THE SIMPLEST GENTILIONIC SYSTEMS

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Fundamental aspects of the gentilionic theory are reanalyzed and significant modifications are introduced in this approach. We show that the state vector of three gentileons has a spinor character and that its basic symmetry properties are described by the intermediate, S_3 and SU(3) groups. As an intermediate and natural result of our theoretical analysis, we show how essential observed properties of composed hadrons can be predicted from first principles assuming quarks as gentileons.

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1. Introduction

In recent papers [1–3] we have proposed, according to the postulates of quantum mechanics and the principle of indistinguishability, that three kinds of particles could exist in nature: bosons, fermions and gentileons. In our theory [1–3] the following statement is taken as a principle (Statistical Principle): "Bosons, fermions and gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively". Bosonic and fermionic systems are described by one-dimensional totally symmetric (Ψ_S) and totally anti-symmetric (Ψ_A) wavefunctions, respectively. Gentilionic systems are described by wavefunctions (Y) with mixed symmetries. Since they are represented by intermediate Young shapes only three or more identical gentileons can form a system of indistinguishable particles. This means that two identical gentileons are *prohibited* for a system of indistinguishable particles.

Let us indicate by YD(n, j) all possible different intermediate Young diagrams ($j = 1, 2, 3, \ldots$) that can be constructed for n-particle systems. For instance, for $n = 3$ there is only one possibility YD(3, 1) and for $n = 4$ there are three possibilites YD($4, j$), where $j = 1$, 2 and 3. As is well known [4–7] there is a one-to-one correspondence between the Young diagrams YD(n, j) and the irreducible representations $Y(n, j)$ of the permutation group in Hilbert space. The state functions $Y(3, j)$, $Y(4, i)$, $Y(5, k)$... have completely different symmetry properties that are defined by the permutations and by the algebraic invariants [2–7] associated with the symmetric groups S_3, S_4, S_5, ... In a n-particle system represented by $Y(n, j)$ sub-systems with m particles do not have a $Y(m, i)$ symmetry. From the above properties very important consequences are deduced:

(*1*) *There is an infinite number of different gentileons.* Indeed, if there were only one kind of gentileon, 3, 4, 5... particles would form systems represented by $Y(3, j)$, $Y(4, i)$, $Y(5, k)$, ..., respectively. Thus, let us consider a given system composed of n gentileons and let us divide it into sub-systems with m particles ($m = n$-1, n-2, ..., 5, 4, 3). Since these m particles are indistinguishable these sub-systems would be necessarily represented by $Y(m, i)$, which is impossible. Consequently, there must be an infinite number of different gentileons g_k ($k = 1, 2, 3, \ldots$). Gentileons g_1 would be associated with YD(3, 1) = $\begin{smallmatrix}\square&\square\\ \square&\end{smallmatrix}$, g_2 with YD(4, 1) = $\begin{smallmatrix}\square&\square\\ \square&\square\end{smallmatrix}$, g_3 with YD(4, 2) = $\begin{smallmatrix}\square&\square&\square\\ \square&&\end{smallmatrix}$, g_4 with YD(4, 3) = $\begin{smallmatrix}\square&\square\\ \square&\\ \square&\end{smallmatrix}$, and so on. In other words, gentileons g_1 would form only 3-particle systems represented by $Y(3, 1)$, g_2 would form only 4-particle systems represented by $Y(4, 1)$ and so on.

(*2*) *Gentilionic systems cannot coalesce.* Two systems of n identical gentileons with each one represented by $Y(n, j)$ cannot form a system with $2n$ indistinguishable entities that would be described by $Y(2n, i)$. Indeed, if coalescence were possible it would be possible to obtain from $Y(2n, i)$ sub-systems with n particles described by $Y(n, j)$, which is prohibited. Thus, system A and B, $[gg\ldots g]_A$ and $[gg\ldots g]_B$ cannot coalesce into a system of indistinguishable particles $[gggg\ldots g]$. Only bound states $[gg\ldots g]_A$-$[gg\ldots g]_B$ could be formed. Then, gentileons from different systems must be distinguishable which means that gentileon wavefunctions from different systems must be non-overlapping.

(*3*) *Gentileons are confined entities.* To see this we must note that a system composed of n gentileons $[gggg\ldots g]$ cannot be created step by step from vacuum because the systems $[g]$, $[gg]$, $[ggg]$, ..., $[ggg\ldots g]$, with 1, 2, 3, ..., n-1 particles, respectively, are not allowed. By the same argument we see also that this system cannot be annihilated by steps. This means that gentilionic systems must be created or annihilated at once. Consequently, no gentileon can escape from the system and no gentileon can enter the system.

Taking into account non-coalescence and confinement properties we see that no gentileons can be subtracted or added to a gentilionic system and that it must have sharp boundaries outside of which gentilionic wavefunctions vanish.

In the above quoted paper [1] only systems of identical gentileons have been considered. Let us now consider systems composed of two different kinds of gentileons, g and G. Taking into account the Statistical Principle we must expect that systems like $[gG]$ are allowed. On the other hand, systems like $[ggG]$, $[gGG]$ and $[ggGG]$ are prohibited because $[gg]$ and $[GG]$ are not allowed. Of course, non-coalescence and confinement properties are also valid for mixed systems, as can be easily verified.

