GRAVITY AND THE CONCEPT OF ENERGY

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The hypothesis is justified that the gravitational field does not carry energy like the electromagnetic field transferring the interaction between the electric charges does not carry charge. It is shown that the pseudotensor approach is inapplicable to the problem of localization of the energy-momentum characteristics of the gravitational field. The results are obtained by embedding the metric tensor describing the gravitation in the affinely connected space. Hence, it is evident that the formal field approach to the General Relativity is invalid for the gravitational energy-momentum problem.

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1. *Introduction*

The problem of energy-momentum of the gravitational field attracts attention since the early days of General Relativity theory (GR). In spite of considerable progress in this direction connected mainly with the definition of the integral energy of the isolated system, the question of localization of the energy-momentum characteristic is still unsolved [1] although a certain advance is observed in recent years [2–5].

In the present paper it is shown that the gravitational field does not transfer energy just as the electromagnetic field transferring the interaction between the charges carries no charge. Moreover, it will be shown that the pseudotensor approach is inapplicable to the problem of localization of the gravitational energy-momentum characteristics.

Now we shall briefly sketch the content of this paper. At first, we shall see that a correct Lagrangian approach requires introduction of the background affine connection without torsion. It is intuitively clear that the existence of the integral conservation laws must be closely connected with the mobility of the background object. If it permits the r-parameter group of motions, there must exist r conservation laws. This is really so, but these laws degenerate into the trivial form and do not describe the local dynamics of the gravitational field. The triviality here means that conserved Noether's currents have a specific

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structure. The physical meaning of this structure is that the field being the force carrier has no corresponding charge, i.e., the gravitational field which transfers the interaction between different energy-momentum characteristics of the external (nongravity) matter has not these characteristics.

2. *The gravitation action functional*

Let us consider vacuum Einstein's equations

$$G_{ik} = 0, \tag{1}$$

where $G_{ik} = R_{ik}-(1/2)Rg_{ik}$ is the Einstein tensor, $R = g^{ab}R_{ab}$ is the curvature scalar, $R_{ik} = R^p_{pik}$ is the Ricci tensor, $R^p_{lik} = \partial_l\Gamma^p_{ik}-\partial_i\Gamma^p_{lk}+\Gamma^p_{ls}\Gamma^s_{ik}-\Gamma^p_{is}\Gamma^s_{lk}$ is the Riemann tensor, $\Gamma^p_{ik} = (1/2)g^{pa}(\partial_i g_{ak}+\partial_k g_{ai}-\partial_a g_{ki})$ is Christoffel's symbol.

Usually, these equations are derived by varying the Hilbert action [6]:

$$S_H = \int \sqrt{-g}\, R d^4x, \tag{2}$$

where $g = \det(g_{ik})$. Equations (1) are of the second order, and the Hilbert Lagrangian

$$L_H = \sqrt{-g}\, R, \tag{3}$$

contains the second-order derivatives too. This leads to the known difficulties [7] because the variational problem for (2) is inconsistent with the order of the Euler-Lagrange equations. When we begin to vary (2), the surface term is produced. In order that the surface term vanishes, it is necessary to set values of the functions g_{ik} and their first derivatives at the boundary of the integration range, which results in overdetermination of the boundary--value problem for the Euler-Lagrange equations.

Instead of (3) the noncovariant Einstein Lagrangian is often used

$$L_E = \sqrt{-g}\, g^{mn}(\Gamma^a_{mb}\Gamma^b_{an}-\Gamma^a_{sa}\Gamma^s_{mn}), \tag{4}$$

which differs from L_H by the divergence term

$$L_H-L_E = \partial_i\omega^i \tag{5}$$

where

$$\omega^i = \sqrt{-g}\,(g^{in}\Gamma^m_{mn}-g^{mn}\Gamma^i_{mn}). \tag{6}$$

Noncovariance of L_E in fact means that the background object is present in the theory [4, 5]. It is the affine connection without torsion. We shall denote the background connection coefficients by $\check{\Gamma}^k_{mn}$.

