HADRON SPECTRA FROM SEMILEPTONIC DECAYS OF HEAVY QUARKS*

By A. Czarnecki

Institute of Physics, Jagellonian University, Cracow**

M. Ježabek

Institute of Nuclear Physics, Cracow***

and J. H. Kühn

Max-Planck-Institut f. Physik u. Astrophysik, Werner Heisenberg Institut f. Physik, München, West Germany

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We give a formula for the distribution of the total energy of hadrons from a semileptonic decays of a heavy quark. The effects of W-propagator, a non-zero mass of the quark in the final state, and $O(\alpha_s)$ QCD correction are incorporated.

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1. Introduction

In recent papers [1, 2, 3] we have studied total rates and energy distributions of leptons in semileptonic decays of quarks. Those calculations where performed in the framework of the QCD improved parton model up to the first order in α_s. It is well known that the lepton spectra in semileptonic decays are free from QCD infrared divergences. In the present paper we calculate the distribution of the total energy of hadrons E_h, another quantity free from infrared divergences. Infrared finiteness of this distribution follows from energy conservation and infrared finiteness of the energy distribution for the virtual W.

In contrast to the case of lepton spectra the kinematic boundaries are different for the final states with and without hard gluons. Thus, studying energy distribution of hadrons

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** Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

*** Address: Instytut Fizyki Jądrowej, Zakład V, Kawiory 26a, 30-055 Kraków, Poland.

or, equivalently, of the virtual W, one can obtain non-trivial information on the masses of the quarks involved as well as on α_s.

The top quark has not been discovered yet. However there are some indirect indications that its mass m_t should be between 50 and 200 GeV. Thus, the effects of the W propagator are of crucial importance in the case of the top quark decay. In particular, if m_t is large enough, production of real W dominates and in Born approximation the decay becomes quasi two-body. In this case the shape of the E_h distribution reflects the shape of the W and QCD corrections modify the corresponding Breit-Wigner distribution.

The formulae which we present describe decays of free quarks. In order to compare them with the experiment one has to include bound state effects, for example the Fermi motion, see e.g. [4, 5, 1].

2. *Formulae*

Our notation is described in [3]. We define also

$$x_h = \frac{E_h}{m_t}. \tag{1}$$

In the Born approximation the energy distribution of hadrons is given by the following formula:

$$\frac{d\Gamma^{(0)}}{dx_h} = \frac{G_F^2 m_t^5}{12\pi^3} \frac{\bar{p}_3 \bar{u}}{(1-\xi\bar{y})^2+\gamma^2}, \tag{2}$$

where

$$y = y(x_h, z) = 1+z-2x_h$$

$$u(x, z) = x(4x-3)-z(3x-2)$$

and $\bar{y} = y(x_h, \varepsilon^2)$, $\bar{u} = u(x_h, \varepsilon^2)$[1].

For the first order QCD correction we find:

$$\frac{d\Gamma^{(1)}}{dx_h} = \frac{G_F^2 m_t^5}{12\pi^3} \frac{2\alpha_s}{3\pi} \mathcal{G}_1(x_h). \tag{3}$$

For $\varepsilon \leqslant x \leqslant (1+\varepsilon^2)/2$

$$\mathcal{G}_1(x) = \left\{ \bar{Y}_p \left[F_1 - 4x\bar{u} \ln \frac{4(x^2-\varepsilon^2)}{\varepsilon^2} \right] + \bar{p}_3 [F_2 \ln \varepsilon + F_3 + 4\bar{u} \ln 4(x^2-\varepsilon^2)] \right.$$

$$+ x\bar{u} \left[2\,\mathrm{Li}_2 \left(1 - \frac{\bar{p}_- \bar{w}_-}{\bar{p}_+ \bar{w}_+} \right) - 2\,\mathrm{Li}_2 \left(1 - \frac{\bar{w}_-}{\bar{w}_+} \right) - 3\,\mathrm{Li}_2 \left(\frac{2\bar{p}_3}{\bar{p}_3 - x} \right) \right.$$