Confinement and non-coalescence are intrinsic properties of gentileons, deduced from the Statistical Principle and from the symmetry properties of the intermediate states $Y(n, j)$, not depending on their physical interpretation. Thus, they could correspond to real particles or to dynamical entities as quantum collective excitations. However if gentileons were real particles there must be some kind of mechanism to explain these properties: a very peculiar interaction potential, an impermeable bag or something else. But any acceptable mechanism must be conceived under the imposition of agreeing exactly with the intermediate symmetry. It is difficult to understand gentileons as real particles; they seem to be some kind of quantum collective excitation.

In Section 2 we present a detailed study of the symmetry properties of the state vector $Y(3, 1)$ representing $[g_1 g_1 g_1]$ systems. It is shown that $Y(3, 1)$ has a spinor character.

As well known, half-odd-integral and integral spin particles are described, from the point of view of the Lorentz group, by spinorial and tensorial irreducible representations, respectively. According to the celebrated Pauli theorem [8–10] if creation and annihilation particle operators obey bilinear commutative (anti-commutative) relations these particles have integral (half-odd-integral) spin. By using bi-linear commutative or anti-commutative relations, consistent local, Lorentz invariant quantum field theories are developed. In Section 3 commutations relations for gentileons g_1 are analysed in order to establish a connection between spin and statistics. It is seen, in Pauli's context, that gentileons g_1 are half-odd-integral spin particles.

In Section 4 we show that the fundamental symmetry properties of the state vector $Y(3, 1)$ are described by the S_3 and SU(3) groups. In Section 5 we summarize the basic features predicted for the $[g_1 g_1 g_1]$ systems. In Section 6, assuming quarks as g_1 gentileons, our theoretical approach is applied to investigate some aspects of hadronic physics. Finally, in Section 7 a quantum chromodynamics is proposed where, instead of fermions, g_1 gentileons interact with gluons.

2. *Symmetry properties of the gentilionic state vector Y(3, 1)*

We present in this Section a detailed study of the symmetry properties of the state vector $Y(3, 1)$ of a system composed of three first kind gentileons g_1. Thus, according to our general results [1] the symmetry properties of $Y(3, 1)$, also indicated by $Y(123)$, is completely described in terms o three quantum states α, β and γ. In terms of α, β and γ the g_1 system will be represented by $Y_+(3, 1)$ or $Y_-(3, 1)$, two equivalent irreducible representations of the symmetric group [3–6, 11] S_3,

$$Y_+(3, 1) = Y_+(\alpha\beta\gamma) = Y_+(123) = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_1(123) \\ Y_2(123) \end{pmatrix},$$

$$Y_-(3, 1) = Y_-(\alpha\beta\gamma) = Y_-(123) = \frac{1}{\sqrt{2}}\begin{pmatrix} Y_3(123) \\ Y_4(123) \end{pmatrix}, \tag{2.1}$$

where

$$Y_1(123) = (|\alpha\beta\gamma\rangle + |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle - |\gamma\beta\alpha\rangle)/\sqrt{4},$$

$$Y_2(123) = (|\alpha\beta\gamma\rangle + 2|\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle + |\gamma\alpha\beta\rangle - 2|\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{12}$$

$$Y_3(123) = (-|\alpha\beta\gamma\rangle + 2|\alpha\beta\gamma\rangle - |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle + 2|\beta\gamma\alpha\rangle - |\beta\gamma\alpha\rangle)/\sqrt{12},$$

$$\text{and } Y_4(123) = (|\alpha\beta\gamma\rangle - |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle + |\gamma\beta\alpha\rangle)/\sqrt{4}.$$

In preceding papers [3, 12] the state function $Y(3, 1)$ was taken as a "bi-spinor" in Dirac's sense $Y(3, 1) = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$. Although it is a possible interpretation for the state function $Y(3, 1)$ it has no rigorous support within the framework of group theory. Thus, in what follows, the g_1 system will be represented by $Y_+(123)$ or $Y_-(123)$, indicated simply by $Y(123)$. It is worthwhile to note that, in this context, our theory differs drastically [3] from para-statistics.

Our intention in this Section is to show explicitly the spinor character of $Y(123)$ and to establish fundamental properties of the g_1 system that can be deduced from this spinor character. In this way we remember that, due to the six permutation operators P_j of the group S_3, the $Y_\pm(123)$ are transformed to [1]:

$$Y' = Y'_\pm = P_j Y_\pm = \eta_j Y_\pm \tag{2.2}$$

where η_j ($j = 1, 2, 3, \ldots, 6$) are 2×2 matrices given by,

$$\eta_1 = \eta\begin{pmatrix}123\\123\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = I; \quad \eta_2 = \eta\begin{pmatrix}123\\213\end{pmatrix} = \begin{pmatrix}-1/2 & \sqrt{3}/2\\ -\sqrt{3}/2 & -1/2\end{pmatrix}$$

$$\eta_3 = \eta\begin{pmatrix}123\\231\end{pmatrix} = \begin{pmatrix}-1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & -1/2\end{pmatrix}; \quad \eta_4 = \eta\begin{pmatrix}113\\213\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\eta_5 = \eta\begin{pmatrix}123\\132\end{pmatrix} = \begin{pmatrix}-1/2 & \sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{pmatrix} \quad \text{and} \quad \eta_6 = \eta\begin{pmatrix}123\\321\end{pmatrix} = \begin{pmatrix}-1/2 & -\sqrt{3}/2\\ -\sqrt{3}/2 & 1/2\end{pmatrix}. \tag{2.3}$$