The difference between the connection coefficients

$$P^k_{mn} = \check{\Gamma}^k_{mn}-\Gamma^k_{mn} \tag{7}$$

is a tensor. It is named the affine-deformation tensor. Let us consider a Lagrangian

$$\tilde{L} = \sqrt{-g}\, g^{mn}(P^a_{mb}P^b_{an} - P^a_{sa}P^s_{mn}). \tag{8}$$

For the action functional

$$\tilde{S} = \int \tilde{L}\, d^4x \tag{9}$$

the variational derivative

$$\tilde{\psi}^{mn} = 2\,\frac{\delta \tilde{S}}{\delta g_{mn}}$$

has been calculated in [4]:

$$\tilde{\Psi}^{mn} = \sqrt{-g}\, g^{ma}g^{nb}(\check{R}_{ab} + \check{R}_{ba} - \check{R}_{ij}g^{ij}g_{ab} - 2G_{ab}), \tag{10}$$

where $\check{R}_{ik} = \check{R}^p_{pik}$ is the Ricci tensor; $\check{R}^p_{lik} = \partial_l \check{\Gamma}^p_{ik} - \partial_i \check{\Gamma}^p_{lk} + \check{\Gamma}^p_{ls}\check{\Gamma}^s_{ik} - \check{\Gamma}^p_{is}\check{\Gamma}^s_{lk}$ is the Riemann tensor for the background connection. If $\check{R}_{(ik)} = 0$, then the equations

$$\tilde{\Psi}^{mn} = 0 \tag{11}$$

coincide with the Einstein equations (1), and

$$L_{\mathrm{H}} - \tilde{L} = \check{\nabla}_i F^i, \tag{12}$$

where $\check{\nabla}_i$ is a covariant derivative with respect to the background connection and

$$F^i = \sqrt{-g}\,(g^{mn}P^i_{mn} - g^{ih}P^m_{mn}) \tag{13}$$

is the vector density of weight one.

If $\check{R}^i_{klm} = 0$ one can choose the coordinate map in which all $\check{\Gamma}^i_{km} = 0$. Then, P^i_{mn} turns into $-\Gamma^i_{km}$, $\tilde{L}$ turns into L_{E}, F^i into ω^i and (12) is transformed into (5). Converting L_{H} into L_{E} by formula (5) is in fact converting L_{H} into $\tilde{L}$ with the fixation of the background connection whose coefficients in this map are assumed to be zero. Hence, it follows that in this theory it is necessary to use Lagrangian $\tilde{L}$.

3. The generally covariant representation of the energy-momentum pseudotensor of the gravitation field

A number of pseudotensor objects were proposed for determining the gravitational energy [8, 9]. Their tensor representations can be found by this method [4, 5, 10, 12]. The geometrical meaning of pseudotensors is that these objects are the tensor functionals of the background connection.

Let us consider a general Lagrangian

$$L = L(g_{mn}; \partial_k g_{mn}; \check{\Gamma}^k_{mn}). \tag{14}$$

It is supposed that the background connection is symmetric. Let the following terms be defined as

$$t^k_a = \frac{\partial L}{\partial g_{mn,k}} \check{\nabla}_a g_{mn} - L\delta^k_a, \tag{15}$$

$$\sigma^{jk}_a = \frac{\partial L}{\partial g_{mn,j}} (g_{ma}\delta^k_n + g_{an}\delta^k_m), \tag{16}$$

$$\Psi^{mn} = 2\frac{\delta S}{\delta g_{mn}} = 2\left(\frac{\partial L}{\partial g_{mn}} - \partial_j \frac{\partial L}{\partial g_{mn,j}}\right), \tag{17}$$

$$\Theta^{mn}_k = \frac{\delta S}{\delta \check{\Gamma}^k_{mn}} = \frac{\partial L}{\partial \check{\Gamma}^k_{mn}}, \tag{18}$$

where

$$S = \int L d^4 x \tag{19}$$

is the action functional; comma before index means the partial derivative. All the defined terms are the tensor densities of weight one. $(1/\sqrt{-g})t^k_a$ may be assumed as a canonical energy-momentum tensor, the term $(1/\sqrt{-g})\,\Theta^{mn}_k$ was first introduced in [10] and named the pre-energy tensor.

