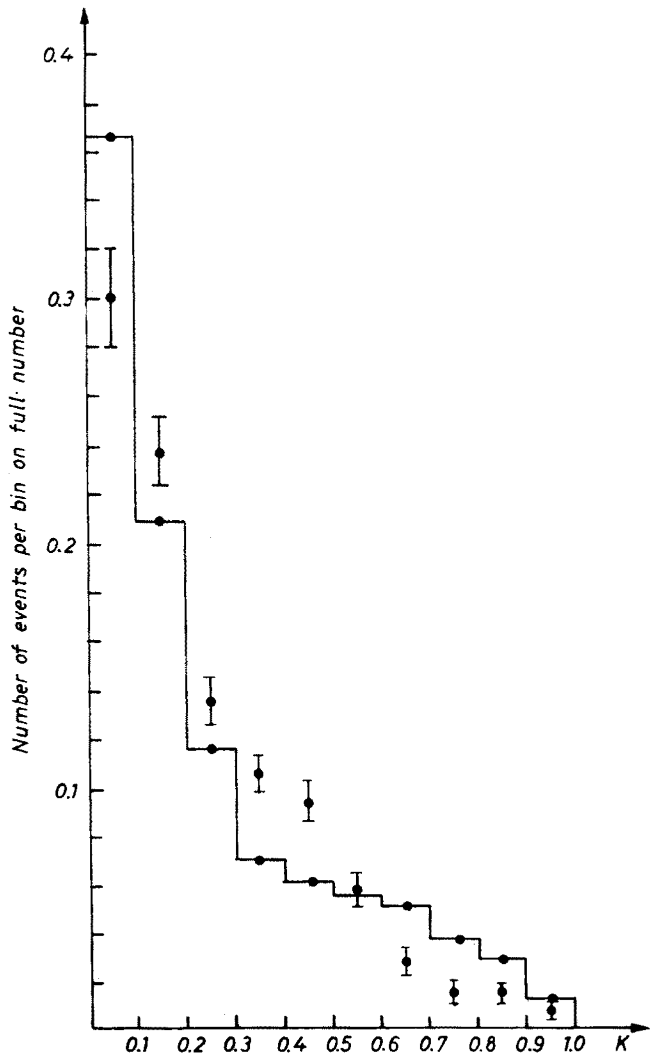

Fig. 3. Distribution of partial number of particles in one detector for a threshold of $N_c \geq 170$ relativistic particles. Points are data from Ref. [1], histogram — our calculation. The distribution is normalized to the total number of particles

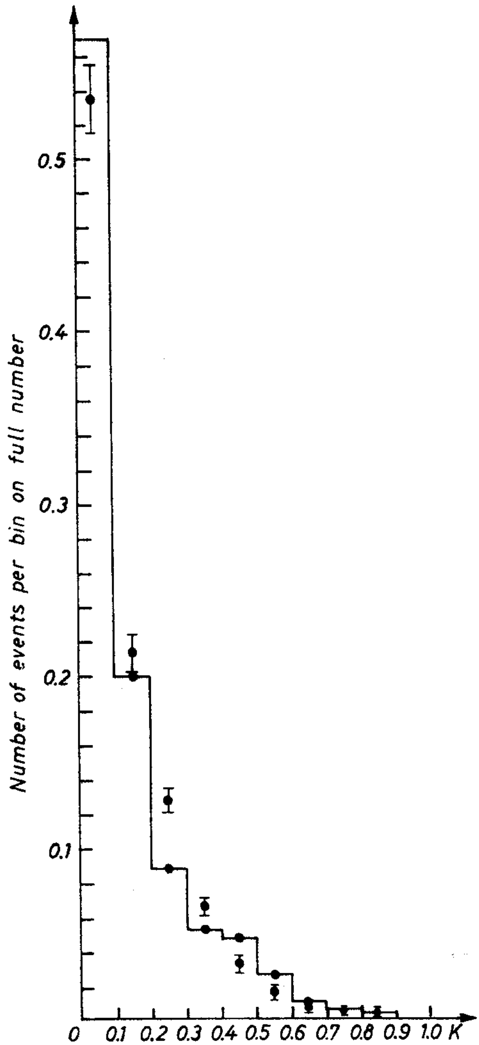

Fig. 4. Distribution of partial number of particles in one detector for a threshold of $N_c \geqslant 625$ relativistic particles. Points — from Ref. [1], histogram is our calculation. The distribution is normalized to the total number of particles

statistical, the histogram is calculated; the statistics used for calculations and the experimental statistics are about the same). As one can see from these figures the calculation is in qualitative agreement with the experimental data. For thresholds $N_c \geqslant 170$, 625 the mean values of shower characteristics (age, height expressed in radiation units, energy) are the following:

	$\bar{s}$	$\bar{H}$	$\bar{E}$ (GeV)
$N_c \geqslant 170$	0.6	4.9	$1.7 \cdot 10^3$
$N_c \geqslant 625$	0.83	10.1	$4.3 \cdot 10^3$

The complete picture obtained at the experimental installation represents a supersposition of a large number of showers and therefore the image corresponding to the mean shower differs quite significantly from a superpositional picture.

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