EVOLUTION OF A SYSTEM OF CLOSED COSMIC STRINGS

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A system of closed cosmic strings is studied. We present a brief discussion of dynamics of strings. We find self intersections of a family of cosmic strings and investigate the distribution of daughter loops. A numerical model of evolution of a system of cosmic strings (low density gas of strings) is proposed and discussed. It is found that the energy spectrum of strings is not affected by the evolution and remains scale invariant.

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1. *Introduction*

Strings and other topological defects are predicted by many grand unification models. Such objects are formed during phase transitions in the early Universe. Domain walls should decay very rapidly lest they dominate the energy density of the Universe. Since this is not the case they are not relevant for cosmology. A domain wall within the present horizon would, moreover, produce a very high anisotropy in the background radiation. The experimental bounds on this anisotropy allow to conclude that there are no massive domain walls at present. The problem of evolution of strings, however, is being widely investigated as strings may solve some important cosmological problems.

Several mechanisms of loosing energy by strings have been found. These are gravitational radiation, electromagnetic radiation (Witten, 1985) in the case of superconducting strings and intercommuting of strings. These processes have been recently reviewed by Vilenkin (1985). The intercommuting is the less known of all the above mentioned processes. Answers to the following questions are still unknown: what impact intercommutings have on the evolution of a system of cosmic strings?, how probable is a self-intersection and whether it leads to a decay of the string?

Strings may give rise to matter density perturbations needed to form the presently observed large scale structure of the Universe (Vilenkin, 1985). The perturbations could be caused by gravitational attraction of matter by strings. Another scenario predicts that electromagnetic radiation from superconducting strings would blow up huge voids in

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the primeval plasma and condensate matter on the borders of such bubbles (Ostriker, Thompson, Witten, 1986). The value of matter density fluctuations predicted by these scenarios is:

$$\frac{d\varrho}{\varrho} \sim 10^{-5}. \tag{1.1}$$

This happens to be the upper bound (imposed by the isotropy of the microwave background) on the value of the initial matter density perturbations. The spectrum of this fluctuations is scale invariant, since the energy distribution of strings is scale invariant, unless this property is disturbed during the evolution of a system of cosmic strings.

There still remains the problem of evolution of a system of cosmic strings. The motion of a string is described by an oscillator-like law with nonlinear constraints. Only a few families of exact solutions of string equations of motion have been found (Turok, 1984; Kibble, Turok, 1982). The numerical simulations on the other hand have met many difficulties (Smith, Vilenkin, 1987, Bennet, and Bouchet, 1987). These are: the above mentioned nonlinearities, the fact that velocity of a string is not continuous at certain points called kinks, the difficulties with numerical detection of intersections of strings and the fact that tracing strings motion is hardly possible if its size is comparable to the lattice. In the numerical models mentioned above a fraction of strings fragment into smallest possible loops.

First we discuss the dynamics of strings and present different families of exact solutions of the equations of motion. The intersections of a certain family of strings are found. We compare these intersections with the results of Shellard (1987) and Matzner (1987). A conclusion can be drawn that most of the self-intersections lead to production of daughter loops. We propose other model of evolution of a system of cosmic strings. In this model it is assumed that self-intersections of strings are similar to the self-intersections of the discussed family of solutions. The intercommuting of different strings is neglected.

A general discussion of formation and properties of different topological defects can be found in the review article by Vilenkin (1985).

2. *Dynamics of strings*

Strings motion can be described by a parametrization of world-sheet just like the motion of a material point is described by a world-line:

$$x_\mu = x_\mu(\xi_1, \xi_2). \tag{2.1}$$

The space like coordinate ξ_1 (ξ_2 is the time like coordinate) is usually the linear energy density of the string. We shall, however, call it length of a string, although the physical length is different and changes with time.

