REVISED GAUGE THEORY OF GRAVITATION WITH GRAVITATIONAL LAGRANGIAN $L_g = \alpha(\Omega^i_{.k} \wedge \eta_i^{\ k} + \Theta^i \wedge * \Theta_i) + \beta\Omega^i_{.k} \wedge * \Omega^k_{.i}$*

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The revised model of the gauge gravitational theory presented by author in previous papers cited in References is considered. This revised model has simplified macroscopic limit owing to the natural condition that the magnitude of the microscopic spin vanishes in macroscopic limit, i.e., when Planck's constant tends to zero. The model restricted by the algebraic constraints $Q_l := Q^i_{.li} = 0$ is also considered.

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1. Introduction

Gauge theory of gravitation having gravitational Lagrangian

$$L_g = \alpha(\Omega^i_{.j} \wedge \eta_i^{\ j} + \Theta^i \wedge * \Theta_i) + \beta\Omega^i_{.j} \wedge \Omega^j_{.i},$$

where

$$\alpha = \frac{c^4}{16\pi G}, \quad \beta = \alpha A = \alpha\frac{K^2 hG}{c^3} = \frac{K^2 hc}{16\pi}, \quad K \in R^+ \tag{1}$$

(most probably $K = 1$), was studied by author in [1]. $\Omega^i_{.j}$ and Θ^i denote here the curvature 2-form and the torsion 2-form of the Riemann-Cartan connection $\omega^i_{.k}$ respectively and $\eta_i^{\ j} = g^{jk}\eta_{ik}$ is the pseudotensorial 2-form introduced by Trautman [2]. $*$ means the Hodge-star-operator [3]. The latin indices run over 0, 1, 2, 3 and metric signature is $(+ - - -)$. h denotes Planck's constant, c is the value of the light velocity in vacuum and G denotes Newtonian gravitational constant. The Lagrangian (1) has very good physical and geometrical motivation (see [1]).

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Performing standard calculation we get the following gravitational field equations

$$D * \Omega^{j}_{.i} = (-)\frac{\alpha}{2\beta}(\vartheta^{j} \wedge * \Theta_{i} - \vartheta_{i} \wedge * \Theta^{j}) - \frac{\alpha}{2\beta}\Theta_{k} \wedge \eta_{i}^{jk} - \frac{S_{i}}{4\beta}, \tag{2}$$

$$D * \Theta_{l} = (-)\Omega^{jk} \wedge \eta_{ljk} + \left[\left(Q^{b}_{.lr}Q_{b}^{;pr} - \frac{\delta^{p}_{t}}{4}Q^{btr}Q_{btr}\right)\right.$$

$$\left. + \frac{\beta}{\alpha}\left(\frac{\delta^{p}_{l}}{4}R^{ijrm}R_{ijrm} - R^{ij}_{\ \ lt}R_{ij}^{\ \ pt}\right)\right]\eta_{p} - \frac{t_{l}}{2\alpha}. \tag{3}$$

Equations (2)–(3) have the same form as equations of a gauge field with sources given by the right hand sides. After decomposing the field equations (2)–(3) in the basis given by the 3-forms η_i we get the following tensorial equations

$$\nabla_{m}R_{li}^{\ \ pm} + R_{li}^{\ \ pt}Q_{t} + \tfrac{1}{2}R_{li}^{\ \ tn}Q^{p}_{.tn} = (-)\frac{\alpha}{2\beta}(Q^{p}_{i.l} - Q^{p}_{l.i}$$

$$+ Q_{l}\delta^{p}_{i} - Q_{i}\delta^{p}_{l} + Q^{p}_{.il}) - \frac{1}{4\beta}S^{p}_{.i}l, \tag{4}$$

$$\nabla_{k}Q_{bp}^{\ \ k} + Q_{bp}^{\ \ k}Q_{k} + \tfrac{1}{2}Q_{b}^{\ lk}Q_{plk} = G_{pb} - \left(\frac{g_{bp}}{4}Q^{itr}Q_{itr}\right.$$

$$\left. - Q^{i}_{.br}Q_{ip}^{\ \ r}\right) - \frac{\beta}{\alpha}\left(R^{ij}_{\ \ bt}R_{ijp}^{\ \ \ t} - \frac{g_{bp}}{4}R^{ijrt}R_{ijrt}\right) - \frac{t_{pb}}{2\alpha}. \tag{5}$$

In the above formulae D means the exterior covariant derivative, ϑ^i denotes the coreper and $R^{i}_{.klm} = (-)R^{i}_{.kml}$ and $Q^{i}_{.kl} = (-)Q^{i}_{.lk}$ are the curvature and torsion components respectively; ∇ means the covariant derivative and $G_{pb} := R_{pb} - \frac{g_{pb}}{2}R$ are the components of the Einstein tensor. t_i and $S_i^{\ j}$ denote the canonical energy-momentum 3-form and the canonical spin 3-form of the microscopic matter (fundamental particles) respectively. The canonical tensors $t^{p}_{.i}$ and $S^{p}_{.il} = (-)S^{p}_{.li}$ are defined by the decompositions

$$t_{i} = \eta_{p}t^{p}_{.i}, \qquad S_{i}^{\ j} = \eta_{p}S^{p\ j}_{.i}. \tag{6}$$

The field equations of the theory are 2-nd order differential equations w.r.t. coreper ϑ^i and connection $\omega^{i}_{.k}$ and, simultaneously, they are 3-rd order differential equations w.r.t. metric and torsion components (or w.r.t. metric and defect components).

In tensor notation and inside of matter we have the system of 40 nonlinear, partial 3-rd order differential equations on the 40 unknown functions: 6 intrinsic metric components, 24 intrinsic torsion or defect components and 10 functions describing phenomenologically microscopic matter.

In the framework of the theory there exist the so-called "differential identities" (DI) ([1, 2, 4, 5]) having the following form

$$DP_i^{\,j} = \vartheta_i \wedge e^j - \vartheta^j \wedge e_i, \tag{7}$$

$$De_i = Q^j_{.i} \wedge e_j + \tfrac{1}{2} R^j_{.ki} \wedge P_j^{\,k}. \tag{8}$$

The 1-forms $Q^j_{.i}$ and $R^j_{.ki}$ are defined by

$$Q^j_{.i} := Q^j_{.ik}\vartheta^k, \tag{9}$$

$$R^j_{.ki} := R^j_{.kil}\vartheta^l, \tag{10}$$

and $P_i^{\,j}$ and e_i denote the 3-forms constituting geometrical part of the field equations, i.e.,

$$P_i^{\,j} = 4\beta D * \Omega^j_{.i} + 2\alpha(\vartheta^j \wedge * \Theta_i - \vartheta_i \wedge * \Theta^j + \Theta_k \wedge \eta_i^{\,jk}), \tag{11}$$

$$e_l = 2\alpha\left[D * \Theta_l + \tfrac{1}{2}\Omega^{jk} \wedge \eta_{ljk} - \left(Q^b_{.lr}Q_b^{.pr} - \frac{\delta_l^p}{4} Q^{btr}Q_{btr}\right)\eta_p\right]$$

$$-2\beta\left(\frac{\delta_l^p}{4} R^{ijrt}R_{ijrt} - R^{ij}_{\;\;lt}R_{ij}^{\;\;pt}\right)\eta_p. \tag{12}$$

The differential identities result from the invariance properties of the Lagrangian (1). In vacuum, the differential identities give, in general, 10 relationships between 40 vacuum field equations. In consequence, we have only 30 independent field equations in vacuum on 30 unknown functions: 6 intrinsic metric components and 24 intrinsic torsion components (or defect).