[1] Quite generally "barred" quantities are evaluated for $z = \varepsilon^2$. However, in the present paper x_h is kept fixed.

$$+3\,\mathrm{Li}_2\left(\frac{2\bar{p}_3}{\bar{p}_3+x}\right)+4\bar{Y}_w\ln\varepsilon\Bigg]\Bigg\}\frac{1}{(1-\xi\bar{y})^2+\gamma^2}$$

$$+\int\limits_{\varepsilon^2}^{x^2}\frac{dz}{(1-\xi y)^2+\gamma^2}\left\{h(x,z)+\frac{4\xi u[2-\xi(y+\bar{y})]}{(1-\xi\bar{y})^2+\gamma^2}\left[p_3(z)\,(1+\varepsilon^2/z)/2-xY_p(z)\right]\right\}, \qquad (4)$$

where

$$F_1 = 2[x\bar{u}(2\bar{Y}_w+\bar{Y}_p)+6x^2(2-3x)+\varepsilon^2(8x-3-3\varepsilon^2)]$$

$$F_2 = 3[x(5-8x)+\varepsilon^2(7x-4)]$$

$$F_3 = 2/3[x(12x^2-20x+9)+11\varepsilon^2(3x-2)]$$

$$\begin{aligned} h(x,z) = {} & Y_p(z)/2[2z^2+(3-12x-\varepsilon^2)z+4x(2x+1)-\varepsilon^2(3+\varepsilon^2)] \\ & +p_3(z)\,\{8z^3+2(1-9x+\varepsilon^2)z^2+[3x(4x-3) \\ & +\varepsilon^2(2-x+2\varepsilon^2)]z+x\varepsilon^2(4x-3-\varepsilon^2)\}/2z^2, \end{aligned} \qquad (5)$$

whereas for $(1+\varepsilon^2)/2 < x \leqslant 1$

$$\mathscr{G}_1(x) = \int\limits_{2x-1}^{x^2}\frac{dz}{(1-\xi y)^2+\gamma^2}\left\{h(x,z)+\frac{2u}{z-\varepsilon^2}\left[p_3(z)\left(1+\frac{\varepsilon^2}{z}\right)-2xY_p(z)\right]\right\}. \qquad (6)$$

We checked the absence of mass singularities ($\varepsilon \to 0$) for G. As another cross check we compared the results for total semileptonic decay rates following from Eqs. (3), (4) and (6) with those given in [2]. We found that they are in perfect numerical agreement.

In the four fermion limit $m_W \to \infty$, i.e. ξ, $\gamma \to 0$, the integrals in Eqs. (4), (6) can be calculated analytically. For $\varepsilon \leqslant x \leqslant (1+\varepsilon^2)/2$

$$\begin{aligned} \mathscr{G}_1(x) = {} & \bar{Y}_p\left[\tilde{F}_1+2x\bar{u}\left(2\bar{Y}_w+\bar{Y}_p-2\ln\frac{4(x^2-\varepsilon^2)}{\varepsilon^2}\right)\right]+\bar{p}_3[\tilde{F}_2\ln\varepsilon+\tilde{F}_3+4\bar{u}\ln 4(x^2-\varepsilon^2)] \\ & +x\bar{u}\left[2\,\mathrm{Li}_2\left(1-\frac{\bar{p}_-\bar{w}_-}{\bar{p}_+\bar{w}_+}\right)-2\,\mathrm{Li}_2\left(1-\frac{\bar{w}_-}{\bar{w}_+}\right)-3\,\mathrm{Li}_2\left(\frac{2\bar{p}_3}{\bar{p}_3-x}\right)\right. \\ & \left.+3\,\mathrm{Li}_2\left(\frac{2\bar{p}_3}{\bar{p}_3+x}\right)+4\bar{Y}_w\ln\varepsilon\right], \end{aligned} \qquad (7)$$