The dynamics of strings is determined by the action. The string lagrangian, first proposed by Nambu, (Nielsen, Olesen, 1973) is:

$$L = \mu[\dot{x}^2 x'^2 - (\dot{x}x')^2]^{1/2}, \tag{2.2}$$

where dots and comas represent derivatives with respect to time-like and space-like coordinates respectively. The equations of motion can be obtained in the standard way:

$$\frac{\partial}{\partial \xi_a}\left(\frac{\partial L}{\partial x^{\mu}_{,a}}\right) - \frac{\partial L}{\partial x_{\mu}} = 0. \tag{2.3}$$

These equations can be simplified by the appropriate choice of coordinates. A two-dimensional surface (i.e. the world-sheet) can be parametrized by the so called normal coordinates. Tangent vectors corresponding to the coordinates are perpendicular:

$$\dot{x}x' = 0 \tag{2.4}$$

and normalized:

$$\dot{x}^2 + x'^2 = 0. \tag{2.5}$$

This choice of parameters allows to simplify the lagrangian and the equations of motions. These two conditions are constraints to the motion of the string. The equations of motion after having taken the above constraints into account become:

$$\ddot{x} - x'' = 0. \tag{2.6}$$

It is easy to check that the constraints are preserved by the equations of motion:

$$(\dot{x}x') = \ddot{x}x' + \dot{x}\dot{x}' = \tfrac{1}{2}(\dot{x}^2 + x'^2)' = 0,$$
$$(\dot{x}^2 + x'^2) = 2\ddot{x}\dot{x} + 2x'\dot{x}' = 2(\dot{x}x')' = 0. \tag{2.7}$$

Equations of motion are solved generally by two arbitrary functions a anb b:

$$x_{\mu} = \tfrac{1}{2}\left[a_{\mu}(\xi_1 + \xi_2) + b_{\mu}(\xi_1 - \xi_2)\right]. \tag{2.8}$$

The constraints applied to the above equation mean that squares of tangent vectors of these functions vanish:

$$a'^2 = b'^2 = 0. \tag{2.9}$$

A general method of finding such functions, called zero curves, was found by Hughston, Shaw (1988).

In the case when the string is closed:

$$x_{\mu}(\xi_1, \xi_2) = x_{\mu}(\xi_1, \xi_2 + L) \tag{2.10}$$

its motion is periodic and the period is $L/2$ (Kibble, Turok, 1982).

Conditions (2.4) and (2.5) do not determine the parametrization uniquely. Two most commonly used parametrizations are: the standard and the light-cone one.

In the standard parametrization the time axis is chosen as ξ_1 parameter, and the energy along string as the ξ_2 parameter. The equations of motion can be written in the three dimensional form:

$$\bar{x}'' - \ddot{\bar{x}} = 0. \tag{2.11}$$

The solution may of course be written in terms of three dimensional functions $\bar{a}$ and $\bar{b}$, and the constraints become:

$$\bar{a}'^2 = \bar{b}'^2 = 1 \tag{2.12}$$

which means that the tangent vectors lie on a unit sphere.

In the case of a closed string these functions must also satisfy the following condition:

$$\int a' + \int b' = p, \int a' - \int b' = 0, \tag{2.13}$$

where p is the total momentum of the string.

Minkowski space (t, x_1, x_2, x_3) can be parametrized by light-cone coordinates $(x_+ = t + x_1, x_- = t - x_1, x_2, x_3)$. In the light-cone parametrization (Goddard, Goldstone, Rebbi, Thorn, 1973) of strings motion the timelike parameter is x_+ and the spacelike one is the energy along the string as in the above example. This choice of coordinates allows to simplify and, in fact, solve the constraints:

$$\dot{x}_- = x'_\perp \dot{x}_\perp, \tag{2.14}$$

$$x'_- = \dot{x}_\perp^2 + x_\perp'^2,$$

where the coordinates x_2, x_3 are denoted by $x_\perp$. The transformation back to the rest frame, however, is hardly possible so this solution has little importance if the properties of motion of string are considered. This parametrization is very convenient, for example, to count string states.