A suitable combination of the differential identities and field equations of the theory leads us to the following covariant, differential conservation laws for matter ([1, 2, 4, 5])

$$DS^{ij} = \vartheta^i \wedge t^j - \vartheta^j \wedge t^i, \tag{13}$$

$$Dt_i = Q^j_{.i} \wedge t_j + \tfrac{1}{2} R^{jk}_{..i} \wedge S_{jk} \tag{14}$$

or, in tensor notation

$$\nabla_l S^l_{.ij} + S^l_{.ij}Q_l = t_{ij} - t_{ji}, \tag{15}$$

$$\nabla_j t^j_{.i} + t^j_{.i}Q_j = Q^j_{.il}t^l_{.j} + \tfrac{1}{2} R^{jk}_{..il}S^l_{.jk}. \tag{16}$$

In order to get the integral conservation laws (non-covariant) for gravity and matter one must transform the field equations of the theory to the so-called superpotential form [1]

$$d(-2\alpha * \Theta_l) = \Bigg[\alpha\Omega^{jk} \wedge \eta_{ljk} - 2\alpha\omega^k_{.l} \wedge * \Theta_k + 2\alpha\left(\frac{\delta_l^p}{4} Q^{btr}Q_{btr}\right.$$

$$-Q^{b}_{.lr}Q_{b}^{;pr}\Big)\eta_p+2\beta\left(R^{ij}_{..lt}R_{ij}^{\ \ pt}-\frac{\delta^p_l}{4}R^{ijrm}R_{ijrm}\right)\eta_p+t_l\Bigg], \tag{17}$$

$$d(-4\beta*\Omega^{l}_{.i}) = [2\alpha(\vartheta^l\wedge*\Theta_i-\vartheta_i\wedge*\Theta^l)-4\beta(\omega^{p}_{.i}\wedge*\Omega^{l}_{.p}$$
$$-\omega^{l}_{.p}\wedge*\Omega^{p}_{.i})+2\alpha\Theta_k\wedge\eta_i^{\ lk}+S_i^{\ l}]. \tag{18}$$

From (17) and (18) there follow the following integral conservation laws

$$\int_{\partial\Omega}\Bigg[t_l+\alpha\Omega^{jk}\wedge\eta_{ljk}-2\alpha\omega^{k}_{.l}\wedge*\Theta_k+2\alpha\Bigg(\frac{\delta^p_l}{4}Q^{btr}Q_{btr}$$
$$-Q^{b}_{.lr}Q_{b}^{;pr}\Bigg)\eta_p+2\beta\left(R^{ij}_{..lt}R_{ij}^{\ \ pt}-\frac{\delta^p_l}{4}R^{ijrm}R_{ijrm}\right)\eta_p\Bigg] = 0. \tag{19}$$

$$\int_{\partial\Omega}[S_i^{\ l}+2\alpha(\vartheta^l\wedge*\Theta_i-\vartheta_i\wedge*\Theta^l)-4\beta(\omega^{p}_{.i}\wedge*\Omega^{l}_{.p}-\omega^{l}_{.p}\wedge*\Omega^{p}_{.i})$$
$$+2\alpha\Theta_k\wedge\eta_i^{\ lk}] = 0. \tag{20}$$

Here $\partial\Omega$ is the 3-dimensional boundary of an arbitrary 4-dimensional domain Ω in the space-time.

From the physical point of view, the theory based on the Lagrangian (1) is an example of the microscopic gravitational theory with microscopic sources (fundamental particles). We will denote this theory (in short) by MicGT. The material tensors t_{pb} and $S^{p}_{.il}$ which are present in the field equations (4)–(5) denote the canonical, non-symmetric in general, energy-momentum tensor and canonical spintensor of the microscopic matter (fundamental particles) respectively. The components $S^{p}_{.il}$ of the spintensor and the components of the spin 3-form $S_i^{\ j}$ have magnitudes proportional to the Planck constant h and, therefore, their values vanish in the macroscopic limit $h \to 0$. In consequence, if $h \to 0$ ($\equiv \beta \to 0$), then the equations (4)–(5) of the MicGT take the following form

$$Q^{i}_{.kl} = 0, \tag{21}$$

$$G_{pb} = \frac{1}{2\alpha}\,{}^{\mathrm{M}}t_{(pb)}, \qquad {}^{\mathrm{M}}t_{[pb]} = 0, \tag{22}$$

i.e., they take form of the Einstein equations of GR. ${}^{\mathrm{M}}t_{pb}$ means here the energy-momentum tensor obtained from the microscopic tensor t_{pb} by limiting process $h \to 0$. Thus, the Einsteinian general relativity (GR) still remains correct gravitational theory but only in macroscopic domain and the both pure gauge terms $\beta\Omega^{i}_{.k}\wedge*\Omega^{k}_{.i}$ and $\alpha\Theta^i\wedge\Theta_i$ of the Lagrangian (1) are valid only in microscopic domain.

The macroscopic limit of the MicGT given by (21) and (22) is simpler than the limit obtained in our previous papers (see [1]) without using the fact that the magnitude of the microscopic spin of fundamental particles vanishes in macroscopic limit $h \to 0$. Now we think that it was incorrect.

Useful property of the MicGT are special torsion constraints. We obtain them in the following way. Let us consider the antisymmetric part of the field equations (5). We get from them

$$R_{[pb]} = \nabla_k Q_{[bp]}{}^k + Q_{[bp]}{}^k Q_k + \frac{1}{2\alpha} t_{[pb]}. \tag{23}$$

On the other hand, the torsion Bianchi identities impose the following structure on $R_{[pb]}$

$$R_{[pb]} = \tfrac{1}{2} \nabla_k Q^k_{.bp} + \nabla_{[b} Q_{p]} + \tfrac{1}{2} Q_n Q^n_{.bp}. \tag{24}$$

Comparing (23) and (24) we get the following torsion constraints

$$\nabla_k Q_{[bp]}{}^k + Q_{[bp]}{}^k Q_k + \frac{1}{2\alpha} t_{[pb]} = \tfrac{1}{2} \nabla_k Q^k_{.bp} + \nabla_{[b} Q_{p]} + \tfrac{1}{2} Q_n Q^n_{.bp}. \tag{25}$$

The constraints (25) are rewritten (with the help of the torsion Bianchi identities) antisymmetric part of the field equations (5). They admit torsionless solutions in vacuum and inside microscopic matter having symmetric energy-momentum tensor only. The constraints (25) are very useful if we want to investigate existence of the solutions with torsion (in a given situation) in vacuum or inside matter having a symmetric energy-momentum tensor. Then they take the simpler form

$$\nabla_k Q_{[bp[}{}^k + Q_{[bp]}{}^k Q_k = \tfrac{1}{2} \nabla_k Q^k_{.bp} + \nabla_{[b} Q_{p]} + \tfrac{1}{2} Q_n Q^n_{.bp}. \tag{26}$$

We have the following Criterion "C" [1].

In a given vacuum or interior problem with a symmetric energy-momentum tensor the solutions to the field equations (4)–(5) having dynamical torsion may exist when: (*i*) The constraints (26) are identically fulfilled (reduce to the form 0 = 0) and the system of the field equations (with not entirely vanishing torsion) which must be solved is not overdetermined, or

(*ii*) The constraints (26) immediately follow from the field equations of the theory (the trivial consistency of the constraints with the field equations) and the system of the field equations is not overdetermined.

The Criterion "C" gives the conditions under which torsion Bianchi identities are compatible with the antisymmetric part of the field equations (5). In all the cases when the solutions having dynamical torsion exist this Criterion is satisfied. If the Criterion is not fulfilled, then the torsion Bianchi identities contradict the antisymmetric part of the field equations (5) and there exist only torsionless solutions (no counter-example is known). We have used the Criterion "C" when we have investigated the existence of the solutions with torsion to the field equations (4)–(5) in the spherical symmetry problems and in cosmology (see [1]).

