HOW GOOD ARE GOOD FITS TO THE MULTIPLICITY DISTRIBUTIONS?*

By K. Fiałkowski**

Laboratoire de Physique Théorique et Hautes Energies***, Université de Paris XI, bâtiment 211, 91405 Orsay, France

(*Received June 19, 1989*)

Higher order moments of multiplicity distributions in rapidity intervals are calculated from lowest order moments for few simple distributions used commonly to describe the data. It is shown that systematical small deviations of data from the fitting curves may lead to significant discrepancies between the values of moments determined directly from data and values calculated using the fits. Additional tests for models which seem of fit well the multiplicity distributions are thus strongly recommended.

PACS numbers: 13.85.-t

The multiplicity distributions in (pseudo-) rapidity intervals became recently a subject of vigorous experimental and theoretical investigations. The negative binomial distributions (NBD), proposed originally to describe data for full phase-space [1] were found to fit as well data for rapidity bins of changing size in various processes [2–3–4]. This has prompted many people to look for the possible physical interpretation of NBD and its parameter values [5–6]. In particular, conditions for NBD to hold in all subdomains of the considered part of phase-space were investigated [7] and found experimentally not always valid [3], although it did not spoil too much reasonable NBD fits. On the other hand, many competing distributions were proposed and claimed similar successes [6]. Moreover, moments of distributions in very small bins were found recently to be of special interest, exhibiting power-like increase [8–9] predicted earlier in analogy with turbulent phenomena ("intermittency") [10].

Thus, although obviously any model "explaining" NBD or other simple fits should be developed to give predictions for many other measurable quantities, the first step should be to check if the multiplicity distributions are really well described by the proposed formula. The aim of this note is to show that it is not enough to find reasonable χ^2/ND ratios

* Research supported in part by CPBP 01.03 and CPBP 01.09.

** On leave from Inst. of Physics, Jagellonian University, Reymonta 4, 30-059 Kraków, Poland.

*** Laboratoire associé au Centre National de la Recherche Scientifique.

in fits to multiplicity distributions. Systematical small over- and underestimation of data in various ranges of multiplicities may lead to significant discrepancies between values of higher moments in fits and data even when χ^2 value is close to ND. Thus fits to multiplicity distributions should be always supplemented by calculation of few moments, preferably those most sensitive to the detailed shape of the distribution.

The first example we will consider are the EMC data [4] on multiplicity distribution on μp interactions at 280 GeV/c. They were shown to fit well the NBD

$$P(n) = \frac{\Gamma(n+k)}{n!\Gamma(k)}\left(\frac{k}{\bar{n}+k}\right)^k\left(\frac{\bar{n}}{\bar{n}+k}\right)^n. \tag{1}$$

For 48 distributions of charged hadrons for energy bins between 4 and 20 GeV and central rapidity bin widths between 1 and 7 units in rapidity, only in 7 cases χ^2/ND is uncomfortably high. Similar results are found for negative particles and for bins shifted to one CM hemisphere. Recently, similarly good agreement was found for so-called Poisson-type distributions, in which Poisson distribution is assumed for clusters with narrow decay multiplicity distributions [11].

Fortunately, published EMC data include not only $\bar{n}$ and k values, but also values of average multiplicity $\langle n\rangle$, dispersion $D^2 = \langle(n^2-\langle n\rangle)^2\rangle$, skewness $\gamma_3 = \langle(n-\langle n\rangle)^3\rangle/(D^2)^{3/2}$ and kurtosis $\gamma_4 = \langle(n-\langle n\rangle)^4\rangle/(D^2)^2-3$ for central rapidity bin $|y| \leqslant 1$ for 8 energy bins [4] (see Erratum!). It is easy to check that first two moments agree perfectly well with values calculated from NBD parameter values

$$\langle n\rangle = \bar{n}, \quad D^2 = \frac{\bar{n}(k+\bar{n})}{k}. \tag{2}$$

For Poisson-type distributions D^2 and $\langle n\rangle$ are simply used to determine average cluster multiplicity $\bar{n}_c$ and cluster decay multiplicity $\bar{k}$ from formulae [11]

$$\langle n\rangle = \bar{k}\bar{n}_c, D^2 = \frac{1+\eta/\bar{k}}{\bar{n}_c}, \tag{3}$$

where $\eta = 0$ if dispersion of cluster decay distributions is neglected (case marked PDD), and $\eta = 1$ if Poisson distribution is assumed also for decay (case marked PPD in what follows).