The spinor character of $Y(123)$, as seen in Eqs. (2.3), is obvious since the matrices η_1, η_2 and η_3 have det $= +1$ and η_4, η_5 and η_6, det $= -1$. We will show that it is correct interpreting the transformation of Y in terms of rotations of an equilateral triangle in an Euclidean space E_3. That is, we assume E_3 as a space where the quantum states that can be occupied by g_1 are defined by three orthogonal coordinates (X, Y, Z). It is also assumed that, in E_3, the states α, β and γ occupy the vertices of an equilateral triangle taken in the (X, Z) plane, as seen in Fig. 1. The unit vectors along the X, Y and Z axes are indicated, as usual, by $\vec{i}$, $\vec{j}$ and $\vec{k}$. In Fig. 1, the unit vectors $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ are given by, $\vec{m}_4 = -\vec{k}$, $\vec{m}_5 = -(\sqrt{3}/2)\vec{i} + (1/2)\vec{k}$ and $\vec{m}_6 = (\sqrt{3}/2)\vec{i} + (1/2)\vec{k}$, respectively.

We represent by $Y(123)$ the state whose particles 1, 2 and 3 occupy the vertices α, β and γ, respectively. Thus, we see that the true permutations, (312) and (231), are obtained from (123) under rotations by angles $\theta = \pm 2\pi/3$ around the unit vector $\vec{j}$. As one can easily verify, the matrices η_2 and η_3, that correspond to these permutations are represented by:

$$\eta_2 = -I/2 + i(\sqrt{3}/2)\sigma_y = \exp\left[i\vec{j}\cdot\vec{\sigma}(\theta/2)\right] \quad \text{and}$$

$$\eta_3 = -I/2 - i(\sqrt{3}/2)\sigma_y = \exp\left[i\vec{j}\cdot\vec{\sigma}(\theta/2)\right], \tag{2.4}$$

where σ_x, σ_y and σ_z are Pauli matrices.

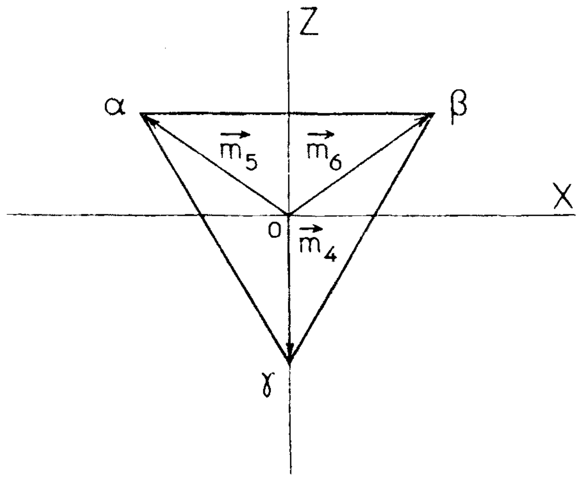

Fig. 1. The equilateral triangle in the Euclidean space (X, Y, Z) with vertices occupied by the states α, β and γ

Similarly, the transpositions (213), (132) and (321) are obtained under rotations by angles $\Phi = \pm\pi$ around the axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. The corresponding matrices are given by:

$$\eta_4 = \sigma_z = i\exp\left[i\vec{m}_4\cdot\vec{\sigma}(\Phi/2)\right],$$
$$\eta_5 = (\sqrt{3}/2)\sigma_x-(1/2)\sigma_z = i\exp\left[i\vec{m}_5\cdot\vec{\sigma}(\Phi/2)\right] \quad \text{and}$$
$$\eta_6 = -(\sqrt{3}/2)\sigma_x-(1/2)\sigma_z = i\exp\left[i\vec{m}_6\cdot\vec{\sigma}(\Phi/2)\right]. \tag{2.5}$$

According to our preceding papers [2, 3] there is an algebraic invariant, $K^{[2,1]}_{(2,1)}$, with a zero eigenvalue, associated with the S_3 gentilionic states. In analogy with continuous groups, this invariant will be named "AS_3 Casimir". For permutations represented by matrices with det $= +1$, the invariant is given by $K_{rot} = \eta_1+\eta_2+\eta_3$. For transpositions for which matrices have det $= -1$, it is given by $K_{inv} = \eta_4+\eta_5+\eta_6$. Taking into account $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ and Eqs. (2.5) we see that, $K_{inv} = \eta_4+\eta_5+\eta_6 = (\vec{m}_4+\vec{m}_5+\vec{m}_6)\cdot\vec{\sigma} = 0$. This means that the invariant K_{inv} can be represented geometrically, in the plane (X, Z), by $\vec{M} = \vec{m}_4+\vec{m}_5+\vec{m}_6 = 0$, and that the equilateral symmetry of the S_3 representation is an intrinsic property of $K_{inv} = 0$.

Eqs. (2.4) and (2.5) permit us to interpret Y_+ and Y_- as spinors. Here, by using another arguments [3, 12], we show that this interpretation is correct. It is well known that the non--relativistic spinor can be introduced in several ways [13]. The interrelation of the various approaches is not obvious and can lead to misconceptions. In order to overcome the necessity of enumerating several approaches, let us stick to a geometrical image, recalling the very fundamental result of group isomorphism [14]: $S_3 \sim PSL_2(F_2)$, where $PSL_2(F_2)$ is the projective group associated with the special group SL_2 defined over a field F_2 with only two elements. Obviously, $PSL_2(F_2) \sim SL_2(F_2)/SL_2(F_2) \cap Z_2$, where the group in the denominator is the centre of SL_2 and corresponds to the central homotheties, since Z_2 is the intersection of the collineation group with SL_2.