The exact MicGT is satisfactory from the formal point of view because: it is causal and deterministic, it admits Hamiltonian formulation, satisfies Birkhoff's theorem and it attributes energy-momentum tensors to the gravitational field (see [1]). Moreover, in the framework of the theory there exist interesting cosmological solutions without singularities [1] and the global quantities of an isolated system (especially global energy) are equal to zero (see Sect. 2). However, the linearized theory admits tachyons and ghosts connected with the vectorial part $Q_l = Q^i_{.li}$ of torsion. The linearized theory is considered in Section 3. The covariant algebraic constraints $Q_l = 0$ exclude tachyons and ghosts connected with the vectorial torsion. Therefore, we proposed previously [1] to confine to the restricted model (RMicGT) of the theory with the L_g given by (1). The suitable restrictions are given by the covariant constraints $Q_l = 0$ imposed on Riemann-Cartan geometry and excluding vectorial torsion (see Section 4). But now we think that no limitation of the theory is necessary because the global energy of an isolated, finite system is nonnegative also in the framework of the linearized theory.

2. *The global quantities of an isolated, finite system*

Let us consider an isolated, non-radiative, and finite material system. Space-time around of such a system is asymptotically flat or asymptotically Newtonian [7, 14], i.e., the gravitational field has, at very great distances from the sources, spherically symmetric leading terms of the order $O\left(\frac{1}{r^2}\right)$. Substituting to the field equations (19)–(20) the g_{ik} determined by the asymptotic form of the line element

$$ds^2 = ds_0^2 - \frac{2GM}{c^2 r}(dr^2 + c^2 dt^2) + O\left(\frac{1}{r^2}\right), \tag{27}$$

where

$$r = \sqrt{x^2+y^2+z^2}, \qquad dr^2 = dx^2+dy^2+dz^2,$$

$$M \text{ is the mass of the system,} \tag{28}$$

and ds_0^2 is the Minkowskian line element and the torsion components

$$Q^i_{.kl} = \frac{\overset{0}{Q}{}^i_{.kl}}{r^2} + O\left(\frac{1}{r^3}\right), \tag{29}$$

where $\overset{0}{Q}{}^i_{.kl} = \text{const}$, we get the following equations to the lowest order $O\left(\frac{1}{r^2}\right)$

$$\overset{0}{Q}{}_i{}^p{}_l - \overset{0}{Q}{}_l{}^p{}_i + \overset{0}{Q}_l \delta^p_i - \overset{0}{Q}_i \delta^p_l + \overset{0}{Q}{}^p_{.il} = 0. \tag{30}$$

The remaining equations are identically fulfilled to the lowest order $O\left(\frac{1}{r^2}\right)$ by the gravitational field given by (27)–(29).

From (30) it follows that $\overset{0}{Q}{}^{i}_{.kl} = 0$; therefore $Q^{i}_{.kl} = O\left(\frac{1}{r^3}\right)$. We conclude from that the asymptotically flat space-time of an isolated, finite material system is, in the framework of the MicGT, torsion-free to the lowest order $O\left(\frac{1}{r^2}\right)$, i.e., for such a space-time

$$Q^{i}_{.kl} = O\left(\frac{1}{r^3}\right), \quad R^{i}_{.klm} = O\left(\frac{1}{r^3}\right) \Rightarrow \Theta_l = O\left(\frac{1}{r^3}\right), \quad \Omega^{i}_{.k} = O\left(\frac{1}{r^3}\right). \tag{31}$$

From (17) and (18) we have the following integrals representing the global energy-momentum

$$P_l = (-)2\alpha \oint\!\!\oint_{\Sigma(\infty)} * \Theta_l \tag{32}$$

and global spin

$$S^{i}_{.l} = (-)4\beta \oint\!\!\oint_{\Sigma(\infty)} * \Omega^{i}_{.l} \tag{33}$$

of gravitation and matter. $\Sigma(\infty)$ denotes here the 2-dimensional boundary of a spatial hypersurface $x^0 = \text{const}$. We will take this boundary in the form of the sphere having infinite radius R. Then it follows from (31), (32) and (33) for an isolated, finite material system

$$P_l = (-)2\alpha \oint\!\!\oint_{\substack{\text{over sphere}\\ \text{with } R\to\infty}} * \Theta_l = (-)2\alpha \lim_{R\to\infty} \oint\!\!\oint_{\substack{\text{over}\\ \text{sphere}}} O\left(\frac{1}{R^3}\right) R^2 d\Omega = 0, \tag{34}$$

$$S^{i}_{.l} = (-)4\beta \oint\!\!\oint_{\substack{\text{over sphere}\\ \text{with } R\to\infty}} * \Omega^{i}_{.l} = (-)4\beta \lim_{R\to\infty} \oint\!\!\oint_{\substack{\text{over}\\ \text{sphere}}} O\left(\frac{1}{R^3}\right) R^2 d\Omega = 0, \tag{35}$$

i.e., that the global quantities, especially global energy, vanish for such a system.

If we use for energy-momentum the superpotentials

$$d(-2\alpha * \Theta_l - \alpha\omega^{j}_{.k} \wedge \eta_{lj}{}^{k}) \tag{36}$$

which differ from the superpotentials $(-)2\alpha d * \Theta_l$ by the exact form $(-)\alpha d(\omega^{j}_{.k} \wedge \eta_{lj}{}^{k})$, then we get for an isolated, finite material system (in the coordinates in which the system is globally at rest)

$$P_0 = \frac{E}{c} = Mc, \quad P_\alpha = 0, \tag{37}$$

i.e., the same result as in the framework of GR.

The superpotentials determined by (17)–(18) are exactly the same as the superpotentials for a gauge field. Using them we have then the following result: for an isolated, finite material system, gravitational and matter energy globally cancel.

3. Linearization

Let us consider the linearized equations of the theory[1]. Generalizing the standard linearization procedure of the GR equations in holonomic coordinates [6] on the space-time with torsion (see also [8]) one can linearize the MicGT equations (4)–(5) and obtain the following system of the linearized field equations (LFE)

$$\Box \Psi^{pb} = (-)\frac{1}{\alpha}\, t^{(pb)} - 4\partial_j Q^{(bp)j} + 2\partial^{(b}Q^{p)} - 2\overset{(0)}{g}{}^{pb}\partial_a Q^a, \tag{38}$$

$$\partial^{[b}Q^{p]} - \partial_j Q^{[bp]j} + \tfrac{1}{2}\,\partial_j Q^{jbp} = \frac{1}{2\alpha}\, t^{[pb]}, \tag{39}$$

$$\partial^l \,\Box\, h^{pi} - \partial^i \,\Box\, h^{pl} - \partial_m\partial^p (Q^{lim} + Q^{iml} + Q^{mil})$$

$$+ \Box\, (Q^{lip} + Q^{ipl} + Q^{pil} + \frac{\alpha}{\beta}(Q^{ipl} - Q^{lpi}$$

$$+ Q^l \overset{(0)}{g}{}^{ip} - Q^i \overset{(0)}{g}{}^{lp} + Q^{pil}) = (-)\frac{1}{2\beta}\, S^{pil}, \tag{40}$$

$$\partial_k \Psi^{ik} = \partial_k \Psi^{ki} = 0. \tag{41}$$

In the above equations $g_{ik} = \overset{(0)}{g}_{ik} + h_{ik}$,

$$g^{ik} = \overset{(0)}{g}{}^{ik} - h^{ik} = \overset{(0)}{g}{}^{ik} - \overset{(0)}{g}{}^{il}\,\overset{(0)}{g}{}^{km}\, h_{lm}, \tag{42}$$

where $|h_{ik}| \ll 1$ and sufficiently many of their partial derivatives. $\overset{(0)}{g}_{ik} = \overset{(0)}{g}{}^{ik} = \mathrm{diag}\,(1, -1, -1, -1)$ is the flat background metric and

$$\Psi_{pb} := h_{pb} - \tfrac{1}{2}\,\overset{(0)}{g}_{pb}\,\overset{(0)}{g}{}^{ik}\, h_{ik} = h_{pb} - \tfrac{1}{2}\,\overset{(0)}{g}_{pb}\, h. \tag{43}$$

We have put on the flat metric corrections $\psi_{pb} = \psi_{bp}$ the gauge conditions (41) the same as in the framework of the GR.