Now, however, we can easily calculate also values of γ_3 and γ_4 from known values of $\bar{n}$, k for NBD and from $\langle n\rangle$, D^2 for PDD and PPD. We find for NBD

$$\gamma_3 = (1/\bar{n}+2/k)/\sqrt{1/\bar{n}+1/k}, \tag{4}$$

$$\gamma_4 = (1/\bar{n}^2+6/\bar{n}k+6/k^2)/(1/\bar{n}+1/k), \tag{5}$$

for PDD

$$\gamma_3 = \sqrt{D^2}/\langle n\rangle, \tag{6}$$

$$\gamma_4 = D^2/\langle n\rangle^2, \tag{7}$$

and for PPD

$$\gamma_3 = \sqrt{D^2}/\langle n\rangle + 1/\sqrt{D^2} - 1/\langle n\rangle\,(D^2)^{3/2}, \tag{8}$$

$$\gamma_4 = D^2/\langle n\rangle^2 + 3/\langle n\rangle - 2/D^2 - \frac{1}{3\langle n\rangle (D^2)^2}\,. \tag{9}$$

The results are compared with measured values in Fig. 1. We see that formula (5) overestimates systematically the experimental values, yielding χ^2 above 20 for 8 data points (confidence level below 1%). On the other hand, formulae (6), (7) underestimate data with similarly poor χ^2. Only formulae (8), (9) agree with data within errors.

These results show that investigating higher moments we may indeed find systematical deviations overlooked in simple fits to multiplicity distributions. The NBD fits were in fact good for the discussed data, yielding χ^2 below N^D in six cases and 16.5/10, 13/11 in two worst cases for eight energy bins [4]. For PDD χ^2 values were not quoted, but the visual agreement for $P(n)$ was equally good [11]. Our results indicate, however, that neither NBD nor PDD can be regarded as really good description of data, unless unknown systematical errors are present in experiment or data processing. Only PPD passes this test. Unfortu-

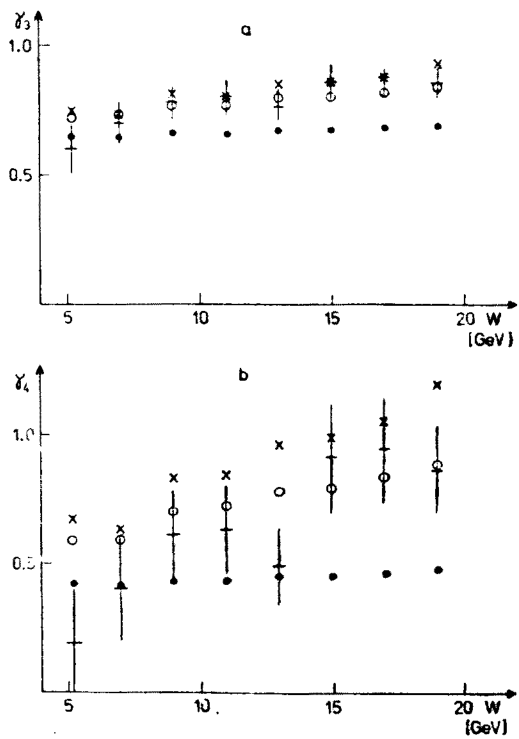

Fig. 1a. Skewness γ_3 and b. Kurtosis γ_4 of multiplicity distributions in CM rapidity bins $|y| < 1$ for μp collisions as functions of hadronic CM energy W. Crosses denote data of Ref. [4] with error bars, crosses result from NBD (4), (5), dots from PDD (6), (7) and circles from PPD (8), (9), respectively

nately, no reliable data for higher moments exist for other lh or e^+e^- experiments, and PPD is not supposed to hold for hadronic reactions. Thus we cannot check how universal is this success.

Our second example concerns the compatibility of NBD with the increase of factorial moments for small rapidity bins. We will discuss here the NA22 data from 250 GeV/c π^+p collisions [8]. It is obvious from formula (1) that for NBD following relations should hold

$$\overline{n!/(n-i)!}/\bar{n}^i \equiv F_q = \sum_{L=1}^{q-1} [L(F_2-1)+1]. \tag{10}$$

The increase of moments is usually parametrized by slopes φ_q in the relation

$$\ln F_q = \alpha_q - \varphi_q \ln \delta, \tag{11}$$

where δ is the rapidity interval length. If δ is so small that the increase resulted in F_2 significantly bigger than one, the leading term in (10) will be that proportional to F_2^{i-1}, and we expect

$$\varphi_q/\varphi_2 = q-1. \tag{12}$$

In real data experimental resolution does not allow to go below $\delta \approx 0.1$, where the increase of moments must stop [12] and F_2 is never bigger than 1.5. Then relations (10) are obviously incompatible with exact linearity (11) for more than one value of q. However, in the finite range of δ characterized by

$$\gamma = \ln(\delta_{max}/\delta_{min}), \tag{13}$$

we can always define the effective slope φ_q as

$$\varphi_q = \frac{1}{\gamma} \ln [F_q(\delta_{min})/F_q(\delta_{max})]. \tag{14}$$