If we consider the matrices (2.3) as representing transformations in a two-dimensional complex space characterized by homogeneous coordinates Y_1 and Y_2,

$$\begin{pmatrix} Y_1' \\ Y_2' \end{pmatrix} = \frac{1}{\varrho} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}, \tag{2.6}$$

where ϱ is an arbitrary complex constant and the latin letters substitute the coefficients taken from (2.3), it is clear that (2.3) constitute a homographic (or projective) group.

Making use of definition (2.6) we can see from (2.3) that, apart from the identity η_1, the two matrices η_2 and η_3, which have det $= +1$, are elliptic homographies with fixed points $\pm i$. If we translate these values to the variables of E_3, we see that η_2 and η_3 correspond to finite rotations around the $\vec{j}$ axis by an angle $\theta = \pm 2\pi/3$, agreeing thus with Eqs. (2.4). The remaining matrices η_4, η_5 and η_6 are elliptic involutions, with det $= -1$. They correspond to space inversions in E_3, considered as rotations of $\pm\pi$ around the three axes $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. These matrices completely define the axis of inversion and the angle $\pm\pi$, as is seen from Eqs. (2.5). It is an elementary task to establish the correspondence, via stereographic projection, between the transformations in the two spaces $Y_+(Y_-)$ and E_3.

A topological image can help us to see the 4π invariance of Y_+ and Y_-. If we consider the rotation angle $\theta(\Phi)$ as the variable describing an Euclidean disc, the covering space associated with this disc is a Moebius strip [15]. Adjusting correctly the position of the triangles we have a vivid picture of the rotation properties for each axis. This construction allow us to visualize the double covering of the transformation in E_3 and is a convincing demonstration of the spinor link between E_3 and $Y_\pm$.

We observe that the same transformation properties of Y_+ and Y_- can be obtained if, instead of the equilateral triangle shown in Fig. 1, we consider the triangle drawn in Fig. 2.

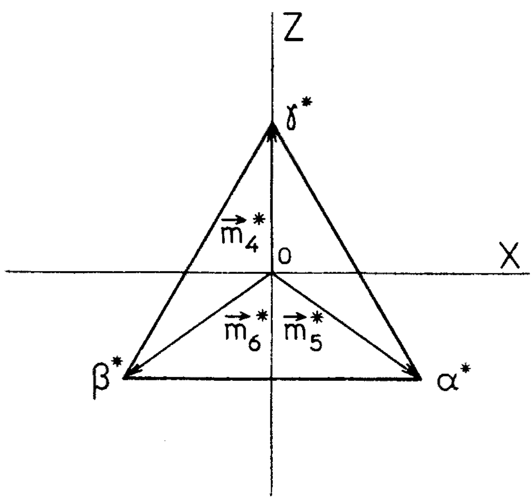

Fig. 2. The equilateral triangle in the Euclidean space (X, Y, Z) with vertices occupied by the states α^*, β^* and γ^*

In the vertices of the equilateral triangle of Fig. 2 we have the states α^*, β^* and γ^*. The unit vectors $\vec{m}_4^*$, $\vec{m}_5^*$ and $\vec{m}_6^*$ are given by $\vec{m}_4^* = -\vec{m}_4$, $\vec{m}_5^* = -\vec{m}_5$ and $\vec{m}_6^* = -\vec{m}_6$. This means that, in this case, K_{inv} is represented geometrically by $\vec{M}^* = \vec{m}_4^*+\vec{m}_5^*+\vec{m}_6^* = 0$. This two-fold possibility for depicting the triangle corresponds, as will be seen in Section 4, to the 3 and 3* representations, respectively, of the SU(3) group.

When two particles occupy the same state as $\alpha = \beta$, for instance, we verify [1, 7] that there is only one 2-dimensional irreducible sub-space associated with gentileons that are now represented by $y(123)$,

$$y(123) = y(\alpha\alpha\gamma) = \frac{1}{\sqrt{2}}\begin{pmatrix} y_1(123) \\ y_2(123) \end{pmatrix}, \tag{2.7}$$

where, $y_1(123) = (|\alpha\alpha\gamma\rangle - |\gamma\alpha\alpha\rangle)/\sqrt{2}$ and $y_2(123) = (2\,|\alpha\gamma\alpha\rangle - |\alpha\alpha\gamma\rangle - |\gamma\alpha\alpha\rangle)/\sqrt{6}$.

Since the $y(123)$ transformations due to the permutation operator P_j are given by the same matrices $\eta_j(j = 1, 2, \ldots, 6)$ defined by Eqs. (2.3) we can conclude that: (a) $y(123)$ is a spinor and (b) $Y(123)$ and $y(123)$ are associated with the same AS_3 Casimir.

In this degenerate case ($\alpha = \beta$) it is not possible to represent permutations as rotations in E_3. Consequently it is not possible to get a geometrical interpretation for the AS_3 Casimir as was seen for gentileons occupying three different states α, β and γ.