As it has been shown above the solution of the equations of motion in the case of standard parametrization can be reduced to finding two functions a' and b' lying on the sphere with the radius of unity. These functions can be thought of as the functions on the surface of such sphere. A solution of the equations of motion is determined by two such curves (satisfying conditions 2.13) with an initial point chosen on each of them. The simplest continuous functions (or curves) on a sphere are large circles:

$$\bar{a}'_1 = \begin{pmatrix} \sin \xi \\ \cos \xi \\ 0 \end{pmatrix}. \tag{2.15}$$

It can be easily proved that there exists only one family of such functions (curves) with two modes, if the curves which can be transformed onto each other by rotation or choice of initial point of parametrization, are identified. The ratio of frequency of this two modes must be 1 : 3 and the family is:

$$\bar{a}'_2 = \begin{pmatrix} \alpha \sin \xi + (1-\alpha) \sin 3\xi \\ \alpha \cos \xi + (1-\alpha) \cos 3\xi \\ 2(\alpha(1-\alpha))^{1/2} \cos \xi \end{pmatrix}. \tag{2.16}$$

We are now ready to write down explicitly several simple string trajectories. The simplest one is:

$$\bar{x}(\sigma, \tau) = \tfrac{1}{2}\begin{bmatrix} \sin\sigma_+ + \sin\sigma_- \\ \cos\sigma_+ + \cos\phi\cos\sigma_- \\ \sin\phi\cos\sigma_- \end{bmatrix}, \tag{2.17}$$

where we have introduced new variables $\sigma_+ = (\sigma+t)/L$ and $\sigma_- = (\sigma-t)/L$. A little more complicated family of solutions, consisting of single and double mode functions, is (Chen, DiCarlo, Hotes, 1988):

$$\bar{x}(\sigma, \tau) = \tfrac{1}{2}\,[\bar{a}_1(\sigma_+) + R(\theta, \phi)\bar{a}_2(\sigma_-)]. \tag{2.18}$$

Combining two double-mode solutions of the type (2.17) one obtains a five parameter family of solutions of equations (2.11) and (2.12):

$$\bar{x}(\sigma, \tau) = \tfrac{1}{2}\,[\bar{a}_2(\sigma_+) + R(\theta, \phi, \delta)\bar{a}_2(\sigma_-)]. \tag{2.19}$$

In this paper a two-parameter sub-family of (2.18) will be investigated:

$$\bar{x}(\sigma, \tau) = \tfrac{1}{2}\begin{bmatrix} (1-\alpha)\cos\sigma_+ \ \frac{\alpha}{3}\cos 3\sigma_+ + \sin\sigma_- \\ -(1-\alpha) - \frac{\alpha}{3}\sin 3\sigma_+ \sin\phi\cos\sigma_- \\ -2[\alpha(1-\alpha)]^{1/2}\sin\sigma_+ + \cos\phi\cos\sigma_- \end{bmatrix}. \tag{2.20}$$

We will call this family of strings the Kibble family of strings.

3. *Self intersections of the Kibble family of strings*

We will consider solutions of the equation:

$$\bar{x}(\sigma_1, \tau) = \bar{x}(\sigma_2, \tau) \tag{3.1}$$

since they represent self-intersections of a string.

Daughter loops produced in a decay of a string satisfy the constraints (2.4) and (2.5) and thus they are "good" solutions of the equations of motion. This can be checked by introducing appropriate parametrizations of the daughter loops and taking into account the fact that the constraints are preserved by the initial string during its motion.

The intersections of the family (2.20) were found. New variables (Turok, 1984) were introduced:

$$\Psi = \frac{\sigma_1+\sigma_2}{2} \qquad \chi = \frac{\sigma_1+\sigma_2}{2} + t \qquad \delta = \frac{\sigma_1+\sigma_2}{2} - t \tag{3.2}$$

and the following system of equations has been obtained:

$$2[\alpha(1-\alpha)]^{1/2}\sin\chi + \sin\phi\cos\delta = 0$$

$$\tfrac{3}{4}(\cos\chi+\cos\delta) = \alpha\cos\chi(-4\cos^2\chi\cos^2\Psi+3\cos^2\Psi\cos^2\delta),$$

$$(3-2\alpha)\sin\chi+3\cos\phi\sin\delta = 4\alpha\sin\chi(-\cos^2\Psi\cos^2\chi+\cos\chi). \tag{3.3}$$

This system of equations was solved numerically for 14400 pairs of parameters (α, ϕ). The map of intersection in the space of parameters is shown in Fig. 1. The intersections region constitutes only about 2 percent of the parameter space. These results are consistent with (Zembowicz, 1988) and (Chen, DiCarlo, S. A. Hotes, 1988). Their results indicate however that self intersections fill approximately half of the parameter space if more general families of solutions are considered.