In the weak-field approximation we also put $|Q^{i}_{.kl}| \ll 1$ and sufficiently many of its partial derivatives; indices are raised and lowered with the flat metrics $\overset{(0)}{g}{}^{ik}$ and $\overset{(0)}{g}_{ik}$.

$$\Box := \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta = \overset{(0)}{g}{}^{ik}\,\partial_i\partial_k = \partial^i\partial_i \tag{44}$$

is the d'Alembert wave operator.

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[1] Our approach to linearization is different and simpler than that presented in [7].

The LFE (38)–(41) satisfy in vacuum ($t^{ik} = S^{i}_{.kl} = 0$) 10 linearized differential identities (LDI)

$$\partial_a P^{a}_{.ij} = e_{ij} - e_{ji}, \tag{45}$$

$$\partial_a e^{a}_{.i} = 0, \tag{46}$$

where

$$p^{ail} = 2\beta\Big[\partial^l \Box h^{ai} - \partial^i \Box h^{al} - \partial_m \partial^a (Q^{lim} + Q^{iml} + Q^{mil})$$

$$+ \Box (Q^{lia} + Q^{ial} + Q^{ail}) + \frac{\alpha}{\beta}(Q^{ial} - Q^{lai} + Q^l \overset{(0)}{g}{}^{ia} - Q^i \overset{(0)}{g}{}^{la} + Q^{ail})\Big], \tag{47}$$

and

$$e^{ai} = \alpha(\Box \Psi^{ai} + 3\partial_j Q^{iaj} - 2\partial^i Q^a + \partial_j Q^{aij} - \partial_j Q^{jia} + 2 \overset{(0)}{g}{}^{ai} \partial_b Q^b). \tag{48}$$

We have obtained the LDI (45)–(46) by linearization of the exact differential identities (7)–(8). As a consequence of the LFE and LDI we get the linearized conservation laws (LCL)

$$\partial_i t^{i}_{.k} = 0, \tag{49}$$

$$t_{ik} - t_{ki} = \partial_l S^{l}_{.ik}. \tag{50}$$

Linearized torsion constraints (LTC) obtained from (26) have the form

$$t^{[pb]} = 2\alpha(\partial^{[b} Q^{p]} - \partial_k Q^{[bp]k}) + \alpha \partial_k Q^{kbp} \tag{51}$$

and they are identical with (39). Therefore, the LTC are not important in linear approximation of the theory.

Using (38) one can rewrite the equations (40) in the form of the 2-nd order equations

$$\Box (2Q^{[li]p} + Q^{pil}) - \partial_m \partial^p (2Q^{[li]m} + Q^{mil})$$

$$-2 \overset{(0)}{g}{}^{p[i} \partial^{l]} \partial_a Q^a + 4\partial_j \partial^{[i} Q^{l]pj} + 4\partial_j \partial^{[i|} Q^{p|l]j}$$

$$+2\partial^p \partial^{[l} Q^{i]} + \frac{\alpha}{\beta}(2Q^{[li]p} + 2 \overset{(0)}{g}{}^{p[i} Q^{l]} + Q^{pil})$$

$$= (-)\frac{S^{pil}}{2\beta} + \frac{1}{\alpha}(\partial^l t^{(pi)} - \partial^i t^{(pl)}) + \frac{1}{\alpha} \overset{(0)}{g}{}^{p[} \partial^{i]} t, \tag{52}$$

where

$$t = t^{i}_{.i} := \overset{(0)}{g}{}^{ik} t_{ik}.$$

Finally, we take the LFE of the MicGT in the form of the following 2-nd order system of the partial differential equations

$$\Box \Psi^{pb} = (-)\frac{1}{\alpha} t^{(pb)} - 4\partial_j Q^{(bp)j} + 2\partial^{(b}Q^{p)} - 2 \overset{(0)}{g}{}^{pb}\, \partial_a Q^a, \tag{53}$$

$$t^{[pb]} = 2\alpha(\partial^{[b}Q^{p]} - \partial_j Q^{[bp]j}) + \alpha\partial_j Q^{jbp}, \tag{54}$$

$$\begin{aligned} &\Box (2Q^{[li]p} + Q^{pil}) - \partial_m \partial^p (2Q^{[li]m} + Q^{mil}) \\ &- 2 \overset{(0)}{g}{}^{p[i}\, \partial^{l]}\partial_a Q^a + 4\partial_j \partial^{[i} Q^{l]pj} + 4\partial_j \partial^{[i|} Q^{p|l]j} + 2\partial^p \partial^{[l} Q^{i]} \\ &+ \frac{\alpha}{\beta}(2Q^{[li]p} + 2 \overset{(0)}{g}{}^{p[i}Q^{l]} + Q^{pil}) = (-)\frac{1}{2\beta} S^{pil} \\ &+ \frac{1}{\alpha}(\partial^l t^{(pi)} - \partial^i t^{(pl)} + \overset{(0)}{g}{}^{p[l}\, \partial^{i]} t), \end{aligned} \tag{55}$$

$$\partial_k \Psi^{ik} = \partial_k \Psi^{ki} = 0. \tag{56}$$

The linearized field equations (53)–(56) are equivalent to the following system if $S^{i}_{\cdot il} = 0$[2]

$$\Box \Psi^{pb} = (-)\frac{1}{\alpha} t^{(pb)} - 4\partial_j Q^{(bp)j} + 2\partial^{(b}Q^{p)} - 2 \overset{(0)}{g}{}^{pb}\, \partial_a Q^a, \tag{57}$$

$$\partial^{[b}Q^{p]} - \partial_j Q^{[bp]j} + \tfrac{1}{2}\, \partial_j Q^{jbp} = \frac{1}{2\alpha} t^{[pb]}, \tag{58}$$

$$\left(\Box - \frac{\alpha}{\beta}\right) Q^l = \frac{1}{2\alpha}\partial^l t + \frac{1}{\alpha}\partial_i t^{[il]} = \frac{1}{\alpha}\partial^{[l} t^{i]}{}_i, \tag{59}$$

$$\left(\Box + \frac{\alpha}{\beta}\right) Q^{[lip]} = (-)\frac{1}{2\beta} S^{[pil]} - \frac{1}{\alpha}\partial^{[l} t^{pi]}, \tag{60}$$

$$\begin{aligned} \left(\Box + \frac{\alpha}{\beta}\right) Q^{(pi)l} &= (-)\frac{1}{4\beta} S^{(pi)l} + \frac{1}{2\alpha} \overset{(0)}{g}{}^{l(p}\partial^{i)} t \\ &- \frac{1}{2\alpha} \overset{(0)}{g}{}^{pi}\, \partial^l t + \tfrac{1}{4}\, \partial^i \Box \Psi^{lp} + \tfrac{1}{4}\, \partial^p \Box \Psi^{li} - \tfrac{1}{2}\, \partial^l \Box \Psi^{ip} \end{aligned}$$

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[2] The condition $S^{\cdot i}_{\cdot il} = 0$ slightly simplifies the right hand side of the equations (59). If $S^{\cdot i}_{\cdot il} =: (-) S_l \neq 0$ then we have on the right hand side of the (59) the additional term $(-)\frac{1}{2\beta} S_l$.