It is easy to check that for F_2 changing as prescribed by (11) linearity for $q > 2$ is also quite well preserved (φ_q depends only weakly on γ) if we have

$$q[F_2(\delta_{min})-F_2(\delta_{max})] \ll q[F_2(\delta_{max})-1]+1. \tag{15}$$

This condition is well fulfilled in all existing data, and in particular in [8]. The values of effective φ_q/φ_2 resulting from (10) are, however, systematically above $q-1$. For F_q close to one

$$(i-1)\,[F_2(\delta_{min})-1] \ll 1 \tag{16}$$

one gets from relations (10) simple upper limit

$$\varphi_q/\varphi_2 \lesssim \frac{q(q-1)}{2}. \tag{17}$$

It is interesting to note that this limit corresponds to the relation obtained for Gaussian approximation of the random cascade models [13] and recently, for Bose-Einstein interference effects [14].

If the multiplicity distributions are well described by NBD, the ratios of effective slopes φ_q/φ_2 should be always between the limiting value (12) and (17). This is the case for data from nuclear collisions [9], but not for π^+p data considered here [8] where one finds

$$\varphi_q/\varphi_2 = 3.9 \pm .3,\ 11.6 \pm .8,\ 25.8 \pm 2.0 \quad \text{for} \quad q = 3, 4, 5, \tag{18}$$

whereas upper limits for NBD allowed by (17) are

$$\varphi_q/\varphi_2 \lesssim 3, 6, 10. \tag{19}$$

The disagreement is obvious. It looks strange, since the same data for central rapidity intervals of width down to $\delta = 0.5$ were successfully fitted to NBD [3]. To see more clearly what happens, we show in Fig. 2 the values of F_3 and F_4 calculated from (10) and compare them with values calculated directly from data [8]. We do not show F_5 values, for which errors are very large. For transparency, the errors of values calculated from (10) are omitted; they are always smaller than errors of moments shown. We see that discrepancies are never really big. In fact, they do not exceed three standard deviations for any point. Thus reason-

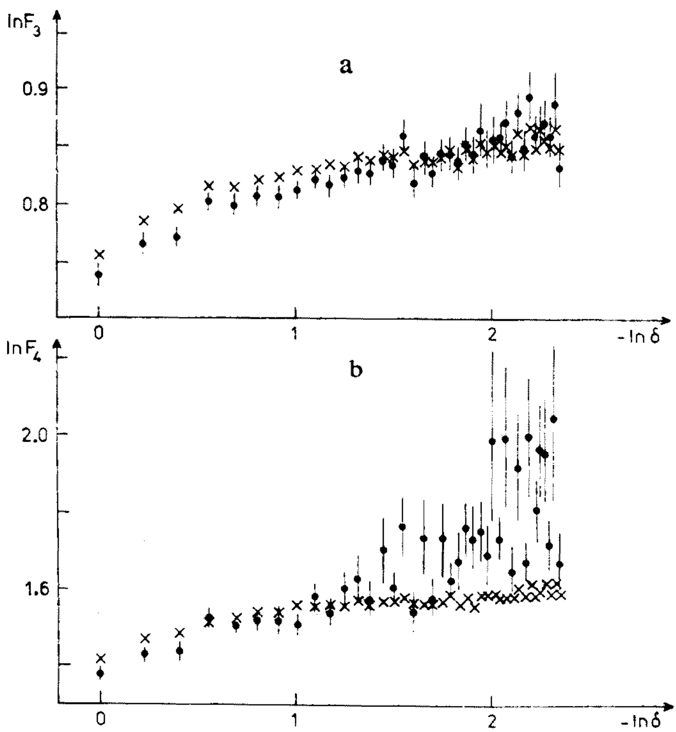

Fig. 2. Left-hand-side (points with error bars) and right-hand-side (crosses) of Eq. (10) for a — $q = 3$ and b — $q = 4$ from the data of Ref. [8] as functions of interval width δ on double logarithmic scale

able NBD fits are always possible. However, the δ-dependence is indeed quite different for two sides of equation (10) (which should be equal, if NBD holds). We can expect that NBD fits will become much worse for very small (accessible for improved rapidity resolution), if this trend holds.