It is important from the cosmological point of view whether a self intersection of a string leads to decay and production of two daughter loops. The question of the result of intersection of two strings was analyzed numerically by Shellard (1987) for global U(1) strings and Matzner (1987) for local strings. The results of these two papers are consistent, although

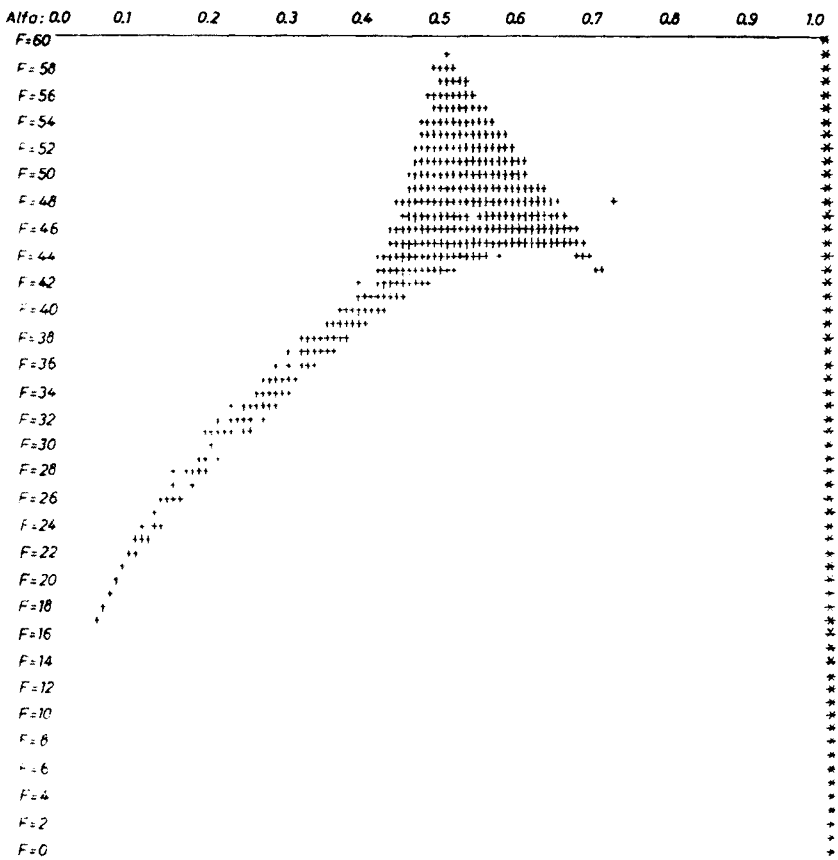

Fig. 1. Map of self intersections of the Kibble string in the parameter space. $F = 60\cos(\text{fi})$. There are no self interactions for $\cos(\text{fi}) < 0$

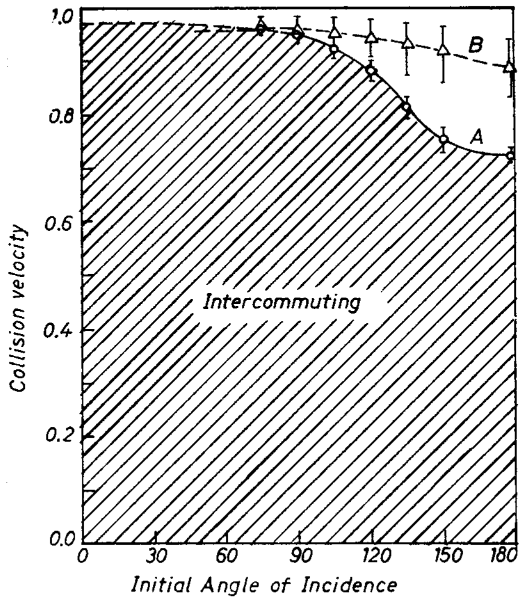

Fig. 2. The intercommuting of global U(1) strings. (Fig. 11 from Shellard, 1987)