In a preceding paper [16] we have shown that gentilionic, bosonic and fermionuic states have completely different topological properties. In particular it was shown that the topological properties of the $Y(3, 1) = Y(123)$ symmetries are clearly exhibited by a T^2 torus generated by two angular variables ϕ and θ that appear in discrete rotations, $R(\phi)$ $R(\theta) = i \exp\left[i\vec{m}\cdot\vec{\sigma}\,\frac{\phi}{2}\right] \exp\left[i\vec{j}\cdot\vec{\sigma}\,\frac{\theta}{2}\right]$, given by Eqs. (2.4) and (2.5). From this work [16] we can see that different state vectors $Y(n, j)$ present different topological properties.

3. Spin and statistics

In this Section the commutation relations for creation (a_α^*) and annihilation (a_α) operators for g_1 gentileons are analysed in order to establish a connection between spin and statistics in Pauli's context [8–10]. It is very important to remark that, according to the Statistical Principle, the number of particles in the $[g_1 g_1 g_1]$ system is constant. Thus, commutation relations for a_α^* and a_α and matrix elements involving gentilionic states are calculated [1] taking into account this fundamental property. We show that when gentileons occupy three different quantum states, a_α^* and a_α obey bilinear anti-commutative relations. Indeed, when two gentileons do not occupy the same quantum state, that is, when $\alpha \neq \beta \neq \gamma \neq \alpha$, we see that the gentilionic commutation relations are given by [1]:

$$[a_i^*, a_j]_+ = \delta_{ij}, \quad [a_i^*, a_i^*]_+ = [a_i, a_i]_+ = 0,$$

$$a_i a_j a_k = a_\alpha a_\beta a_\gamma \eta \begin{pmatrix} kji \\ \gamma\beta\alpha \end{pmatrix} \quad \text{and} \quad a_i^* a_j^* a_k^* = \eta \begin{pmatrix} \alpha\beta\gamma \\ ijk \end{pmatrix} a_\alpha^* a_\beta^* a_\gamma^*, \tag{3.1}$$

where the indices i, j and k can assume the values α, β and γ and $\eta(\ldots)$ are the 2×2 matrices shown in Eqs. (2.3). From the above tri-linear relations one can deduce the bilinear relations seen below applied on gentilionic states Y,

$$a_\beta a_\alpha Y(\alpha\beta\gamma) = Y(00\gamma), \quad a_\alpha a_\beta Y(\alpha\beta\gamma) = \eta\begin{pmatrix}\alpha\beta\gamma\\ \beta\alpha\gamma\end{pmatrix} Y(00\gamma),$$

$$a_\beta a_\alpha Y(\alpha\gamma\beta) = \eta\begin{pmatrix}\alpha\gamma\beta\\ \alpha\beta\gamma\end{pmatrix} Y(00\gamma), \quad a_\alpha a_\beta Y(\alpha\gamma\beta) = \eta\begin{pmatrix}\alpha\gamma\beta\\ \beta\alpha\gamma\end{pmatrix} Y(00\gamma),$$

$$a_\beta a_\alpha Y(\beta\alpha\gamma) = \eta\begin{pmatrix}\beta\alpha\gamma\\ \alpha\beta\gamma\end{pmatrix} Y(00\gamma), \quad a_\alpha a_\beta Y(\beta\alpha\gamma) = Y(00\gamma),$$

$$a_\beta a_\alpha Y(\gamma\alpha\beta) = \eta\begin{pmatrix}\gamma\alpha\beta\\ \alpha\beta\gamma\end{pmatrix} Y(00\gamma), \quad a_\alpha \alpha_\beta Y(\gamma\alpha\beta) = \eta\begin{pmatrix}\gamma\alpha\beta\\ \beta\alpha\gamma\end{pmatrix} Y(00\gamma),$$

$$a_\beta a_\alpha Y(\beta\gamma\alpha) = \eta\begin{pmatrix}\beta\gamma\alpha\\ \alpha\beta\gamma\end{pmatrix} Y(00\gamma), \quad a_\alpha a_\beta Y(\beta\gamma\alpha) = \eta\begin{pmatrix}\beta\gamma\alpha\\ \beta\alpha\gamma\end{pmatrix} Y(00\gamma),$$

$$a_\beta a_\alpha Y(\gamma\beta\alpha) = \eta\begin{pmatrix}\gamma\beta\alpha\\ \alpha\beta\gamma\end{pmatrix} Y(00\gamma), \quad a_\alpha a_\beta Y(\gamma\beta\alpha) = \eta\begin{pmatrix}\gamma\beta\alpha\\ \beta\gamma\alpha\end{pmatrix} Y(00\gamma),$$

$$a_\beta^* a_\alpha^* Y(00\gamma) = \eta\begin{pmatrix}\beta\alpha\gamma\\ \alpha\beta\gamma\end{pmatrix} Y(\alpha\beta\gamma), \quad a_\alpha^* a_\beta^* Y(00\gamma) = Y(\alpha\beta\gamma),$$

$$a_\beta^* a_\alpha^* Y(0\gamma 0) = \eta\begin{pmatrix}\alpha\gamma\beta\\ \alpha\beta\gamma\end{pmatrix} Y(\alpha\beta\gamma), \quad a_\alpha^* a_\beta^* Y(0\gamma 0) = \eta\begin{pmatrix}\beta\gamma\alpha\\ \alpha\beta\gamma\end{pmatrix} Y(\alpha\beta\gamma),$$

$$a_\beta^* a_\alpha^* Y(\gamma 00) = \eta\begin{pmatrix}\gamma\alpha\beta\\ \alpha\beta\gamma\end{pmatrix} Y(\alpha\beta\gamma) \quad \text{and} \quad a_\alpha^* a_\beta^* Y(\gamma 00) = \eta\begin{pmatrix}\gamma\beta\alpha\\ \alpha\beta\gamma\end{pmatrix} Y(\alpha\beta\gamma), \tag{3.2}$$

remembering that there are six intermediate states $Y(\alpha\beta\gamma)$, $Y(\beta\alpha\gamma)$, $Y(\gamma\alpha\beta)$, $Y(\beta\gamma\alpha)$, $Y(\alpha\gamma\beta)$ and $Y(\gamma\beta\alpha)$ that can be assumed by the g_1 system. The above bi-linear relations have been written in order to calculate the non-null matrix elements of the operators $A^* = [a_\alpha^*, a_\beta^*]_+$ and $A = [a_\alpha, a_\beta]_+$.