The number of identities can be proved by the variational method [11]. In particular,

$$t^k_a + \check{\nabla}_j \sigma^{jk}_a + \Psi^{km} g_{ma} = 0, \tag{20}$$

$$-\check{\nabla}_k t^k_a = \Theta^{mn}_k \check{R}^k_{amn} + (1/2)\sigma^{mn}_k \check{R}^k_{mna} + (1/2)\Psi^{mn}\check{\nabla}_a g_{mn}, \tag{21}$$

$$\check{\nabla}_m \check{\nabla}_n \Theta^{mn}_k + \Theta^{mn}_a \check{R}^a_{kmn} = \nabla_a(\Psi^{am} g_{mk}), \tag{22}$$

where ∇_a is the covariant derivative with respect to Christoffel's symbol;

$$\sigma^{(pk)}_a = -\Theta^{pk}_a. \tag{23}$$

Terms (15)–(18) are defined for the general Lagrangian (14). Let the terms corresponding to the concrete Lagrangian (8) be marked by the tilde ~ above the letter. After simple calculations we get the known relation (10)

$$\tilde{\Psi}^{mn} = \sqrt{-g}\, g^{ma} g^{nb}(\check{R}_{ab} + \check{R}_{ba} - \check{R}_{ij} g^{ij} g_{ab} - 2G_{ab}).$$

If $\check{R}^i_{klm} = 0$ then the coordinate map in which $\check{\Gamma}^k_{mn} = 0$, can be chosen. As it has been shown in [12], in this map the value $(1/\sqrt{-g})\tilde{t}^k_a$ coincides with the Einstein pseudotensor [13].

4. *Noether's theorem and the structure of conserved currents*

Let us consider a system described by the fields φ^A where A is the collective index. Let equations for φ^A follow from the condition of the action functional

$$S = \int L d^4x, \tag{24}$$

where L is the Lagrangian, being stationary.

The statement known as the first Noether theorem was formulated in the first section of the famous Noether paper [14]: If the action is invariant under the r-parameter Lie group G_r, then r linearly independent combinations of the variational derivatives turn into divergences, i.e.

$$\partial_j J^j_{(\lambda)} = \sum_A \Psi_A X^A_{(\lambda)}, \quad \lambda = 1, \ldots, r, \tag{25}$$

where $J^j_{(\lambda)}$ are expressions named the Noether currents, $\Psi_A = \frac{\delta S}{\delta \varphi^A}$ are variational derivatives, $X^A_{(\lambda)}$ are the representation generators corresponding to the transformations of φ^A under G_r.

Let the action be invariant under a continuous group which may be parametrized by p arbitrary functions of the coordinates. We shall denote this group as $G_{p\infty}$. If one singles out a subgroup G_r from the group $G_{p\infty}$, then according to the first Noether theorem, r local conservation laws will take place.

In Sect. 6 of paper [14] it has been formulated and proved that if G_r is a subgroup of the group $G_{p\infty}$, all currents $J^j_{(\lambda)}$ may be represented in the form:

$$J^j_{(\lambda)} = A^j_{(\lambda)} + B^j_{(\lambda)}, \tag{26}$$

where $A^j_{(\lambda)} = 0$ if $\Psi_A = 0$, and $B^j_{(\lambda)}$ satisfies the condition $\partial_j B^j_{(\lambda)} \equiv 0$. We shall call the currents satisfying (26) the trivial currents.