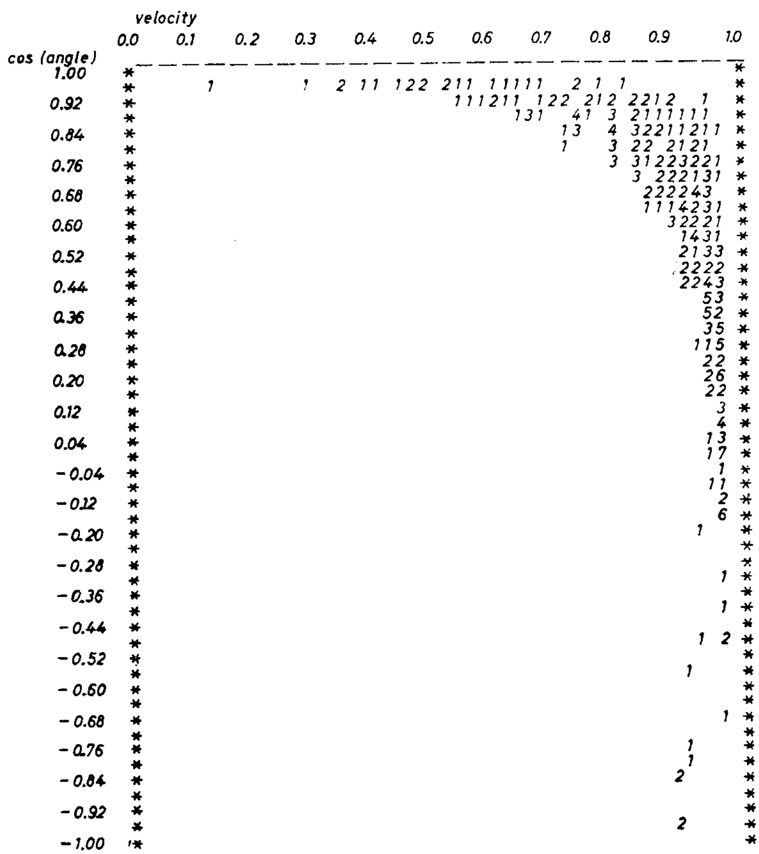

Fig. 3. The intercommuting of the Kibble string. Number of self intersections in the (velocity, angle) coordinates. The angle is expressed in term of its cosine, while the velocity is expressed in the units of speed of light. There are never more than 9 intersections in one grid point

in the second one string intercommutings were calculated only in a few points in the (velocity, angle) plane. Map of possible outcomes of intercommuting of straight strings as a function of velocity and angle of collision is shown in Fig. 2 from the article by Shellard (1987). The intersections of the Kibble string in the same (velocity, angle of collision) coordinates are presented in Fig. 3. The curvature of strings can be neglected since the diameter of a string is much smaller than its macroscopic size. One can see that most of the intersections fall in the decay region of the diagram in Fig. 2. One can thus assume that most of the intersections lead to production of two daughter loops. The distribution of length of shorter daughter loops in the decays of the Kibble string is presented in Fig. 4.

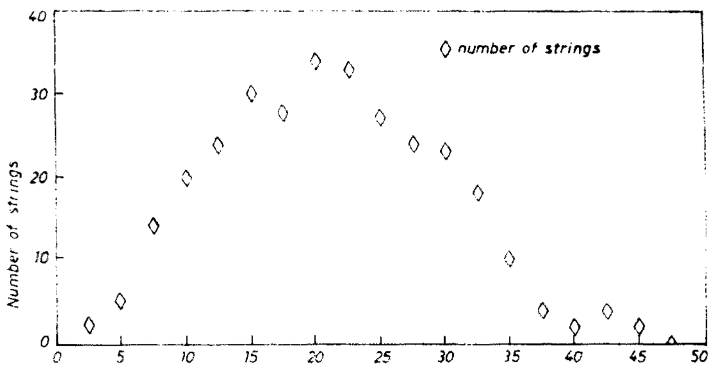

Fig. 4. Length distribution of the produced strings

4. *Model of evolution of a system of closed cosmic loops*

4.1. Numerical technique

A system of non interacting strings was considered. This can be understood as the gas of strings in the limit of low density. The initial distribution of string energies was the scale invariant distribution:

$$dn \sim l^{-5/2}dl. \tag{4.1}$$

Such distribution was obtained in the numerical analysis of cosmological phase transitions and formation of strings (Vachaspati, Vilenkin, 1984). Upper and lower cut-offs of the string length had to be introduced in the numerical calculations. The lower cut-off may be regarded as the smallest relevant length of a string while the upper one corresponds to the fact that there are very few long strings according to the distribution law (4.1).