$$+\frac{\alpha}{\beta}(\overset{(0)}{g}{}^{pi}Q^{l}-\overset{(0)}{g}{}^{l(p}Q^{i)})-\frac{\partial^{i}\partial^{p}Q^{l}}{2}-\frac{\overset{(0)}{g}{}^{ip}}{2}\partial^{l}\partial_{a}Q^{a}$$

$$+\frac{\overset{(0)}{g}{}^{l(p}}{g}\partial^{i)}\partial_{a}Q^{a}+\tfrac{1}{2}\partial^{l}\partial^{(i}Q^{p)}, \tag{61}$$

$$\partial_{i}\Psi^{ik}=\partial_{i}\Psi^{ki}=0. \tag{62}$$

We obtain the system (57)–(62) from the system (53)–(56) by means of the following procedure. We write (55) in the form

$$T^{pil}:=\Box(2Q^{[li]p}+Q^{pil})-\partial_{m}\partial^{p}(2Q^{[li]m}+Q^{mil})$$

$$-2\overset{(0)}{g}{}^{p[i}\partial^{l]}\partial_{a}Q^{a}+4\partial_{j}\partial^{[i}Q^{l]pj}+4\partial_{j}\partial^{[i|}Q^{p|l]j}$$

$$+2\partial^{p}\partial^{[l}Q^{i]}+\frac{\alpha}{\beta}(2Q^{[li]p}+2\overset{(0)}{g}{}^{p[i}Q^{l]}+Q^{pil})$$

$$+\frac{1}{2\beta}S^{pil}-\frac{1}{\alpha}(\partial^{l}t^{(pi)}-\partial^{i}t^{(pl)}+\overset{(0)}{g}{}^{p[l}\partial^{i]}t)=0 \tag{63}$$

and decompose $T^{pil}=(-)T^{pli}$ into irreducible components with respect to $L^{\uparrow}_{+}$

$$T^{pil}=\tfrac{2}{3}({}^{t}T^{pil}-{}^{t}T^{pli})+\tfrac{1}{3}(\overset{(0)}{g}{}^{pi}T^{l}-\overset{(0)}{g}{}^{pl}T^{i})+\eta^{pilm}a_{m}, \tag{64}$$

where

$$T_{l}:=T^{i}_{.il},\qquad a_{m}=\tfrac{1}{6}\eta_{mrst}T^{rst} \tag{65}$$

and

$${}^{t}T^{pil}=T^{(pi)l}+\tfrac{1}{6}(\overset{(0)}{g}{}^{pl}T^{i}+\overset{(0)}{g}{}^{il}T^{p})-\tfrac{1}{3}\overset{(0)}{g}{}^{pi}T^{l}. \tag{66}$$

We see that

$$T^{pil}=0\Leftrightarrow T^{l}=0\wedge a_{m}=0\wedge{}^{t}T^{pil}=0. \tag{67}$$

Using the equations (54) one can bring the equations $T^{l}=0$ to the form (59) and the equations $a_{m}=0$ to the form (60). Indeed, the equations ${}^{t}T^{pil}=0\equiv T^{(pi)l}=0$ if we use $T^{l}=0$, one can bring to the form (61) with the help of the equations (53) and (54). The remaining equations (57), (58) and (62) of the system (57)–(62) are the same as the equations (53), (54) and (56) of the system (53)–(56).

It is seen from the system (57)–(62) that the null hypersurfaces are characteristic surfaces of the LFE and that the linearized theory is deterministic. Moreover, one can see that the propagation law of the vectorial part Q^{l} of torsion given by (59) leads to tachyons (problems with causality) and to negative energy-density states (ghosts) (problems with stability and unitarity). Namely, let us consider the equations (59) in vacuum

$$\left(\Box - \frac{\alpha}{\beta}\right) Q^l = 0, \quad \frac{\alpha}{\beta} > 0. \tag{68}$$

It is easy to check that the particles of the field Q^l described by the plane wave

$$Q^l = \overset{(0)}{Q^l} \cos \frac{2\pi}{h} (Et - \vec{p} \cdot x)\left(\frac{Et}{h} - \frac{\vec{p} \cdot \vec{X}}{h}\right); \quad \overset{(0)}{Q^l} = \text{const},$$

satisfying (68) have imaginary masses $m = i\sqrt{\frac{hc}{G}}$ (we put $K = 1$), positive energy $E > 0$ and velocities $v > c$ i.e., they are tachyons. On the other hand, a real field Φ satisfying Klein-Gordon equation $\left(\Box - \frac{\alpha}{\beta}\right)\Phi = 0$, $\frac{\alpha}{\beta} > 0$ may have negative energy-density. It is easily seen from the $t^0_{.0}$ component of the canonical energy-momentum tensor $t^i_{.k}$ of such a field

$$t^0_{.0} = \tfrac{1}{2}\left(\Phi_{,0}\Phi_{,0} - \frac{\alpha}{\beta}\,\Phi^2\right). \tag{69}$$

Thus, the field Q^l satisfying (68) leads to negative energy density (ghosts), i.e., it generates non-stability of the solutions to the linearized field equations. If we want to have the linearized theory without tachyons and ghosts connected with Q^l, we must confine from the beginning to the restricted MicGT. The suitable restriction is given by the constraints $Q_l = Q^i_{.li} = 0 \equiv {}^{V}\Theta = \vartheta^i \wedge * \Theta_i = 0$. We will consider this restricted version of the MicGT in Section 4 we end this Section with some remarks concerning linearized MicGT.

(*i*) In the framework of the linearized MicGT one can associate with the gravitational field the following particles[3]:

— massless graviton 2^- described by $\Psi^{pb}: \partial_i \Psi^{ik} = 0$, $\Psi = 0$,
— tordion 0^+ described by $Q^i_{.0i}$ } tachyons connected with Q_l
— tordion 1^- described by $Q^i_{.\alpha i}$ } tachyons connected with Q_l
— tordion 1^+ described by $Q_{[0\alpha\beta]}$,
— tordion 2^+ described by $\bar{Q}^{(\alpha\beta)0}$,
— tordion 2^- described by ${}^{T}Q^{\alpha\beta\gamma}$,
— tordion 1^- described by $Q_{(0\beta)0} = \tfrac{1}{2}\, Q_{0\beta 0}$,
— tordion 0^- described by $\varepsilon^{\alpha\beta\gamma} Q_{\alpha\beta\gamma}$.

Tordion 0^- can be eliminated by means of the constraints

$$\partial^{[b} Q^{p]} - \partial_j Q^{[bp]j} + \tfrac{1}{2}\, \partial_j Q^{jbp} = \frac{1}{2\alpha}\, t^{[pb]}. \tag{70}$$

[3] Our definitions of tordions are different and simpler than given in [7]. We decompose the linearized field Q^i_{kl} onto irreducible components with respect to 3-dimensional rotation group and connect particle having suitable spin and parity with every such component.

The greek indices run over 1, 2, 3 and

$$\bar{Q}^{(\alpha\beta)0} := Q^{(\alpha\beta)0} - \frac{\delta^{\alpha\beta}}{3} Q^{\gamma\ 0}_{.\gamma.},$$

$$^{T}Q^{\alpha\beta\gamma} := Q^{(\alpha\beta)\gamma} - \tfrac{1}{2}\eta^{\alpha\beta}Q^{\delta\ \gamma}_{.\delta} + \tfrac{1}{2} Q^{\delta\ (\alpha}_{.\delta}\eta^{\beta)\gamma},$$

$$\eta^{\alpha\beta} = \eta_{\alpha\beta} = \mathrm{diag}(-1, -1, -1). \tag{71}$$

All the above particles, except tachyons 0^+ and 1^- are normal in the sense that they have positive masses and the energy-densities of the "generating" fields Ψ^{ik}, $Q^{[ikl]}$ and $Q^{(ik)l}$ are positive-definite (see Section 4). The masses of the normal particles and the modules of masses of the tachyons 0^+ and 1^- are very large if $K = 1$ and equal to the so-called Planck's mass $m_P = \sqrt{\frac{hc}{G}} \approx 10^{-5}$ g. Therefore, these particles practically do not propagate.