Very similar results may be obtained for Poisson-type distributions (PPD) although, as noted above, they are not really expected to hold for hadron-hadron collisions. In this case relations between scaled factorial moments are

$$F_3 = F_2^2+F_2-1, \quad F_4 = F_2^3+4F_2^2-5F_2+1, \quad F_5 = F_2^4+11F_2^3-14F_2^2+F_2+2. \quad (20)$$

We can see immediately that limiting values for φ_q/φ_2 ratios for $F_2 \gg 1$ and $F_2-1 \ll 1$ are again the same as for NBD, i.e. (12) and (17), respectively. F_q are not so sensitive to change of parametrization as γ_q. Thus RHS of Eq. (20) agrees within errors with RHS of Eq. (10) for $q = 3, 4$. We do not show it to keep Fig. 2 transparent.

In fact, it is not very surprising that simple two-parameter distributions fail to describe in detail the experimental data. It is well known that the effects of energy-momentum and charge conservation, resonance decays and possible other short-range correlation effects may change significantly the distribution assumed for directly produced particles neglecting conservation laws and short-range correlations. It is difficult to imagine that all these complicated effects cancel somehow, yielding again a simple formula for the distribution of observed charged stable hadrons. It is certainly worthwhile to look for physical picture which gives NBD or other simple distribution, but it may be dangerous to disqualify models which do not yield them in a "natural" way.

It should be stressed, however, that the agreement of limits (17) with nuclear data [9] for which the linear formula (11) holds best, is quite intriguing. It may suggest that NBD holds down to very small bins in rapidity when short-range effects are suppressed by superposing the production from many nucleon-nucleon collisions. It would be interesting to investigate this behaviour in more detail.

To conclude, we have shown that models fitting quite well the multiplicity distribution in rapidity intervals may fail to reproduce the values of higher moments and/or their dependence on energy and interval length. We recommend that all the tests of models for multiplicity distributions should not be restricted to fitting the distribution, but should check also the agreement at least for few moments (preferably most sensitive to the shape of distribution). Only the models which agree with data in all representations can be regarded as describing them well.

I would like to thank A. Capella and J. Tran Thanh Van for their warm hospitality in Orsay, which enabled me to prepare these remarks. Special thanks are due to W. Kittel for supplying the data [8] in numerical form and to P. Malecki for drawing my attention to Erratum of Ref. [4].

REFERENCES

[1] M. Garetto, A. Giovannini, *Lett. Nuovo Cimento* **7**, 35 (1973); A. Giovannini et al., *Nuovo Cimento* **24A**, 421 (1974); N. Suzuki, *Prog. Theor. Phys.* **51**, 1629 (1974); W. J. Knox, *Phys. Rev.* **D10**, 65 (1974).
[2] G. J. Alner et al. (UA5 coll.), *Phys. Lett.* **B160**, 193, 199 (1985); **B167**, 476 (1986); R. Ammer et al. (LEBC-MPS coll.), *Phys. Lett.* **B178**, 124 (1986); F. Dengler et al. (NA5 coll.), *Z. Phys.* **C33**, 3304 (1986); M. Derrick et al. (EHS coll.), *Phys. Lett.* **B168**, 299 (1986); *Phys. Rev.* **D34**, 3304 (1986).
[3] M. Adamus et al. (NA22 coll.), *Z. Phys.* **C32**, 475 (1986); *Phys. Lett.* **B177**, 239 (1986).
[4] M. Arneodo et al. (EMC coll.), *Z. Phys.* **C35**, 335 (1987); *Erratum* **C36**, 512 (1987).
[5] A. Giovannini, L. Van Hove, *Z. Phys.* **C30**, 391 (1986); *Acta Phys. Pol.* **B19**, 495 (1988).
[6] P. Carruthers, C. C. Shih, *Int. J. Mod. Phys.* **A2**, 1447 (1987) and references therein.
[7] L. Van Hove, *Physica* **147A**, 19 (1987).
[8] I. V. Ajinenko et al. (NA22 coll.), *Phys. Lett.* B (1989), to be published.
[9] R. Hołyński et al. (KLM coll.), *Phys. Rev. Lett.* **62**, 733 (1989); R. Albrecht et al. (WA80 coll.), *Phys. Lett.* **B221**, 427 (1989).
[10] A. Białas, R. Peschanski, *Nucl. Phys.* **B273**, 703 (1986).
[11] A. Capella, A. V. Ramallo, *Phys. Rev.* **D37**, 1763 (1988).
[12] A. Białas, K. Fiałkowski, R. Peschanski, *Europhys. Lett.* **7**, 125 (1988).
[13] A. Białas, R. Peschanski, *Nucl. Phys.* **B308**, 857 (1988).
[14] M. Gyulassy, Is Intermittency Caused by Bose-Einstein Interference?, Berkeley preprint LBL-26831 (1989).