The probability of a self intersection of a string during its period of motion was assumed to be 0.5. A string was considered stable if it has not intersected during one period of motion. If, however, a self-intersection of a string occurred then it was assumed to decay (see Sect. 3). Decay of a smallest string was considered as its disappearing and no daughter loops were created. The length of produced daughter loops was chosen randomly from the set of previously found decays of the Kibble string. The time step was the period of motion of the shortest string. The probabilities of decay of longer strings in one time step were accordingly smaller since the period of motion of a string of length equal to n lengths of smallest string was n times longer.

4.2. Results of the simulation

The system of strings consisted initially of 20 different lengths of strings and their numbers followed formula (4.1). There were 10000 smallest strings initially. The calculations were performed until the moment when all the strings have become stable. After every time step the exponential formula:

$$dn \sim l^{-\alpha} dl \tag{4.2}$$

was fitted to the distribution.

The time dependance of the coefficient α is shown in Fig. 5. One can see that the scale invariant distribution is not affected by the decays of strings. The relaxation time of the system of strings depends on the probability of self intersection and is in our case about 25 time steps. The initial and the final distribution of strings is shown in Fig. 6 in order to illustrate the dispersion from the ideal exponential curve. The behavior of the system of strings in this model does not vary substantially with the change of cut-off.

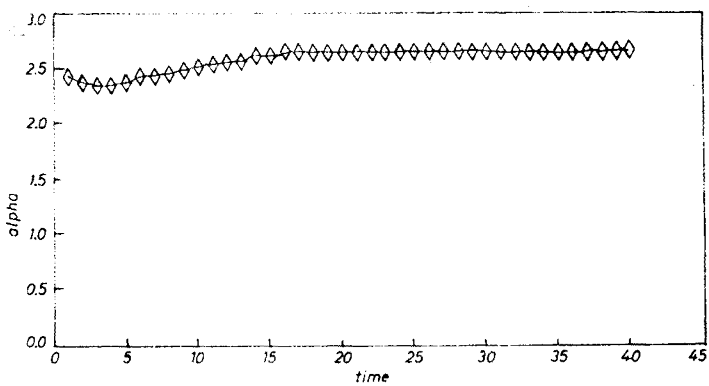

Fig. 5. Time dependence of the parameter α in the simulation

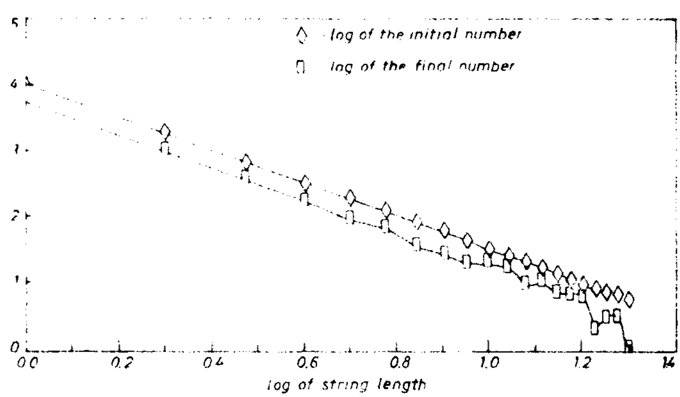

Fig. 6. Initial and final distribution of strings length

5. Conclusions

The model shown above describes the behavior of gas of strings in the low density limit, since no interactions between different strings are considered. The system of strings evolves very quickly. The final state which we believe is the equilibrium state of the strings gas in the scale invariant energy distribution. In the cosmological case this leads to scale invariant initial density fluctuations. There is, however, another difficulty in this model. About half of the number of strings are those produced in decays. Such strings move with the relative velocities of the order of magnitude about the speed of light. On the other hand the presently observed velocities of galaxies are much smaller. We are left with the question whether the model is completely wrong or there exists a very effective mechanism of friction which slows down relativistic strings.

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