Since the six different state vectors Y are equivalent for representing the system, all them must be taken into account to calculate the A^* and A matrix elements. Thus using Eqs. (3.2) the $\eta(\ldots)$ matrices and remembering that Y_1, Y_2, Y_3 and Y_4 are orthonormal functions [1], we verify that the expected values $\langle A^*\rangle$ and $\langle A\rangle$ are equal to zero. That is, for $\alpha \neq \beta$, $\langle [a_\alpha^*, a_\beta^*]_+\rangle = \langle [a_\alpha, a_\beta]_+\rangle = 0$. As only the expected values $\langle A^*\rangle$ and $\langle A\rangle$ have a physical meaning we see, according to the above results and to the bilinear terms of Eqs. (3.1), that the following bilinear commutation relations can be taken as valid for g_1 gentileons in the framework of a quantum field theory,

$$[a_i^*, a_j]_+ = \delta_{ij} \quad \text{and} \quad [a_i^*, a_j^*)_+ = [a_i, a_j]_+ = 0, \tag{3.3}$$

where the indices i, j and k can assume the values α, β and γ.

As g_1 gentileons obey bilinear anti-commutative relations defined by Eqs. (3.3) it is possible to construct for these gentileons a consistent local, Lorentz-invariant quantum field theory. Moreover, we conclude from Eqs. (3.3) and the use of Pauli's theorem [8–10] that g_1 gentileons must be half-odd-integral spin particles.

It is important to note that the above results have been obtained assuming that gentileons occupy three different quantum states, $\alpha \neq \beta \neq \gamma \neq \alpha$. When two gentileons occupy the same quantum state, as one can easily verify [1], the operators a_α^* and a_α do not obey bilinear anti-commutative or commutative relations. Thus, integral or half-odd-integral spin gentileons could not be represented, by state vectors $Y(nnm)$, where $n, m = \alpha, \beta$ and γ. Consequently these states are prohibited in Pauli's context.

4. The S_3 symmetry and the SU(3) eigenstates

In Section 2 we have shown that it was possible to interpret the $Y(123) = Y(\alpha\beta\gamma)$ in terms of rotations, in an Euclidean space E_3, of only two equilateral triangles with vertices occupied by three privileged states $\alpha(\alpha^*)$, $\beta(\beta^*)$ and $\gamma(\gamma^*)$. Y must constitute symmetry adapted kets for S_3. In other words, their disposition in the plane of the triangle must agree with the imposition made by the AS_3 Casimir. According to Fig. 1, these states are defined by, $\alpha = \vec{m}_5 = (-\sqrt{3}/2, 1/2)$, $\beta = \vec{m}_6 = (\sqrt{3}/2, 1/2)$ and $\gamma = \vec{m}_4 = (0, -1)$, and according to Fig. 2, $\alpha^* = \vec{m}_5^* = -\vec{m}_5$, $\beta^* = \vec{m}_6^* = -\vec{m}_6$ and $\gamma^* = \vec{m}_4^* = -\vec{m}_4$. The equilateral triangle symmetry for S_3 plays a fundamental role in E_3, allowing us to obtain a very simple and elegant geometrical interpretation for the invariant $K_{\text{inv}} = 0$. Indeed, since the S_3 symmetry, according to Section 2, implies that $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$ ($\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0$), we conclude that $\vec{M} = 0$ ($\vec{M}^* = 0$), pictured in E_3, is a null constant of motion.

At this point we compare our states α, β and γ with the SU(3) eigenstates [17–19] $|n\rangle$, $|p\rangle$ and $|\lambda\rangle$. These states are eigenstates of the hypercharge Y and of the isospin I_3 both diagonal generators of the algebra of the SU(3). The eigenstates $|n\rangle$, $|p\rangle$ and $|\lambda\rangle$ are written as $|n\rangle = |-1/2, 1/3\rangle$, $|p\rangle = |1/2, 1/3\rangle$ and $|\lambda\rangle = |0, -2/3\rangle$.

Remembering that the SU(3) and the intermediate S_3 fundamental symmetries are defined by equilateral triangles, it is quite apparent that the states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ can be represented by eigenstates of I_3 and Y. Indeed, assuming that the axes X and Z (see Fig. 1) correspond to the axes I_3 and Y, respectively, and adopting the units along these axes as the side and the height of the triangle [18] we verify that $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ would be given by, $|\alpha\rangle = |n\rangle = |-1/2, 1/3\rangle$, $|\beta\rangle = |p\rangle = |1/2, 1/3\rangle$ and $|\gamma\rangle = |\lambda\rangle = |0, -2/3\rangle$. If we have considered the states $|\alpha^*\rangle$, $|\beta^*\rangle$ and $|\gamma^*\rangle$, seen in Fig. 2, we should verify that these states would correspond to the states $|n^*\rangle$, $|p^*\rangle$ and $|\lambda^*\rangle$ of the 3^* representation.