(*ii*) Forces connected with the normal tordions are attractive (short-range, having Yukawa's potentials) and the forces connected with tachyons are repulsive for $r > \sqrt{\frac{\beta}{\alpha}}$ $\left(\sqrt{\frac{\beta}{\alpha}} = l_P \text{ if } K = 1\right)$ and exponentially growing. So, the forces connected with tachyons lead to nonstability of matter.

(*iii*) If torsion vanishes, then the LFE (53)-(56) reduce to the form

$$\Box\, \Psi_{pb} = (-)\frac{1}{\alpha}\, t_{pb}, \qquad t_{pb} = t_{bp}, \tag{72}$$

$$\Psi^{ik}{}_{,k} = \Psi^{ki}{}_{,k} = 0, \tag{73}$$

$$S^{pil} = 2A(2\partial^{[l}t^{i]p} + \overset{(0)}{g}{}^{p[l}\,\partial^{i]}t). \tag{74}$$

In the above system the equations (72) form dynamical system; (73) are constraints and (74) only define spin $S^{pil} = (-)S^{pli}$ which is microscopic and conserved.

In the macroscopic domain ($h = 0$) the system (72)-(74) takes the form

$$\Box\, \Psi_{pb} = (-)\frac{1}{\alpha}\, t_{pb}, \qquad t_{pb} = t_{bp}, \tag{75}$$

$$\Psi^{ik}{}_{,k} = \Psi^{ki}{}_{,k} = 0; \qquad S^{p}_{.il} = 0, \tag{76}$$

identical with the linearized form of GR (see, e.g., [9]). In the so-called "Newtonian limit" (see, e.g., [6]), the equations (75)-(76) take the form of the Poisson equation

$$\Delta\varphi = 4\pi G\varrho \tag{77}$$

with the variable density $\varrho(x, y, z)$.

(*iv*) The so-called "superpotentials" in the framework of the linearized MicGT are the same as in the exact MicGT, i.e., they have the form $(-)2\alpha d * \Theta_i$ and $(-)4\beta d * \Omega^i_{.k}$. In consequence, the global quantities of an isolated, finite material system, especially global energy, vanish owing to the fact that the linearized MicGT (likely as the exact MicGT) admits asymptotically Newtonian solutions (see [14]) having vanishing torsion only.

4. *The restricted model of the MicGT with constraints $Q_l = 0$ and conclusions*

Let us consider the restricted version of the gauge gravitational theory with the gravitational Lagrangian (1). (We will denote this version by RMicGT.) The restriction will be given by the algebraic constraints $Q_l = 0 \equiv \kappa_l = 0$ where $\kappa_l := \kappa^i_{.li}$ and $\kappa^i_{.kl} = \frac{1}{2}(Q^i_{.kl} + Q_{lk}{}^i + Q_{kl}{}^i)$ are defect components. One can introduce the constraints $Q_l = 0 \equiv \kappa_l = 0$ in usual way by adding to the Lagrangian (1) the term $L_{add} = \lambda \wedge {}^V\Theta$ where 1-form $\lambda = \lambda_i \vartheta^i$ is the Lagrange's multiplier and 3-form ${}^V\Theta = \vartheta^i \wedge * \Theta_i = Q^i_{.ji}\eta^j = Q_j \eta^j$. Then we obtain the theory having the following gravitational Lagrangian

$$L_g = \alpha(\Omega^i_{.k} \wedge \eta_i{}^k + \Theta^i \wedge * \Theta_i) + \beta \Omega^i_{.k} \wedge * \Omega^k_{.i} + \lambda \wedge {}^V\Theta. \tag{78}$$

Varying $L_g(\vartheta^i, \omega^i_{.k} \lambda_i)$ we treat ϑ^i, $\omega^i_{.k}$ and λ_i as independent variables. We obtain the following field equations

$$D * \Omega^j_{.i} = (-)\frac{\alpha}{2\beta}(\vartheta^j \wedge * \Theta_i - \vartheta_i \wedge * \Theta^j) - \frac{\alpha}{2\beta}\Theta_k \wedge \eta_i{}^{jk}$$

$$-\frac{S_i{}^j}{4\beta} - \frac{1}{4\beta}[*(\lambda \wedge \vartheta_i) \wedge \vartheta^j - *(\lambda \wedge \vartheta^j) \wedge \vartheta_i], \tag{79}$$

$$D * \Theta_l = (-)\frac{\Omega^{jk}}{2} \wedge \eta_{ljk} + \left(Q^b_{.lr} Q_b^{;pr} - \frac{\delta^p_l}{4} Q^{btr} Q_{btr}\right)\eta_p$$

$$+\frac{\beta}{\alpha}\left(\frac{\delta^p_l}{4} R^{ijrm} R_{ijrm} - R^{ij}{}_{lt} R_{ij}{}^{pt}\right)\eta_p$$

$$-\frac{1}{2\alpha}[\lambda^k Q_{jlk}\eta^j + D * (\lambda \wedge \vartheta_l)] - \frac{t_l}{2\alpha}, \tag{80}$$

$${}^V\Theta = 0 \equiv Q_l = 0, \tag{81}$$

or, in tensor notation (modulo the terms containing Q_l)

$$\nabla_m R_{li}{}^{pm} + \tfrac{1}{2} R_{li}{}^{tn} Q^p_{.tn} = (-)\frac{\alpha}{2\beta}(Q_{i.l}^{\,p} - Q_{l.i}^{\,p} + Q^p_{.il}) - \frac{\bar{S}^p_{.l}}{4\beta}, \tag{82}$$

$$\nabla_k Q_{bp}{}^k + \tfrac{1}{2} Q_b^{;lk} Q_{plk} = G_{pb}$$

$$-\left(\frac{g_{bp}}{4}Q^{itr}Q_{itr}-Q^{i}_{.br}Q_{ip}{}^{r}\right)-\frac{\beta}{\alpha}\left(R^{ij}{}_{bt}R_{ijp}{}^{t}-\frac{g_{bp}}{4}R^{ijrt}R_{ijrt}\right)-\frac{\bar{t}_{pb}}{2\alpha}, \tag{83}$$

$$Q_l = 0, \tag{84}$$

where

$$\bar{S}^{p}_{.il} := S^{p}_{.il}+\lambda_i\delta^p_l-\lambda_l\delta^p_i$$

and

$$\bar{t}_{pb} := t_{pb}+\lambda_{p,b}-\lambda^{k}{}_{,k}g_{pb}.$$

We see that the field equations of the RMicGT originate from the field equations of the MicGT by: 1) omitting all the terms containing Q_l; 2) changing $S^{p}_{.il}\to\bar{S}^{p}_{.il}$, $t_{pb}\to\bar{t}_{pb}$; 3) adding the constraints $Q_l = 0$.