5. *The conservations laws*

If the background object is a metric, then the solution of the problem of integral conservation laws is well known. The metric energy-momentum tensor satisfies the local conservation laws, and if the Killing vector the background metric is present, then the integral conservation law can be obtained by integrating the local conservation law. In the present case, the metric energy-momentum tensor cannot be defined because the background metric is absent. Moreover, in the general case the canonical energy-momentum tensor does not satisfy the local conservation law (see (21)). Therefore, we use the Noether algorithm. The general form of the action variation can be written as [11]

$$\delta S = \int \left[\frac{\partial L}{\partial \check{\Gamma}^k_{mn}} \delta \check{\Gamma}^k_{mn} + \frac{\delta S}{\delta g_{mn}} \delta g_{mn} + \partial_j \left(\frac{\partial L}{\partial g_{mn,j}} \delta g_{mn} + L \delta x^j \right) \right] d^4x. \tag{27}$$

Now we substitute the definitions (15)–(18) into (27) and demand that δS vanish under Lie variations

$$\delta x^j = \varepsilon\xi^j, \tag{28}$$

$$\delta\check{\Gamma}^k_{mn} = -(\check{\nabla}_m\check{\nabla}_n(\varepsilon\xi^k)+\check{R}^k_{amn}(\varepsilon\xi^a)); \tag{29}$$

$$\delta g_{mn} = -(g_{ms}\check{\nabla}_n(\varepsilon\xi^s)+g_{ns}\check{\nabla}_m(\varepsilon\xi^s)+\varepsilon\xi^s\check{\nabla}_s g_{mn}); \tag{30}$$

where ε is an infinitesimal parameter, ξ^j is an arbitrary vector field. Then, we obtain

$$\int [\Theta^{mn}_k\check{\nabla}_m\check{\nabla}_n(\varepsilon\xi^k)+\Theta^{mn}_k\check{R}^k_{amn}\varepsilon\xi^a+(1/2)\varepsilon\xi^a\Psi^{mn}\check{\nabla}_a g_{mn}$$
$$+\Psi^{mn}g_{ma}\check{\nabla}_n(\varepsilon\xi^a)+\partial_j(\sigma^{jk}_a\check{\nabla}_k(\varepsilon\xi^a)+t^j_a\varepsilon\xi^a)]d^4x = 0. \tag{31}$$

Since the range of integration is arbitrary, it follows from (31) that the integrand is zero. By reduction of ε one can obtain

$$\partial_j(\sigma^{jk}_a\check{\nabla}_k\xi^a+t^j_a\xi^a)+\Psi^{mn}((1/2)\xi^a\check{\nabla}_a g_{mn}$$
$$+g_{ma}\check{\nabla}_n\xi^a) = -\Theta^{mn}_k(\check{\nabla}_m\check{\nabla}_n\xi^k+\check{R}^k_{amn}\xi^a). \tag{32}$$

Until now the vector field ξ^a was arbitrary. Further, the background connection will be assumed to permit some group of motions and ξ^a will generate the one-parameter subgroup of this group; consequently, the Lie derivative in the direction ξ^a of the background connection is equal to zero. It means that ξ^a satisfies the equation

$$\check{\nabla}_m\check{\nabla}_n\xi^a+\check{R}^a_{kmn}\xi^k = 0. \tag{33}$$

Under this condition the right-hand side of (32) vanishes. Using (33) we write down (32) in the form

$$\partial_j J^i = X_{mn}\Psi^{mn}, \tag{34}$$

where

$$J^j = \sigma^{jk}_a\check{\nabla}_k\xi^a+t^j_a\xi^a, \tag{35}$$

$$X_{mn} = -(1/2)\xi^a\check{\nabla}_a g_{mn}-g_{ma}\check{\nabla}_n\xi^a. \tag{36}$$

Formula (34) has the same form as (25); therefore, the energy-momentum problem seems to be solved. Indeed, let us condiser the integral

$$A = \int J^j dS_j, \tag{37}$$

where the integration is over any infinite hypersurface including the whole three-space. Relation (34) means that A is conserved if the equations of motion hold. Formula (34) is invariant because J^j is a vector density of weight one.

However, a more careful analysis shows that the problem is still unsolved. For the concrete Lagrangian $\tilde{L}$ defined by (8) the conservation laws following from (34) appear to be trivial.