Thus, if we assume that the states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ correspond to $|n\rangle$, $|p\rangle$ and $|\lambda\rangle$, respectively, each unit vector $\vec{m}_j$ ($j = 4$, 5 and 6) is represented, in the plane (I_3, Y) by the operator $q = I_3 + Y/2$. This means that the vector $\vec{M}$ will be represented by the operator $M = q_1 + q_2 + q_3$, where the indices 1, 2 and 3 refer to the three gentileons of the system. Thus, adopting the SU(3) eigenvalues we see that the expected values $\langle M \rangle = 0$, for the 3 and 3* representations, must be a constant of motion.

We conclude that the fundamental symmetry properties of the state function $Y(\alpha\beta\gamma)$ are described by the intermediate S_3 and SU(3) groups.

We intend to analyse in a forthcoming paper systems composed of four indentical gentileons. Our intention is to determine what kind of groups, besides the intermediate S_4, are necessary to describe the fundamental symmetry properties of these systems. It will be shown, for instance, that the $[g_2g_2g_2g_2]$ state vector symmetries are described by the intermediate S_4 and SU(4) groups.

5. *Fundamental properties of g_1 systems*

Let us summarize the fundamental properties predicted for the g_1 systems:

(1) Gentileons g_1 are not permitted to form systems with more than three entities. Only $[g_1g_1g_1]$ systems can be formed.

(2) Two systems $[g_1g_1g_1]$ and $[g_1g_1g_1]$ cannot coalesce, that is, cannot form a system composed by six indistinguishable particles $[g_1g_1g_1g_1g_1g_1]$.

(3) The state function $Y(3, 1) = Y(123)$ has a spinor character.

(4) Gentileons g_1 are half-odd-integral entities represented by the state vector $Y(123) = Y(\alpha\beta\gamma)$, where α, β and γ are three different quantum states.

(5) The fundamental symmetry properties of $Y(\alpha\beta\gamma)$ are described by the intermediate S_3 and SU(3) groups.

(6) There must exist some conserved physical quantity associated with the AS_3 Casimir $\langle M \rangle = 0$.

As pointed out before, confinement and non-coalescence are intrinsic properties of gentileons: they could correspond to real particles or to dynamical entities as quantum collective excitations. If gentileons g_1 were real particles there must be some kind of mechanism to explain these properties: a very peculiar interaction potential, an impermeable bag or something else. It seems reasonable to expect that this mechanism is intimately related to, or is a consequence of the local SU(3) symmetry. If these astonishing predictions had been done about 30 years ago probably the gentilionic states would be taken as non-physical representations of the permutation group in quantum mechanics and promptly discarded. Today, however, this situation is somewhat modified since, as will be shown in next Section, basic hadronic properties will be explained assuming quarks as g_1 gentileons.

6. *The gentilionic hadrons*

Since g_1 gentileons are spin 1/2 confined entities that cannot form systems with more than three indistinguishable particles and their systems, with symmetry properties described by the SU(3) group, are non-coalescent it seems natural to think quarks q as g_1 gentileons. With this hypothesis we can show that baryons $[qqq]$, that are composed of three indistinguishable gentileons in color space, are represented by wavefunctions [2, 3]

$\psi = \varphi \cdot Y(\text{brg})$. The state $\varphi = (\text{SU}(6) \times \text{O}_3)_{\text{symmetric}}$ corresponds, according to the symmetric quark model of baryons, to a totally symmetric state. The state function $Y(\text{brg})$ corresponds to the intermediate state $Y(123)$ written in terms of the $\text{SU}(3)_{\text{color}}$ eigenstates blue (b), red (r) and green (g). This function that can be taken as $Y_+(\text{brg})$ or $Y_-(\text{brg})$, shown in Section 2, will be named "colorspinor" [3].

From the above results and observing Section 4 we see that in the gentilionic formalism one possibility is to define the individual quark charge as,

$$q = q_f + \tilde{q}_c = (I_3 + Y/2) + \lambda(\tilde{I}_3 + \tilde{Y}/2), \tag{6.1}$$

where $q_f = I_3 + Y/2$ refers to flavor charge $\tilde{q}_c = \lambda(\tilde{I}_3 + \tilde{Y}/2)$ refers to color charge and λ is a constant parameter. With this definition, the total color bayron charge $\tilde{Q}$ is given by $\tilde{Q} = \lambda\langle\tilde{M}\rangle$, where $\tilde{M} = \tilde{q}_1 + \tilde{q}_2 + \tilde{q}_3$, following Section 4. Remembering that the expected value $\langle\tilde{M}\rangle$ is a constant of motion equal to zero, that is, $\langle\tilde{M}\rangle = \text{constant} = 0$, as shown in Section 4 for the state $Y(\text{brg})$, we see that the generalized Gell-Mann-Nishijima relation is automatically satisfied [2, 3] independently of the λ value. However, we must note that to preserve the gentilionic character of the quarks it is necessary to put $\lambda = 0$. Thus, in our approach quarks have fractional charges, in agreement with Gell--Mann results. We see that the baryon color charge $\tilde{Q}$ is the physical conserved quantity associated with the AS_3 Casimir $\langle\tilde{M}\rangle = 0$ that is named "color Casimir" [2, 3].

