LETTERS TO THE EDITOR

ON THE ABSENCE OF THE TEMPERATURE PHASE TRANSITION IN FINITE SUPERSYMMETRIC GUT'S

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It is shown that temperature phase transition does not take place in supersymmetric finite GUT's. An example of such theory is given.

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1. It is well-known that temperature phase transitions take place in gauge theories at finite temperature[1]. [1–3]. Some models of the inflationary universe (for review, see [8]) have been constructed on the basis of the proposition about the possibility of the temperature phase transition in realistic GUT in early universe.

In the recent papers [9–14] the finite theories were discussed. It was shown [11] that there are the realistic finite GUT's for which the renormalization group functions of coupling constants and masses are equal to zero.

In the present paper we consider the supersymmetric finite theories at finite temperature. It is shown that temperature phase transition does not take place in supersymmetric finite theories. Suppose that the very early Universe is described by the realistic finite supersymmetric GUT. Then our results show that in this universe the inflationary epoch based on the temperature phase transition is absent.

2. Let us consider the SU(2) gauge theory [15] with the Lagrangian:

$$L = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu)^2 + \frac{1}{2}(\partial_\mu \varphi^a + g\varepsilon^{acb} A^c_\mu \varphi^b)^2$$
$$- \frac{f}{4!}(\varphi^a\varphi^a)^2 + i\bar{\psi}^a \gamma^\mu D^{ab}_\mu \psi^b - ih\varepsilon^{acb}\bar{\psi}^a \varphi^c \psi^b - \frac{m^2}{2}\varphi^a\varphi^a, \quad (1)$$

where $a = 1, 2, 3$, φ^a, ψ^a, A^a_μ are the scalars, spinors and gauge fields, respectively (see [15]).

[1] The gauge theories at finite temperature have been discussed recently in Refs. [4–7].

The renormalization group function γ_m for the scalar field mass in theory (1) was obtained in Ref. [16]:

$$\gamma_m = \frac{1}{(4\pi)^2}\left(12g^2 - \tfrac{5}{3}f - 8h^2\right). \tag{2}$$

Let us consider the theory with the Lagrangian (1) at finite temperature. The SU(2) symmetry of the theory is assumed to be spontaneously broken at zero temperature so that $m^2 < 0$. We introduce an effective potential at finite temperature $\Phi_\beta(\phi^2)$, where ϕ^a is the background scalar, β is the inverse temperature.

The critical temperature β_c for the theory is defined by the well-known condition [1–3]:

$$m^2(\beta) \equiv 2\left.\frac{\partial V_\beta(\phi^2)}{\partial \phi^2}\right|_{\phi^2=0} = 0, \tag{3}$$

where $\phi^2 = \phi^a\phi^a$. Above the critical temperature the symmetry is restored and when $\beta > \beta_c$ the symmetry is broken.

We will use the gauge;

$$L_{GF} = -\tfrac{1}{2}(\partial_\mu A^{a\mu})^2.$$

The straightforward calculation for the one-loop effective potential gives (for scalar QED see, also [6])

$$V_\beta = \frac{m^2+\delta m^2}{2}\phi^2 + \frac{f+\delta f}{4!}\phi^4$$

$$+\frac{ih}{2}\int [dk]\ln\left[(-k^2+g^2\phi^2)^6\left(k^2-\frac{f\phi^2}{2}\right)\left(k^2-\frac{f\phi^2}{12}\right.\right.$$

$$\left.\left.-\frac{\phi^2}{2}\sqrt{\frac{f^2}{36}-\tfrac{2}{3}fg^2}\right)^2\left(k^2-\frac{f\phi^2}{12}+\frac{\phi^2}{2}\sqrt{\frac{f^2}{36}-\tfrac{2}{3}fg^2}\right)^2(-k^2+h^2\phi^4)^{-4}\right]. \tag{4}$$

In the expression (4) we omit the ϕ^2-independent terms (such as $\ln k^2$). Using Eqs. (3), (4) it is evident that in the high temperature limit

$$\beta_c^2 = -\frac{1}{12m^2}\left(6g^2 - 4h^2 - \tfrac{5}{6}f\right). \tag{5}$$

We compare Eqs. (2) and (5) and get (in one-loop approximation and in high temperature limit)

$$\beta_c^2 = -\frac{2\pi^2}{3m^2}\gamma_m. \tag{6}$$

It is easy to prove the relation (6) for arbitrary gauge theory with one scalar multiplet and scalar mass term like as $m^2\varphi^i\varphi^i$. One can generalize the relation (6) for the gauge theory with some scalar multiplets (tensor scalar mass). In the next Section we will use the relation $\beta = -\text{const}\,\frac{\gamma_m}{m^2}$.

3. It is known [9–13] that there are the supersymmetric finite theories for which $\gamma_m = 0$. If such finite theory contains only one scalar multiplet and mass term is as mentioned above, then

$$\beta_c^2 = 0. \tag{7}$$

Thus, the symmetry of the theory under consideration does not change at any temperature. For example, if the symmetry is broken at zero temperature then the symmetry is broken at any temperature.