It is easy to verify that the right-hand side of (34) can be identically presented in the form

$$\Psi^{mn}X_{mn} = \xi^a\nabla_k\Psi^k_a - \check{\nabla}_k(\Psi^k_a\xi^a), \tag{38}$$

where $\Psi^k_a = \Psi^{km}g_{ma}$. But if the equations

$$\Psi^{mn} = 0 \tag{39}$$

coincide with the Einstein equations, then

$$\nabla_k\Psi^k_a \equiv 0 \tag{40}$$

because of the Bianchi identities. If $\tilde{L}$ is used as L, then (34) is

$$\partial_j\tilde{J}^j = -\check{\nabla}_k(\tilde{\Psi}^k_a\xi^a). \tag{41}$$

Hence, it follows that the current $\tilde{J}^j$ can be presented in the form of (26) and, therefore, it is trivial.

The triviality of $\tilde{J}^j$ is easily seen if its structure is investigated in more detail. Let us consider (35). We have

$$\tilde{J}^j = \check{\nabla}_k(\tilde{\sigma}^{jk}_a\xi^a) - \xi^a\check{\nabla}_k\tilde{\sigma}^{jk}_a - \tilde{t}^j_a\xi^a. \tag{42}$$

Hence, using (20) we can get

$$\tilde{J}^j = \tilde{J}^j_1 + \tilde{J}^j_2 - \tilde{\Psi}^j_a\xi^a, \tag{43}$$

where

$$\tilde{J}^j_1 = \check{\nabla}_k(\tilde{\sigma}^{[jk]}_a\xi^a), \tag{44}$$

$$\tilde{J}^j_2 = \tilde{\sigma}^{(jk)}_a\check{\nabla}_k\xi^a - \xi^a\check{\nabla}_k\tilde{\sigma}^{(jk)}_a. \tag{45}$$

$\partial_j\tilde{J}^j_1 \equiv 0$ independently of assumptions about ξ^a since for any antisymmetrical twice contravariant tensor density of weight one the relation $\tilde{\nabla}_m\tilde{\nabla}_n S^{[mn]} = 0$ is true; $\partial_j\tilde{J}^j_2 = 0$ by virtue of (33) and (40). Indeed, if $\tilde{\sigma}^{jk}_a$ is replaced by $-\tilde{\Theta}^{jk}_a$ (see (23)), then we obtain

$$\tilde{J}^j_2 = \xi^a\check{\nabla}_k\tilde{\Theta}^{jk}_a - \tilde{\Theta}^{jk}_a\check{\nabla}_k\xi^a. \tag{46}$$

As $\tilde{J}^j_2$ is the vector density of weight one, $\partial_j\tilde{J}^j_2 = \check{\nabla}_j\tilde{J}^j_2$, and

$$\partial_j\tilde{J}^j_2 = \xi^a\check{\nabla}_j\check{\nabla}_k\tilde{\Theta}^{jk}_a - \tilde{\Theta}^{jk}_a\check{\nabla}_j\check{\nabla}_k\xi^a. \tag{47}$$

According to (33) $\check{\nabla}_j\check{\nabla}_k\xi^a$ in (47) can be replaced by $-\check{R}^a_{mjk}\xi^m$.

$$\partial_j\tilde{J}^j_2 = \xi^a(\check{\nabla}_j\check{\nabla}_k\tilde{\Theta}^{jk}_a + \check{R}^m_{ajk}\tilde{\Theta}^{jk}_m). \tag{48}$$

By virtue of (22)

$$\partial_j\tilde{J}^j_2 = \xi^a\nabla_k\tilde{\Psi}^k_a. \tag{49}$$

Since (40) holds for $\tilde{\Psi}^k_a$, the relation

$$\partial_j(\tilde{J}^j_1 + \tilde{J}^j_2) \equiv 0$$

is true and the triviality of Noether's current is evident.