In our approach [1–3] mesons are composed of a quark-antiquark pair $[q\bar{q}]$. According to the statistical principle (see Introduction), systems like q, $[qq]$, $[qq\bar{q}]$ and $[qq\bar{q}\bar{q}]$, for instance, are prohibited. Of course baryons with more than three quarks q are also prohibited. Thus, only the systems $[q\bar{q}]$ and $[qqq]$ are allowed in the gentilionic theory.

Since q and $\bar{q}$ are different particles in color space we can conclude, in agreement with our general results [1], that mesons $[q\bar{q}]$ are represented by one-dimensional state functions. This implies, remembering that q and $\bar{q}$ are spin 1/2 particles, that the system $[q\bar{q}]$ is represented in fermionic and gentilionic theories by the same state vector.

According to the gentilionic theory proton must be stable [1–3]. This stability, predicted as a selection rule, is a consequence of the spinor character of the baryon states: proton decay is forbidden because the spinor character of the initial current (proton) would not be present in the final current.

From the above analysis we see that fundamental properties of hadrons can be explained assuming quarks as g_1 gentileons. In spite of our stimulating general results, these remains the crucial problem of determining the intrinsic nature of the quarks and their dynamical properties. In the next Section taking quarks as g_1 gentileons, a quantum chromodynamics is proposed where, instead of fermions, gentileons interact with gluons.

7. *A quantum chromodynamics for gentilionic hadrons*

To construct a quantum field theory for hadrons assuming quarks as g_1 gentileons we must take into account the $\text{SU}(3)_{\text{color}}$ and S_3 symmetries and remember, according to Section 3, that the creation and annihilation operators for g_1 gentileons obey bilinear anti-commutative relations. The gentilionic field approach must be formulated in order to predict, as conservation laws or selection rules, the hadronic properties deduced in Section 6: (a) only $[q\bar{q}]$ and $[qqq]$ hadrons can exist in nature, (b) quark-confinement, (c) non-coalescence of hadrons, (d) proton stability and (e) the hadron color charge is a constant of motion equal to zero. This is a very ambitious and extremely difficult task. Since we were not able, up to now, to develop such a formalism an alternative one will be proposed. In this way, let us suggest as a first approximation the following Lagrangian density for gentilionic quarks interacting with gluons,

$$L = \sum_f \left[i q_a^+ \gamma^\mu \frac{\partial}{\partial x^\mu} q_a + g q_a^+ \gamma^\mu \left(\frac{\lambda_i}{2}\right)_{ab} A_\mu^i q_b - m_f q_a^+ q_a \right]$$

$$-\tfrac{1}{4}\left(\frac{\partial A_\nu^i}{\partial x^\mu} - \frac{\partial A_\mu^i}{\partial x^\nu} + g f_{ijk} A_\mu^i A_\nu^k\right)^2, \qquad (7.1)$$

where the summation is over the flavors $f =$ u, d, s, c, ... The summation over repeated indices $a, b, \ldots$, referring to color is understood. The A_μ^i is the gauge-field, $\lambda_i/2$ are the 3×3 matrix representation of the $SU(3)_{color}$ algebra generators, satisfying the commutation relations $[\lambda_i, \lambda_j] = if_{ijk}\lambda_k/2$, where f_{ijk} are the SU(3) structure constants. The flavor symmetry is only broken by the lack of degeneracy in the quark masses. Finally, the quark free fields $q(x)$ are expanded in terms of positive and negative frequency solutions, $\varphi_{k+}(x)$ and $\varphi_{k-}(x)$, of Dirac's equation,

$$q(x) = \sum_k \{a_{k+}\varphi_{k+}(x) + a_{k-}^* \varphi_{k-}(x)\},$$

where a_i and a_i^* obey fermionic commutation relations.

With the above assumptions, both theories, the usual QCD and the gentilionic QCD, indicated by QCDG, will have the same gluons and the same Lagrangian density. In both approaches the previously mentioned properties (a), (b), ... and (e) appear as additional conditions. In these circumstances, both theories will give identical predictions for hadronic properties. In spite of this we note that they are not equivalent. Indeed, in QCDG, the five conditions cited above appear naturally, deduced from first principles, whereas in QCD they are imposed "ad hoc".

Since in QCDG quarks are taken as real particles it must exist, according to Section 5, some kind of mechanism intimately related with the $SU(3)_{color}$ symmetry that would be responsible for the confinement and non-coalescence properties. Hopes for a theoretical explanation of quark confinement are pinned on the non-Abelian nature of the $SU(3)_{color}$ group which is the gauge invariance group of the quantum chromodynamics. In spite of considerable efforts only indications for confinement have been found. Since no rigorous proof for confinement has been obtained, this problem has been considered, by way of a mathematical analogy, as the "Fermat" theorem of the contemporary particle theory [20].

Finally let us consider hadronic matter in the high density domain that occurs at ultra relativistic hadronic collisions, at the core of neutron stars and at the early stages of the universe. If in these extreme conditions permutation symmetries are preserved we must expect, due to the non-coalescence property of gentilionic systems, that the hadron structure is maintained. That is, hadrons will not be destroyed but only highly compressed. Thus, in these conditions quarks will be so closely packed that the interactions between them will be weak due to the assymptotic freedom. This means that, according to the gentilionic theory, the dense hadronic matter would be constituted of free quarks. These quarks however are confined inside compressed hadrons and not forming an ideal gas (quark plasma) as predicted by the fermionic approach.

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