Let us give the example of the supersymmetric finite GUT for which the temperature phase transition is absent. It is $N = 1$ supergauge theory with the Lagrangian [13]:

$$\begin{aligned} L = & -\tfrac{1}{4} F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\lambda}^a\bar{\delta}^\mu D_\mu\lambda^a + i\bar{\psi}_i\bar{\delta}^\mu D_\mu\psi_i \\ & + (D^\mu\phi_i)^* D_\mu\phi_i + \sqrt{2}\, g(\phi_i^* T^a_{ij}\psi_j\lambda^a + \bar{\lambda}^a\bar{\psi}_i T^a_{ij}\phi_j) \\ & -\tfrac{1}{2}(d_{ijk}\psi_i\psi_j\phi_k + d^*_{ijk}\bar{\psi}_i\bar{\psi}_j\phi^*_k) \\ & -\tfrac{1}{2} g^2(\phi_i^* T^a_{ij}\phi_j)(\phi_k^* T^a_{kl}\phi_l) - \tfrac{1}{4}(d^*_{nik}\phi_i^*\phi_k^*)(d_{nje}\phi_j\phi_e) \\ & -\tfrac{1}{2}(m\psi_i\psi_i + m^*\bar{\psi}_i\bar{\psi}_i) - mm^*\phi_i^*\phi_i - \tfrac{1}{2}(d_{ijk}m^*\phi_i\phi_j\phi_l^* + d^*_{ijk}\phi_i^*\phi_j^*\phi_k). \end{aligned} \tag{8}$$

Here λ^a, ψ^i are the Weyl spinors, ϕ_i are the scalars, T^a_{ij} are the generators of gauge group (see Ref. [13] for details).

The theory with the Lagrangian (8) is one-loop finite (and $\gamma_m = 0$) when [13]:

$$3C(G) = S(T), \qquad d^*_{ikl}d_{jkl} = 4g^2C(T)_{ij}, \tag{9}$$

where the quadratic Casimir operator $C(T)_{ij} = (T^aT^a)_{ij}$, $S(T)$ is the Dynkin index of scalar representation [13]. Thus, we presented an example of the supersymmetric finite GUT for which the critical temperature $T_c \sim \infty$.

It is well-known that non-supersymmetric one-loop finite theories exist. Let such theory contain only one scalar multiplet and $\gamma_m = 0$. Then it is evident that temperature phase transition in this theory also does not take place.

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REFERENCES

[1] D. A. Kirznits, A. D. Linde, *Phys. Lett.* **B42**, 471 (1972); *Ann. Phys.* (*USA*) **101**, 195 (1976).
[2] L. Dolan, R. Jackiw, *Phys. Rev.* **D9**, 3320 (1974).
[3] S. Weinberg, *Phys. Rev.* **D9**, 3357 (1974).
[4] O. K. Kalashnikov, *Fortschr. Phys.* **32**, 525 (1984).
[5] U. Heinz, K. Kajantie, T. Toimela, *Ann. Phys.* (*USA*) **176**, 218 (1987); T. H. Hansson, I. Zahed, *Nucl. Phys.* **B292**, 725 (1987); S. Nadkarni, *Phys. Rev. Lett.* **61**, 396 (1988).
[6] R. Kobes, G. Kunstatter, D. J. Toms, Proceedings of 12th Johns Hopkins Workshop, 1988.

[7] S. D. Odintsov, *Mod. Phys. Lett.* **3**, 1391 (1988).
[8] R. Brandenberger, *Rev. Mod. Phys.* **57**, 1 (1985).
[9] P. S. Howe, K. S. Stelle, P. K. Townsend, *Nucl. Phys.* **B214**, 519 (1983); L. Brink, D. Lindgren, B. E. W. Nilsson, *Nucl. Phys.* **B212**, 401 (1983).
[10] A. J. Parkes, P. C. West, *Phys. Lett.* **138B**, 99 (1984); D. R. T. Jones, L. Mezineescu, *Phys. Lett.* **136B**, 242 (1984); A. I. Vainstein, V. I. Zakharov, M. A. Shifman, *Sov. J. Nucl. Phys.* **43**, 1602 (1986).
[11] S. Hamidi, J. H. Schwarz, *Phys. Lett.* **147B**, 301 (1984); J. E. Björkman, D. R. T. Jones, S. Raby, *Nucl. Phys.* **B259**, 503 (1985); A. V. Ermushev, D. I. Kazakov, O. V. Tarasov, *Nucl. Phys.* **B281**, 72 (1987).
[12] S. Rajpoot, J. G. Taylor, *Phys. Lett.* **147B**, 91 (1984); J. P. Derendinger, S. Ferrara, A. Masiero, *Phys. Lett.* **143B**, 333 (1984).
[13] M. Böhm, A. Denner, *Nucl. Phys.* **B282**, 206 (1987).
[14] S. D. Odintsov, I. L. Shapiro, *JETP Lett.* **49**, 125 (1989).
[15] B. L. Voronov, I. V. Tyutin, *Sov. J. Nucl. Phys.* **23**, 664 (1976).
[16] I. L. Buchbinder, S. D. Odintsov, *Lett. Nuovo Cimento* **42**, 379 (1985).