(82)–(84) form the definite system of 44 equations on 44 intrinsic unknown functions: 6 intrinsic metric components, 24 intrinsic torsion components, 4 Lagrange's multipliers λ_i and 10 functions describing microscopic matter. Physically, we can interprete the Lagrange multipliers as some kind of the new field describing dynamical properties of vacuum (Higg's field)[4]. The energy-momentum pseudotensor of the new field λ^i, as it is easily seen from the (80) or (83), has the form

$$^{\lambda}t_{ik} = \lambda_{i,k}-\lambda^{k}{}_{,k}g_{ik} \tag{85}$$

and the spintensor

$$^{\lambda}S^{p}_{.\,il} = \lambda_i\delta^p_l-\lambda_l\delta^p_i. \tag{86}$$

From the (82)–(84) we get that the linearized equations of the RMicGT have the form

$$\Box\,\Psi^{pb} = (-)\frac{1}{\alpha}\,\bar{t}^{(pb)}-4\partial_jQ^{(bp)j}, \tag{87}$$

$$\partial_jQ^{jbp}-2\partial_jQ^{[bp]j} = \frac{\bar{t}^{[pb]}}{\alpha}, \tag{88}$$

$$\left(\Box+\frac{3\alpha}{\beta}\right)\lambda^l+\frac{1}{\alpha}\,\partial^l\lambda^k{}_{,k} = \frac{1}{2\alpha}\,\partial^lt+\frac{1}{\alpha}\,\partial_it^{[il]}, \tag{89}$$

$$\left(\Box+\frac{\alpha}{\beta}\right)Q^{[lip]} = (-)\frac{1}{2\beta}\,S^{[pil]}-\frac{1}{\alpha}\,\partial^{[l}t^{pi]}, \tag{90}$$

$$\left(\Box+\frac{\alpha}{\beta}\right)Q^{(pi)l} = (-)\frac{1}{4\beta}\,\bar{S}^{(pi)l}+\frac{1}{2\alpha}\,\overset{(0)}{g}{}^{l(p}\partial^{i)}\bar{t}$$

[4] In analogy as the cosmological constant Λ is interpreted in GR.

$$-\frac{1}{2\alpha}\overset{(0)}{g}{}^{pi}\partial^{l}t+\tfrac{1}{2}\partial^{(i}\Box\Psi^{p)l}-\tfrac{1}{2}\partial^{l}\Box\Psi^{pi}, \tag{91}$$

$$\partial_i\Psi^{ik}=\partial_i\Psi^{ki}=0,\quad Q_i=0 \tag{92}$$

with the following constraints on material tensors $t^{i}_{\ k}$ ans $S^{i}_{.ik}$ (LCL)

$$\partial_i t^{i}_{.k}=0, \tag{93}$$

$$t_{ik}-t_{ki}=\partial_l S^{l}_{.ik}. \tag{94}$$

The constraints $Q_l=0$ eliminate tachyons and negative energy densities[5] connected with vectorial torsion Q_l and the remaining fields Ψ^{ik}, $Q^{[ikl]}$ and $Q^{(ik)l}$ have positive--definite energy-densities and the particles connected with them have positive (or null) masses. The positive-definitness of the energy density of the fields $Q^{[ikl]}$ and $Q^{(ik)}$ is seen from the following considerations.

Let us consider the energy-momentum complex ${}_gt_l$ of the gravitational field in the framework of the RMicGT. We have from the equations (80)[6] with constraints $Q_l=0$.

$$\begin{aligned}{}_gt_l={}&\alpha(\omega^{j}_{.k}\wedge\Theta^{i}\eta_{lj.i}^{\ \ \ k}+\omega^{j}_{.k}\wedge\omega^{p}_{.l}\wedge\eta_{pj}^{\ \ k}\\&+\omega^{j}_{.k}\wedge\omega^{p}_{.j}\wedge\eta_{lp}^{\ \ k})-2\alpha\omega^{k}_{.l}\wedge*\Theta_k\\&+2\alpha\left(\frac{\delta^p_l}{4}Q^{btr}Q_{btr}-Q^{b}_{.lr}Q_b^{\cdot pr}\right)\eta_p\\&+2\beta\left(R^{ij}_{\ \ lt}R_{ij}^{\ \ pt}-\frac{\delta^p_l}{4}R^{ijrm}R_{ijrm}\right)\eta_p,\end{aligned} \tag{95}$$

or, in terms of components

$$\begin{aligned}{}_gt^{p}_{.l}={}&\alpha[(\Gamma^{jk}{}_{i}Q^{i}_{.jk}+\Gamma^{jk}{}_{i}\Gamma^{i}{}_{jk}-\Gamma^{ji}{}_{i}\Gamma^{m}{}_{jm})\delta^p_l\\&-\Gamma^{jk}{}_{l}\Gamma^{p}{}_{.jk}+\Gamma^{jp}{}_{l}\Gamma^{m}{}_{jm}+\Gamma^{ji}{}_{i}\Gamma^{p}{}_{.jl}-\Gamma^{jp}{}_{i}\Gamma^{i}{}_{jl}\\&+\Gamma^{ik}{}_{i}\Gamma^{p}{}_{.lk}-\Gamma^{pk}{}_{i}\Gamma^{i}{}_{lk}+\Gamma^{jp}{}_{i}\Gamma^{i}{}_{.lj}-\Gamma^{ji}{}_{i}\Gamma^{p}{}_{.lj}\\&+\Gamma^{pi}{}_{i}\Gamma^{m}{}_{lm}-\Gamma^{ip}{}_{i}\Gamma^{m}{}_{lm}+\Gamma^{ji}{}_{l}Q^{p}_{.ij}+\Gamma^{pj}{}_{i}Q^{i}_{.jl}\\&-\Gamma^{ji}{}_{j}Q^{p}_{.il}-\Gamma^{jp}{}_{i}Q^{i}_{.jl}+\Gamma^{ji}{}_{i}Q^{p}_{.jl}]-2\alpha\Gamma^{k}_{.la}Q_k^{\cdot pa}\\&+2\alpha\left(\frac{\delta^p_l}{4}Q^{btr}Q_{btr}-Q^{b}_{.lr}Q_b^{\cdot pr}\right)\\&+2\beta\left(R^{ij}_{\ \ lt}R_{ij}^{\ \ pt}-\frac{\delta^p_l}{4}R^{ijrm}R_{ijrm}\right),\end{aligned} \tag{96}$$

$$Q_l=0, \tag{97}$$

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[5] Note that our method of elimination of tachyons and ghosts is different and simpler than the methods presented in [7, 10, 11].

[6] ${}_gt_l$ given by (95) is based on the superpotentials (36). The constraints $Q_l=0$ do not matter for the energy densities of the fields $Q^{[ikl]}$ and $Q^{(ik)l}$.

where

$$\Gamma^{i}_{\ kl} = \begin{Bmatrix} i \\ kl \end{Bmatrix} - \kappa^{i}_{.kl} = \begin{Bmatrix} i \\ kl \end{Bmatrix} - \tfrac{1}{2}(Q^{i}_{.kl} + Q_{lk}{}^{i} + Q_{kl}{}^{i}) \tag{98}$$

and

$$R^{i}_{.jkl} = \partial_k \Gamma^{i}_{.jl} - \partial_l \Gamma^{i}_{\ jk} + \Gamma^{i}_{\ rk}\Gamma^{r}_{\ jl} - \Gamma^{i}_{.rl}\Gamma^{r}_{\ jk}. \tag{99}$$