where

$$\tilde{F}_1 = \tfrac{1}{12}\,[36x^2(5-8x)-6\varepsilon^2(10x^2-28x+9)+3\varepsilon^4(20x-19)+5\varepsilon^6]$$

$$\tilde{F}_2 = 3[x(5-8x)+\varepsilon^2(7x-4)]$$

$$\tilde{F}_3 = \tfrac{1}{180}\,[32x^5-168x^4+1170x^3-3720x^2+2430x$$
$$-\varepsilon^2(14x^3-36x^2-4905x+3120)-3\varepsilon^4(31x+256)], \tag{8}$$

and for $(1+\varepsilon^2)/2 < x \leqslant 1$

$$\mathcal{G}_1(x) = \tfrac{1}{180}(1-x)\,[32x^5-136x^4+1034x^3-2946x^2+1899x+312$$
$$-15\varepsilon^2(2x^3-6x^2-267x+208)-90\varepsilon^4(x+4)]+\tfrac{1}{2}\,\varepsilon^2 x(x-1)(3-4x+\varepsilon^2)/(2x-1)$$
$$+\tfrac{1}{24}\ln(2x-1)\,[320x^3-240x^2+24x+5-3\varepsilon^2(8x+1)-36\varepsilon^4 x]$$
$$+2\bar{u}\left\{2\bar{p}_3\ln\left(\frac{\bar{p}_3+1-x}{\bar{p}_3-1+x}\right)+x\left[\ln(2x-1)\ln\left(\frac{\varepsilon^2}{2x-1-\varepsilon^2}\right)\right.\right.$$
$$\left.\left.-\mathrm{Li}_2\left(\frac{2x-1}{\bar{p}_-}\right)-\mathrm{Li}_2\left(\frac{2x-1}{\bar{p}_+}\right)+\mathrm{Li}_2\left(\frac{1}{\bar{p}_-}\right)+\mathrm{Li}_2\left(\frac{1}{\bar{p}_+}\right)\right]\right\}. \tag{9}$$

In the massless limit ($\varepsilon \to 0$) we obtained for $0 \leqslant x \leqslant 1/2$:

$$\mathcal{G}_1(x) = x^2\left\{\tfrac{1}{90}(16x^4-84x^3+585x^2-1860x+1215)\right.$$
$$\left.+(8x-9)\ln 2x+2(4x-3)\left[\frac{\pi^2}{2}+\mathrm{Li}_2(1-2x)\right]\right\}, \tag{10}$$

and for $1/2 < x \leqslant 1$:

$$\mathcal{G}_1(x) = \tfrac{1}{180}(1-x)(32x^5-136x^4+1034x^3-2946x^2+1899x+312)$$
$$-\tfrac{1}{24}\ln(2x-1)(64x^3-48x^2-24x-5)$$
$$+x^2(3-4x)\,[-\pi^2+4\mathrm{Li}_2(2x)+\ln^2(2x-1)]. \tag{11}$$

3. *Results for the top quark decay*

In Fig. 1 the normalized distribution of the total energy of hadrons is shown for $m_b = 5$ GeV and $m_t = 40$, 60 and 80 GeV in the Born approximation. These distributions vanish outside the kinematical boundaries $\varepsilon \leqslant x \leqslant (1+\varepsilon^2)/2$. The first order QCD correction

$$\frac{d\Gamma^{(1)}}{dx_h} = \frac{2\alpha_s}{3\pi}\,\Gamma_b R(x_h), \tag{12}$$

where Γ_b denotes the total semileptonic width in the Born approximation is shown in Fig. 2 for the same three values of m_t. In the region $\varepsilon \leqslant x \leqslant (1+\varepsilon^2)/2$ both real and virtual gluons contribute and the combined correction to the differential rate is negative. In the region $(1+\varepsilon^2)/2 < x \leqslant 1$ only configurations with at least one real gluon are allowed