Physical content of the triviality of Noether's current is that the field has no proper charge. For instance, let us consider the complex scalar field φ interacting with the electromagnetic field A^m. The Lagrangian

$$L_\varphi = \partial_m \varphi^* \partial^m \varphi \tag{50}$$

is invariant under the group $U(1)$

$$\varphi(x) \to e^{-ie\alpha}\varphi(x),$$
$$\varphi^*(x) \to e^{ie\alpha}\varphi^*(x). \tag{51}$$

According to the first Noether theorem this results in conservation of the electromagnetic current

$$j^m = ie(\varphi^* \partial^m \varphi - \varphi \partial^m \varphi^*). \tag{52}$$

If the electromagnetic interaction is switched on, the invariance (51) is extended to the local U(1) one and the conservation law following from the invariance of the complete Lagrangian

$$L_\Sigma = (\partial_m + ieA_m)\varphi^*(\partial^m + ieA^m)\varphi - (1/4)F^{mn}F_{mn} \tag{53}$$

under (51) becomes trivial since the conserved current

$$J_A^m = j^m - 2e^2 A^m \varphi^* \varphi \tag{54}$$

is identically presented in the form

$$J_A^m = \partial_p F^{pm} - \Psi^m, \tag{55}$$

where $F^{pm} = \partial^p A^m - \partial^m A^p$ is the tensor of the electromagnetic field,

$$\Psi^m = \frac{\partial L_\Sigma}{\partial A_m} - \partial_j \frac{\partial L_\Sigma}{\partial A_{m,j}} \tag{56}$$

is the variational derivative of L_Σ with respect to A_m.

The electromagnetic charge of the field φ is conserved "accidentally" because the Lagrangian

$$L_\varphi + L_{\varphi A} = \partial_m \varphi^* \partial^m \varphi - A_m j^m + e^2 A_m A^m \varphi^* \varphi \tag{57}$$

is invariant under (51) with any arbitrary backgroung field A^m. If the field A^m is dynamical it has no charge and the identically conserved current $J_{AI}^m = \partial_p F^{pm}$ describes the asymptotics of A^m at the spatial infinity. The asymptotics must satisfy the condition under which the surface integral $\int F^{0\alpha} d\sigma_\alpha$ over the infinitely remote surface is equal to the total charge of φ.

In the case of gravitation such an "accidental" symmetry of the material Lagrangian with the arbitrary background metric tensor is absent since the Poincaré group does not permit deviation of the metric from the Euclidean form. Therefore, the energy of the mater system is nonconserved and the gravitational field has no energy at all.

6. Concluding remarks

Many attempts were made to solve the problem of localization of the gravitational energy by introducing the nondynamical (background) object [15, 16]. Usually it was a background metric (bimetric theories) and the gravitation was considered as a usual matter field alongside with other fields [17]. The theory remained generally covariant but the dynamical invariance under the diffeomorphism group

$$x^m \to x^m + \xi^m(x),$$

$$h_{mn}(x) \to h_{mn}(x+\xi) + h_{ms}(x+\xi)\xi^s_{,n}$$

$$+ h_{sn}(x+\xi)\xi^s_{,m} + h_{ab}(x+\xi)\xi^a_{,m}\xi^b_{,n},$$

$$F^A(x) \to X^A_B F^B(x+\xi), \tag{58}$$

where h_{mn} is a gravitational field, F^A are the other fields, was violated. In the general case, when the background object is arbitrary the invariance is completely violated, i.e., any residual symmetry is absent. However, if the background object permits the group of motions, the theory is invariant under this group. Usually, the background object is a metric permitting a Poincaré group, and thus the energy-momentum problem seems to be solved. As it has been shown, this is not right. The absence of the concept of gravitation energy is not connected with the physical interpretation of the mathematical terms. It is a consequence of the properties of the Einstein equations. The main source of the difficulties in the classical GR is the invariance of the Einstein equations under the diffeomorphism group. The absence of the physical interpretation of the noncovariant objects such as psudotensors or integrals of spatial components of the tensor densities produced in the process of analysing makes it impossible to correctly close the statement of the mathematical problem by boundary or initial conditions. In the present paper, these difficulties were avoided by defining the suitable (invariant) Lagrangian and replacing the boundary-value problem by the variational one. As a result, we have succeeded in solving the group analysis problem by applying the simple and well-elaborated Noether algorithm. It is important to emphasize that it is classical GR that has been considered in this paper and not any generalization of GR such as bimetric theories.

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