From the expressions (96)–(99) we get the pure torsion part of the component ${}_{g}t_{00}$ of the linearized energy-momentum complex in the form

$$\begin{aligned} {}_{g}t_{00} &= \alpha(\tfrac{3}{4}\, Q^{ijk}Q_{ijk} + \tfrac{1}{4}\, Q^{kij}Q_{ikj} + \tfrac{1}{4}\, Q^{kij}Q_{jik} \\ &+ \tfrac{1}{2}\, Q^{ij0}Q_{ij0} + \tfrac{1}{2}\, Q^{ij0}Q_{0ij} + \tfrac{1}{2}\, Q^{ij0}Q_{ji0} - \tfrac{1}{2}\, Q^{0ij}Q_{0ij}) \\ &+ 2\beta[R^{ijk0}(Q)R_{ijk0}(Q) - \tfrac{1}{4}\, R^{ijkl}(Q)R_{ijkl}(Q)], \end{aligned} \tag{100}$$

$$Q_l = 0.$$

$R^{i}_{.jkl}(Q)$ denotes here linearized, pure trace-free torsion part of the curvature tensor $R^{i}_{.jkl}(\{\} - \kappa)$. From (100) we have for the axial torsion ${}^{\mathrm{A}}Q^{ikl} = Q^{[ikl]}$

$$\begin{aligned} {}^{\mathrm{A}}_{g}t_{00} &= \alpha(\tfrac{1}{4}{}^{\mathrm{A}}Q^{ijk\,\mathrm{A}}Q_{ijk} - {}^{\mathrm{A}}Q^{ij0\,\mathrm{A}}Q_{ij0}) \\ &+ 2\beta[R^{ijk0}({}^{\mathrm{A}}Q)R_{ijk0}({}^{\mathrm{A}}Q) - \tfrac{1}{4}\, R^{ijkl}({}^{\mathrm{A}}Q)R_{ijkl}({}^{\mathrm{A}}Q)] > 0, \end{aligned} \tag{101}$$

because the linearized tensors $\tfrac{1}{4}\, Q^{ijk}Q_{ijk} - Q^{ij0}Q_{ij0}$ and $R^{ijk0}R_{ijk0} - \tfrac{1}{4}\, R^{ijkl}R_{ijkl}$ are positive-definite. Concerning of the linearized field $Q^{(ik)l} =: {}^{\mathrm{s}}Q$ we get from (100)

$$\begin{aligned} {}^{\mathrm{s}}_{g}t_{00} &= \alpha(\tfrac{1}{4}\, Q^{(ij)k}Q_{(ij)k} - Q^{(ij)0}Q_{(ij)0} + \tfrac{5}{4}\, Q^{(ij)k}Q_{(ij)k} \\ &+ 2Q^{(\alpha\beta)k}Q_{(\alpha\beta)k}) + 2\beta[R^{ijk0}({}^{\mathrm{s}}Q)R_{ijk0}({}^{\mathrm{s}}Q) \\ &- \tfrac{1}{4}\, R^{ijkl}({}^{\mathrm{s}}Q)R_{ijkl}({}^{\mathrm{s}}Q)] = \frac{\alpha}{4}(\tfrac{1}{2}\, Q^{(ij)0}Q_{(ij)0} + \tfrac{2}{3}\, Q^{(ij)\alpha}Q_{(ij)\alpha} \\ &+ 2Q^{(\alpha\beta)k}Q_{(\alpha\beta)k}) + 2\beta[R^{ijk0}({}^{\mathrm{s}}Q)R_{ijk0}({}^{\mathrm{s}}Q) - \tfrac{1}{4}\, R^{ijkl}({}^{\mathrm{s}}Q)R_{ijkl}({}^{\mathrm{s}}Q)] > 0 \end{aligned} \tag{102}$$

as the all appearing linearized tensors are positive-definite.

The linearized energy-momentum complex for the massless graviton field $\Psi^{ik} : \Psi = 0$ obtained from (96) if $Q^{i}_{.kl} = 0$ is, by virtue of the linearized field equations with $Q^{i}_{.kl} = 0$ and linearized Bach-Lanczos identity [7], the same as in the framework of linearized GR and it leads to the positive-definite energy density of the field.

However, the energy-density of the new introduced field λ^i (instead of Q_l) is equal to

$$ {}^{\lambda}t_{00} = \lambda_{0,0} - \lambda^{k}_{,k} g_{00} \tag{103}$$

and is not positive-definite.

Thus, we see that the linearized RMicGT is not better than the linearized MicGT: it only moves the bad, local properties of the field Q_l on the "Higg's" field Lagrange's multipliers λ_i.

From the point of view of the exact MicGT any limitation of the Einstein-Cartan geometry, e.g., by the covariant constraint ${}^{V}\Theta = \vartheta^i \wedge * \Theta_i = 0$ is artificial and unfounded because:

1. The exact theory is causal.
2. The exact theory admits only non-negative global energy for an isolated, finite material system, i.e., an isolated system is stable.
3. In the framework of the exact MicGT there exist interesting cosmological solutions having vectorial torsion [1].
4. The constraints ${}^{V}\Theta = 0$ complicate formal structure of the theory:

Moreover:

1. Also in the linearized MicGT the global energy of an isolated, finite material system is non-negative in spite of appearing negative energy-densities.
2. If $K = 1$, then all the tordions are too massive for propagation and problems with cousality vanish.

Summing up the constraints ${}^{V}\Theta = 0$ are not necessary and we think that the MicGT is very good microscopic gauge theory of gravitation (the best model in our opinion).

REFERENCES

[1] J. Garecki, *Acta Phys. Pol.* **B13**, 397 (1982); **B14**, 713 (1983); **B16**, 699 (1985); **B18**, 147 (1987); *Class. Quantum Grav.* **2**, 403 (1985); an article in *On Relativity Theory*, World Scientific, Singapore 1985, p. 232; *Gauge Theory of Gravitation with Gravitational Lagrangian* $L_g = \alpha(\Omega \wedge \eta + \Theta \wedge * \Theta) + \beta\Omega \wedge * \Omega$, Zeszyty Naukowe Politechniki Szczecińskiej. Prace Instytutu Matematyki, Szczecin 1988; *Gauge Theory of Gravitation with Gravitational Lagrangian* $L_g = \alpha(\Omega \wedge \eta + \Theta \wedge * \Theta) + \beta\Omega \wedge * \Omega$. *The Revised Model with Constraints* $Q_l = 0$, University of Szczecin, preprint KFiz(4)88, Szczecin 1988. Some fragments of the preprint are submitted to GRG Journal and will be published in 1990.

[2] A. Trautman, *Symposia Mathematica* **12**, 138 (1973).

[3] W. Thirring, *Acta Phys. Austriaca Suppl.* XXX, 439 (1978); *Use of Differential Forms in General Relativity*, Vienna preprint, Vienna 1978.

[4] W. Adamowicz, Doctoral Dissertation, Warsaw 1980 (in Polish).

[5] W. Kopczyński, *J. Phys.* **A15**, 493 (1982); preprint IFT/8/87, Warsaw 1987.

[6] L. D. Landau, E. M. Lifschitz, *Classical Fields Theory*, PWN, Warsaw 1977 (in Polish).

[7] K. Hayashi, T. Shirafuji, *Prog. Theor. Phys.* **64**, 866 (1980); **64**, 883 (1980); **64**, 1435 (1980); **64**, 2222 (1980); **65**, 525 (1981); **66**, 318 (1981); **66**, 2258 (1981); **73**, 54 (1985); **74**, 852 (1985).

[8] W. Kopczyński, *On the Linearized EC Theory*, preprint ITF, Warsaw 1973.

[9] F. A. E. Pirani, an article in *Lectures on General Relativity*, Prentice-Hall, Inc. Englewood Cliffs, New Jersey 1965.

[10] E. Sezgin, P. van Nieuwenhuizen, *Phys. Rev.* **D21**, 3269 (1980).

[11] R. Kuhfuss, J. Nitsch, *Propagating Modes in Gauge Field Theories of Gravity*, preprint MPA 223, 1986.

[12] M. Blagojevič, I. A. Nikolič, 10th International Conference on GRG, Contributed Papers, Vol. 1, p. 473 and 476, Consiglio Nazionale delle Ricerche-Roma 1983; *Phys. Rev.* **D28**, 2455 (1983).

[13] I. A. Nikolič, *Phys. Rev.* **D30**, 2508 (1984).

[14] Hui-Hua Chen et al., *Prog. Theor. Phys.* **79**, 77 (1988).