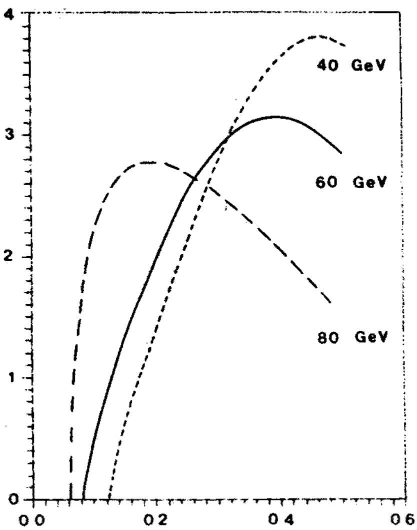

Fig. 1

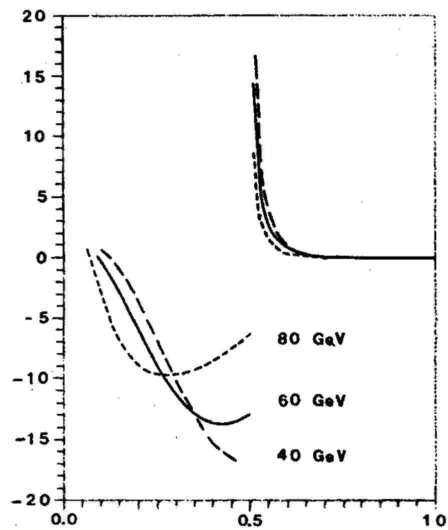

Fig. 2

Fig. 1. Normalized distribution of the total energy of hadrons shown for $m_t = 40$, 60 and 80 GeV in the Born approximation

Fig. 2. First order QCD correction $R(x_h)$ to the distribution of the total energy of hadrons

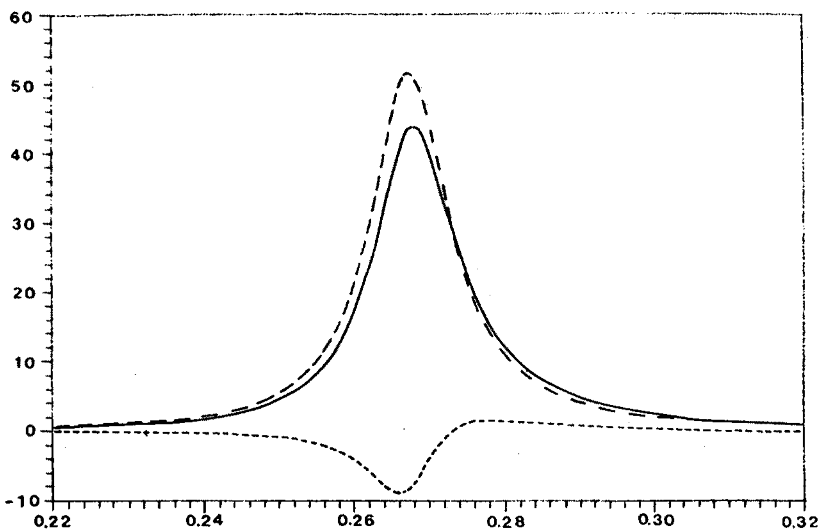

Fig. 3. Distribution of the total energy of hadrons for $m_t = 120$ GeV in the region of the peak

and thus the correction to the width is positive there. The gluon cannot be soft unless x is very close to $(1+\varepsilon^2)/2$, where the correction has a logarithmic singularity.

If the top quark is so heavy that real W production is possible the shape of the hadron energy distribution reflects the shape of the W boson. A few GeV above the threshold real W production becomes the dominant decay mode. In Fig. 3 the distribution of the energy of hadrons is shown for $m_t = 120$ GeV (solid line). The Born approximation (dashed line) and $O(\alpha_s)$ correction (dotted line) are also plotted. In the region below the peak the correction is negative, above it is positive.

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